Explanation of Muon g-2 discrepancy using the Dirac Equation and Einstein Field Equations in Geometric Algebra

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Abstract

In this paper, we will use Geometric Algebra to combine the Einstein Field Equations with the Dirac Equation, arriving to the following equation:

$$\nabla\boldsymbol{\psi}+\psi\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{||\hat{t}||}\frac{\boldsymbol{g}}{||g||}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{||\hat{t}||}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right)-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \quad (1)$$

Where ψ is the wavefunction o the particle and $\mathbf{g}^{1/2}$ is the collection of the basis vectors which square is the collection of the product of the basis vectors by themselves (the metric **g** in Geometric Algebra representation). And the **t** represents the trivector in Geometric Algebra $Cl_{3,0.}$

The original aim of the paper was to demonstrate that the muon g-2 discrepancy could be explained by gravitational effects created by its own muon mass (not even necessary to consider Earth's or Sun's gravity.

The difference of muon g-2 between measured value and the theoretical value is:

$$Difference_{a_\mu}=a_{\mu_{measured}}-a_{\mu_{theor}}=2.89E-9 \quad (9.4)$$

Solving equation (1) is very complicated and depends on the boundary conditions. But the set of the possible solutions will depend only in its coefficients and the boundary conditions.

Applying only the coefficients of equation (1) and as a possible boundary condition the classical radius of the muon, we can get solutions as:

$$Sol_1 = \sqrt{\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{1^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{r_{muon}}} = 2.78E-9 \qquad (9.7)$$

That has less than 4% error compared to the expected value.

$$\frac{Difference_{a_\mu} - Sol_1}{Difference_{a_\mu}} = 3.95\%\ Error \qquad (9.8)$$

There are other possibilities, as:

$$Sol_2 = 4\pi\sqrt{2\sqrt{\frac{2\pi G\hbar}{c^3}}\frac{m_{muon}c}{\hbar}} = 2.617E-9 \qquad (9.9)$$

With a higher error:

$$\frac{Difference_{a_\mu} - Sol_2}{Difference_{a_\mu}} = 9.44\%\ Error \qquad (9.10)$$

None of these solutions gives the exact value, but it has been demonstrated that yes, the muon g-2 discrepancy could be due to gravitational effects (they are in the order of the difference between the measured value and the theoretical value). It is important to remark that the gravitational effects in this regard are due the mass of the muon itself, it is not necessary to include higher gravitational potentials as the Earth or the Sun gravity. Just the muon mass is capable of creating this discrepancy.

Also, we have seen how using Geometric algebra it is possible to combine gravitational equations (generally used in tensor formalism) with the Dirac Equation (normally used in Matrix formalism).

Keywords

Geometric Algebra, Dirac Equation, Einstein Field Equations, Gravitation, Muon g-2 discrepancy

1. Introduction

In this paper, we will use Geometric Algebra to combine the Einstein Field Equations with the Dirac Equation. Also, we will demonstrate how the muon g-2 discrepancy could be explained mainly by gravitational effects related to its own muon mass.

I will not make any introduction regarding Geometric Algebra. To be able to follow the mathematic framework in this paper, I recommend you read the chapters 2 to 8 of [5] or [6] before continuing. It is not strictly necessary to do it, but you will miss some information as how the bivector or trivector can be considered imaginary units in Geometric Algebra and alike.

If you do not know what I am talking about, I strongly recommend you check the masterpiece [1] and the best collection of Geometric Algebra knowledge [3].

2. Partial derivative of a basis vector in Geometric Algebra

In Euclidean metric, the partial derivative of a basis vector is zero. This is because the vector is constant. In non-Euclidean metric, the basis vector changes orientation and even norm, so this is not fulfilled any more. See [2] for more info.

As we made in [2], we can take the partial derivative of a basis vector using the following property. In Geometric Algebra the square of any vector is always a scalar that corresponds to the square of its norm. This means, we can operate this scalar as any number or scalar function (not needing to care about vectors).

Also, in the standard tensor formalism for non-Euclidean metrics, the product of a basis vector by itself is a component of the metric tensor [7] that is called g_{ii}. We will use also this definition to try to merge both worlds as possible.

So, to take the partial derivative of the square of the basis vector $\hat{x}$, we can do it the two following forms:

$$\frac{\partial}{\partial\lambda}(\hat{x}\hat{x}) = \frac{\partial\|\hat{x}\|^2}{\partial\lambda} = \frac{\partial(g_{xx})}{\partial\lambda} = \frac{\partial(g_{xx})}{\partial\lambda} \qquad (2.1)$$

And also using the derivative chain rule, this way:

$$\frac{\partial}{\partial\lambda}(\hat{x}\hat{x}) = \frac{\partial\hat{x}}{\partial\lambda}\hat{x} + \hat{x}\frac{\partial\hat{x}}{\partial\lambda} \quad (2.2)$$

So, joining equations (2.1) and (2.2):

$$\frac{\partial(g_{xx})}{\partial\lambda} = \frac{\partial\hat{x}}{\partial\lambda}\hat{x} + \hat{x}\frac{\partial\hat{x}}{\partial\lambda} \quad (2.3)$$

Now, we can calculate also:

$$\frac{\partial}{\partial\lambda}(\hat{x}\hat{x}^{-1}) = \frac{\partial}{\partial\lambda}(1) = 0 \quad (2.4)$$

And we get the result zero. But we can calculate the derivative another way:

$$\frac{\partial}{\partial\lambda}(\hat{x}\hat{x}^{-1}) = \frac{\partial\hat{x}}{\partial\lambda}\hat{x}^{-1} - \hat{x}\frac{1}{\|\hat{x}\|^2}\frac{\partial\hat{x}}{\partial\lambda} = \frac{\partial\hat{x}}{\partial\lambda}\frac{\hat{x}}{\|\hat{x}\|^2} - \hat{x}\frac{1}{\|\hat{x}\|^2}\frac{\partial\hat{x}}{\partial\lambda} = \frac{1}{\|\hat{x}\|^2}\left(\frac{\partial\hat{x}}{\partial\lambda}\hat{x} - \hat{x}\frac{\partial\hat{x}}{\partial\lambda}\right)$$
$$= \frac{1}{g_{xx}}\left(\frac{\partial\hat{x}}{\partial\lambda}\hat{x} - \hat{x}\frac{\partial\hat{x}}{\partial\lambda}\right) \qquad (2.5)$$

So, if above expression has to be zero, and we consider that g_{xx} should be different to zero, we have:

$$\frac{1}{g_{xx}}\left(\frac{\partial\hat{x}}{\partial\lambda}\hat{x} - \hat{x}\frac{\partial\hat{x}}{\partial\lambda}\right) = 0 \qquad (2.6)$$
$$\frac{\partial\hat{x}}{\partial\lambda}\hat{x} - \hat{x}\frac{\partial\hat{x}}{\partial\lambda} = 0 \qquad (2.7)$$
$$\frac{\partial\hat{x}}{\partial\lambda}\hat{x} = \hat{x}\frac{\partial\hat{x}}{\partial\lambda} \qquad (2.8)$$

Putting this into (2.3) we have:

$$\frac{\partial(g_{xx})}{\partial\lambda} = \frac{\partial\hat{x}}{\partial\lambda}\hat{x} + \hat{x}\frac{\partial\hat{x}}{\partial\lambda} = 2\hat{x}\frac{\partial\hat{x}}{\partial\lambda} \quad (2.9)$$

Multiplying both sides by $\frac{1}{2}\hat{x}^{-1}$ we have:

$$\frac{1}{2}\hat{x}^{-1}\frac{\partial(g_{xx})}{\partial\lambda} = \frac{1}{2}\hat{x}^{-1}2\hat{x}\frac{\partial\hat{x}}{\partial\lambda} = \frac{1}{2}2\hat{x}^{-1}\hat{x}\frac{\partial\hat{x}}{\partial\lambda} = \frac{\partial\hat{x}}{\partial\lambda} \quad (2.10)$$

Where, above, we have used the property that scalars are commutative and are free to move inside the products, and also that a vector by its inverse is equal to the number 1.

Reversing the sides and continuing operating:

$$\frac{\partial\hat{x}}{\partial\lambda} = \frac{1}{2}\hat{x}^{-1}\frac{\partial(g_{xx})}{\partial\lambda} = \frac{1}{2}\frac{\hat{x}}{\|\hat{x}\|^2}\frac{\partial(g_{xx})}{\partial\lambda} = \frac{1}{2}\frac{\hat{x}}{g_{xx}}\frac{\partial(g_{xx})}{\partial\lambda} \quad (2.11)$$

Here, we have considered that:

$$\hat{x}^{-1} = \frac{\hat{x}}{\|\hat{x}\|^2} = \frac{\hat{x}}{g_{xx}} \quad (2.12)$$

It is not clear if this holds as a general case, but yes it holds for diagonal metrics (this means basis which basis vectors are orthogonal -although not necessarily orthonormal-). So, any time you see a relation like that applied, take into account that could only be valid for orthogonal metrics.

3. Covariant derivative of a vector in Geometric Algebra

The definition of covariant derivative [9] can be summarized as applying the operator del/nabla ∇ [10] not only to the coefficients of a vector but also to the basis vectors which accompany them.

As an example, if we have a vector **A** in two dimensions:

$$\boldsymbol{A} = a\hat{x} + b\hat{y} \quad (3.1)$$

And we want to apply the following del/nabla ∇ operator both to coefficients and to its basis vectors.

$$\nabla = \frac{\partial}{\partial x}\hat{x}^{-1} + \frac{\partial}{\partial y}\hat{y}^{-1} = \frac{\partial}{\partial x}\frac{\hat{x}}{\|\hat{x}\|^2} + \frac{\partial}{\partial y}\frac{\hat{y}}{\|\hat{y}\|^2} = \frac{\partial}{\partial x}\frac{\hat{x}}{g_{xx}} + \frac{\partial}{\partial y}\frac{\hat{y}}{g_{yy}} \quad (3.2)$$

Again, we have used equation (2.12) that could be only valid for diagonal -orthogonal metric- and not for a general case.

One thing to remark is that the units of the nabla operator are length^{-1}. The reason for this is that we are deriving (dividing) by a unit of space -length-. That is the reason why the basis vectors have to be also to the power of -1. Because the length they are referring to is dividing. Check [2] for more details.

In Geometric Algebra the inverse of a basis vector is something similar or equivalent to what it is called a one-form [11] in standard algebra/tensor formalism.

Equation (3.2) is a reduced version of the del/nabla operator for non-Euclidean metric in two dimensions in Geometric Algebra (See [4][5][6] for a general definition of del/nabla operator), but we will use it as example to simplify.

The covariant derivative [9] would be:

$$D(\boldsymbol{A}) = D(a\hat{x} + b\hat{y}) \quad (3.3)$$

Now, we will substitute the D by the nabla operator but knowing that in this case, it has to apply not only to the coefficients but also to the basis vectors:

$$\begin{aligned} D(\boldsymbol{A}) = D(a\hat{x} + b\hat{y}) = \nabla(a\hat{x} + b\hat{y}) = \nabla(a\hat{x}) + \nabla(b\hat{y}) = \\ = \nabla(a)\hat{x} + a\nabla(\hat{x}) + \nabla(b)\hat{y} + b\nabla(\hat{y}) = \\ = \nabla(a)\hat{x} + \nabla(b)\hat{y} + a\nabla(\hat{x}) + b\nabla(\hat{y}) \quad (3.4) \end{aligned}$$

If we establish a nomenclature of an accent in the components that the nabla applies to, another way of writing the previous expression would be:

$$\begin{aligned} D(\boldsymbol{A}) = \nabla(a)\hat{x} + \nabla(b)\hat{y} + a\nabla(\hat{x}) + b\nabla(\hat{y}) = \acute{\nabla}\acute{a}\hat{x} + \acute{\nabla}\acute{b}\hat{y} + \acute{\nabla}a\acute{\hat{x}} + \acute{\nabla}b\acute{\hat{y}} \\ = \acute{\nabla}\left(\acute{a}\hat{x} + \acute{b}\hat{y}\right) + \acute{\nabla}\left(a\acute{\hat{x}} + b\acute{\hat{y}}\right) \quad (3.5) \end{aligned}$$

We can make the following nomenclature to separate conceptually the coefficients of the vector **A** with the basis vectors. This separation is just conceptual as it cannot be made explicitly using expressions as in equation (3.5).

$$\boldsymbol{A} = A\boldsymbol{g}^{\frac{1}{2}} \quad (3.6)$$

Where A represents the scalar coefficients of the vector and the factor $\boldsymbol{g}^{\frac{1}{2}}$ represents the basis vectors part of the vector. As commented, this separation, at this stage, cannot be done explicitly as something similar done in equation (3.5) separating $\hat{x}$ and $\hat{y}$ from the coefficients. It is just a conceptual representation.

The reason of using this strange ½ exponent, we will see later. In tensor formalism the tensor metric g is a kind of collection of the scalar products of the different basis vectors. This exponent of ½ means that this $\boldsymbol{g}^{\frac{1}{2}}$ represents one of the components of the factorization of this metric tensor g (which components are scalar) in two vectors (whose geometric product gives an element similar to the g metric tensor, but in geometric algebra formalism). See Annex A1 for more information regarding $\boldsymbol{g}^{\frac{1}{2}}$ in Geometric Algebra.

So, considering this conceptual factorization for vector A we can now represent the covariant derivative much more easily:

$$D(\boldsymbol{A}) = (DA)\boldsymbol{g}^{\frac{1}{2}} + AD\left(\boldsymbol{g}^{\frac{1}{2}}\right) = (\nabla A)\boldsymbol{g}^{\frac{1}{2}} + A\nabla\,\boldsymbol{g}^{\frac{1}{2}} \qquad (3.7)$$

That is another way of representing the equation (3.5).

In this chapter we have talked about nabla ∇ in two dimensions. If we go to the nabla in 4 dimensions we would have:

$$\nabla = \frac{\partial}{\partial x}\hat{x}^{-1} + \frac{\partial}{\partial y}\hat{y}^{-1} + \frac{\partial}{\partial z}\hat{z}^{-1} - \frac{1}{c}\frac{\partial}{\partial t}\hat{t}^{-1} \quad (3.8)$$

Normally, this is represented as □. But in this paper, we will use the nomenclature nabla ∇ independently of the number of dimensions. If you want to check a complete definition of nabla in Geometric Algebra and non-Euclidean metric, you can check [26].

We can see that dimension units in all the elements are length^{-1}, for the same reasons we commented in (3.2).

4. Einstein field equations in Geometric Algebra (I)

In tensor formalism the Einstein field equation [12] is:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad (4.1)$$

Putting the Ricci scalar R as function of the Ricci tensor $R_{\mu\nu}$ [14][15] we have:

$$R_{\mu\nu} - \frac{1}{2} g^{\alpha\beta} R_{\alpha\beta} g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (4.2)$$

To make the conversion to geometric algebra we will pretty straight forward.
The Ricci tensor is considered the Laplacian of the metric tensor [15]. This would mean that in Geometric Algebra the representation of the Ricci tensor would be something like:

$$\boldsymbol{R} = \nabla^2 \boldsymbol{g} \qquad ?? \quad (4.3)$$

Being **g** a multivector that somehow represents the metric tensor in geometric algebra. We will talk about **g** later. But if you have been working in geometric algebra you would know that the Laplacian is not represented that way in geometric algebra. Geometric Algebra is in general non-commutative. And it works always symmetrically when using operators. The reason that normally the Laplacian is the square of the del operator in standard algebra is because the standard algebra is commutative so you can move the nabla to the beginning of the equation and multiply it by itself.

In Geometric Algebra an expression with a Laplacian like (4.3) would read as follows:

$$\boldsymbol{R} = \nabla \boldsymbol{g} \nabla \quad (4.4)$$

This would keep the symmetry in the products (that as commented are not commutative). You can check [5] for example to see how this works for the Dirac equation for example.

The next thing we can see is that we see in equation (4.2) the metric tensor is in two forms (the contravariant and the covariant form). This in geometric algebra corresponds to the inverse of the metric (when the indices are up) and to the metric (with the lower indices).

This means:

$$g^{\alpha\beta} \to \boldsymbol{g}^{-1} = \frac{\boldsymbol{g}}{\|g\|^2} \qquad (4.5)$$

$$g_{\alpha\beta} \to \boldsymbol{g} \qquad (4.6)$$

So, we have a multivector called **g,** its inverse $\mathbf{g}^{-1}$ and its norm $\|g\|$ (that it is scalar related to the norm -similar of what it could be the determinant or a trace in a matrix-). It is important to know that the norm is a scalar and is commutative (it can be moved to different positions in a product).

And if we call the stress-energy tensor in Geometric Algebra as **T** (we will see later what this means), we will have the following equation in GA equivalent to what (4.1) is in tensor formalism:

$$\boldsymbol{R} - \frac{1}{2} \frac{\boldsymbol{g}}{\|g\|^2} \boldsymbol{R} \boldsymbol{g} + \Lambda \boldsymbol{g} = \frac{8\pi G}{c^4} \boldsymbol{T} \qquad (4.7)$$

$$\nabla \boldsymbol{g} \nabla - \frac{1}{2}\frac{\boldsymbol{g}}{\|g\|^2}\acute{\nabla}\acute{\boldsymbol{g}}\acute{\nabla}\boldsymbol{g} + \Lambda \boldsymbol{g} = \frac{8\pi G}{c^4}\boldsymbol{T} \qquad (4.8)$$

I have introduced accents for two del operators and on a **g** to remark that these operators (partial equations) apply to the **g** in the middle (this is the **g** that comes from the Ricci tensor) and not to the last one of the product. Remind that in Geometric Algebra (non-commutative) the order matters (the del operator has its own basis vectors that cannot be moved as our wish). So, we have to keep the del operator in the right, in that position even if it is affecting the **g** on its left.

Do not worry, this representation will be much simplified, and accents will not be even necessary.

In the next chapter we will work with stress-energy tensor to create something that we can use in Geometric Algebra. Before going there, the last comment is that the cosmological constant Λ is a scalar (we will need to use this information in the future).

5. The stress-energy tensor in Geometric Algebra

The components of the stress-energy tensor are either pressures (Force/surface) or Energy density (Energy/volume).

The nabla operator divides whatever quantity by units of length. This means a makes a derivative with respect to a unit of length. So, the unit vector associated with this length is also in its inverse form (this is, to the power of -1), as we have seen in equation (3.2) and subsequent paragraphs.

The units of the stress-energy tensor elements [16] are (depending on the position):

$$\frac{Force}{Surface} = \frac{N}{m^2} = kg\frac{m}{s^2m^2} = \frac{kg}{s^2m} \qquad (5.1)$$

$$\frac{Energy}{Volume} = \frac{J}{m^3} = kg\frac{m^2}{s^2m^3} = \frac{kg}{s^2m} \qquad (5.2)$$

As we can see, they correspond to the same elemental units.

If we go to the definition in equation (5.1), we see that it is a force by unit of surface. Let's consider two orthogonal directions that compose a surface. Having a magnitude per unit of that surface is the same as making two derivatives per unit of length (one in each of the directions). Considering this magnitude, the force, we would have:

$$\frac{dF}{dS} = \frac{d}{dx}\frac{d}{dy}(F) \qquad (5.3)$$

If we consider another magnitude (E/meter) -we will see later where this comes from-, we can consider a similar situation:

$$\frac{d\left(\frac{E}{meter}\right)}{dS} = \frac{d}{dx}\frac{d}{dy}\left(\frac{E}{meter}\right) \qquad (5.4)$$

Considering a situation where the derivative can be interchanged by the partial derivative and trying to keep the symmetry and using the rule that in Geometric Algebra a Laplacian (or double partial derivative) corresponds with a partial derivative pre-multiplying and the second post-multiplying (see 4.3 and 4.4), this would show (5.3) as:

$$\frac{\partial}{\partial x}(F)\frac{\partial}{\partial y} \quad (5.5)$$

And equivalently equation (5.4) as:

$$\frac{\partial}{\partial x}\left(\frac{E}{meter}\right)\frac{\partial}{\partial y} \quad (5.6)$$

Generalizing this, using all the possible partial derivatives with respect to lengths in the different directions including the direction of time (ct) we would have:

$$\nabla(F)\nabla \qquad (5.7)$$

So somehow, the stress Energy tensor is related to the following magnitude:

$$\mathrm{T} \rightarrow \nabla(F)\nabla \qquad (5.8)$$

Where the definition of ∇ is the complete one including time as commented in (3.8). The equivalent one in tensorial formalism would be □, as commented in (3.8).

The same for equation (5.6) would lead to:

$$\nabla\left(\frac{E}{meter}\right)\nabla \quad (5.9)$$

Again, somehow the energy-stress tensor should be related to above expression:

$$\mathrm{T} \rightarrow \nabla\left(\frac{E}{meter}\right)\nabla \quad (5.10)$$

We will come back later regarding this point. Let's just advance a little more before.

Let's go one step further. We know that the force is the derivative of the momentum with respect to time:

$$F = \frac{dp}{dt} \quad (5.11)$$

On the other side, the strange magnitude (E/meter) corresponds to the derivative of the energy through a direction (through a length). Let's put x and example:

$$\frac{E}{meter} = \frac{dE}{dx} \qquad (5.12)$$

Again, we will consider the derivatives as equivalent to partial derivatives as commented in (5.4) and (5.5).

$$F = \frac{\partial p}{\partial t} \quad (5.13)$$

$$\frac{E}{meter} = \frac{\partial E}{\partial x} \qquad (5.14)$$

Ok, if we go further, we know that the momentum in Quantum mechanics [17] (as an example in x axis applying to a wavefunction ψ) corresponds to:

$$p_x = -i\hbar\frac{\partial\psi}{\partial x} \qquad (5.15)$$

So, coming back to (5.13) we have:

$$F = \frac{\partial p}{\partial t} = \frac{\partial}{\partial t}\left(-i\hbar\frac{\partial\psi}{\partial x}\right) \quad (5.16)$$

If we want to generalize this equation using del operators (∇) we need to introduce the speed of light c in the denominator multiplying by ∂t because the del operator is defined that way (3.8). So, we multiply and divide by c.

$$F = c\frac{\partial}{c\,\partial t}\left(-i\hbar\frac{\partial\psi}{\partial x}\right) \quad (5.17)$$

And now, yes, we can generalize converting the partial derivatives with respect certain dimensions (ct and x) to all of them. Also, we will apply the rule in Geometric Algebra of pre and post multiplying the operators. The scalars are commutative so we can put them when we want in the product. We will put them in the beginning at this stage:

$$F = -ic\hbar\nabla(\boldsymbol{\psi})\nabla \quad (5.18)$$

We will follow a similar path for the Energy to demonstrate that arrive to the same place. The Energy applied to a wavefunction ψ in Quantum Mechanics is [18]:

$$E = i\hbar\frac{\partial\psi}{\partial t} \quad (5.19)$$

Now we apply:

$$\frac{E}{meter} = \frac{\partial E}{\partial x} = \frac{\partial}{\partial x}\left(i\hbar\frac{\partial\psi}{\partial t}\right) \quad (5.19)$$

Again, we have to multiply and divide by c to be able to generalize to the del operators ∇ (3.8):

$$\frac{E}{meter} = c\frac{\partial}{\partial x}\left(i\hbar\frac{\partial\psi}{c\,\partial t}\right) \quad (5.20)$$

$$\frac{E}{meter} = ic\hbar\nabla(\boldsymbol{\psi})\nabla \quad (5.21)$$

So, we can see that following both paths we arrive to the same equation for the Force or for the Energy/meter, these magnitudes that appear in the stress-energy tensor divided by surface.

So, coming to equation (5.8):

$$\mathrm{T} \rightarrow \nabla(F)\nabla = \nabla(-ic\hbar\nabla(\boldsymbol{\psi})\nabla)\nabla = -ic\hbar\nabla\nabla\boldsymbol{\psi}\nabla\nabla \quad (5.22)$$

If we do the same with the equations (5.8) and (5.21):

$$\mathrm{T} \rightarrow \nabla\left(\frac{E}{meter}\right)\nabla = \nabla(ic\hbar\nabla(\boldsymbol{\psi})\nabla)\nabla = ic\hbar\nabla\nabla\boldsymbol{\psi}\nabla\nabla \quad (5.23)$$

We can see that we arrive to the same equation with different sign for the momentum part and the energy part, as it is common also in tensor formalism.

So, the logic move would be just to say that the stress-energy tensor in Geometric Algebra can be represented as:

$$\mathrm{T} = ic\hbar\nabla\nabla\boldsymbol{\psi}\nabla\nabla \quad ?? \quad (5.24)$$

It is difficult to explain why this move is incorrect (or could it be correct? See Annex A2). In fact, for example, in the Klein-Gordon Equation Laplacians and double derivatives are applied to the simple wavefunction as above.

Not to fill the main body of the paper with philosophical questions and examples, I will leave all the discussion in Annex A2, and I invite you to read it to understand, what we are going to the next.

Ok, you have read Annex A2, so probably you agree that the correct move seems (5.25) instead of (5.24):

$$\mathrm{T} = ic\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (5.25)$$

And as commented in Annex A3, the imaginary unit in Geometric Algebra can be exchanged by the trivector (when no preferred direction is implied in the equation, see Annex A3). In the general case (non-Euclidean metric), we have to divide it by its norm to keep square equal to 1 and not distorting the result (Annex A3).

$$\mathrm{T} = \frac{\hat{t}}{\|\hat{t}\|}c\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (5.26)$$

The $\tilde{\psi}$ could be the be the reverse of ψ (being the reverse commented in [3]), or any other operation that affects the sign of the certain components of the wavefunction multivector (or reverse the order of its basis vectors). Or it could be exactly the same ψ. Really, it does not matter as we will see later. In standard quantum mechanics algebra, it corresponds to the conjugate of ψ as usual.

6. Einstein field equations in Geometric Algebra (II)

Coming back to equation (4.8), we had:

$$\nabla\boldsymbol{g}\nabla - \frac{1}{2}\frac{\boldsymbol{g}}{\|g\|^2}\acute{\nabla}\acute{\boldsymbol{g}}\acute{\nabla}\boldsymbol{g} + \Lambda\boldsymbol{g} = \frac{8\pi\mathrm{G}}{c^4}\boldsymbol{T} \qquad (6.1)$$

Putting **T** as in equation (5.25), we have:

$$\nabla\boldsymbol{g}\nabla - \frac{1}{2}\frac{\boldsymbol{g}}{\|g\|^2}\acute{\nabla}\acute{\boldsymbol{g}}\acute{\nabla}\boldsymbol{g} + \Lambda\boldsymbol{g} = \frac{8\pi\mathrm{G}}{c^4}\frac{\hat{t}}{\|\hat{t}\|}c\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (6.2)$$

$$\nabla\boldsymbol{g}\nabla - \frac{1}{2}\frac{\boldsymbol{g}}{\|g\|^2}\acute{\nabla}\acute{\boldsymbol{g}}\acute{\nabla}\boldsymbol{g} + \Lambda\boldsymbol{g} = \frac{8\pi\mathrm{G}}{c^3}\frac{\hat{t}}{\|\hat{t}\|}\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (6.3)$$

Now, we want to make all the elements of the equation as symmetric as possible in the left and the right, we will see later why. So, we start, just dividing the scalars in the left and in the right to be the same (taking square roots if necessary):

$$\nabla\boldsymbol{g}\nabla - \frac{1}{\sqrt{2}\|g\|}\boldsymbol{g}\acute{\nabla}\acute{\boldsymbol{g}}\acute{\nabla}\boldsymbol{g}\frac{1}{\sqrt{2}\|g\|} + \sqrt{\Lambda}\boldsymbol{g}\sqrt{\Lambda} = 2\sqrt{\frac{2\pi\mathrm{G}\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla 2\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\sqrt{\frac{2\pi\mathrm{G}\hbar}{c^3}} \qquad (6.4)$$

Even we can divide the **g**'s in the product $\mathbf{g}^{1/2}\mathbf{g}^{1/2}$. The second $\mathbf{g}^{1/2}$ could be the reverse or a similar operation to the $\mathbf{g}^{1/2}$. The same as we commented for the wavefunction ψ before. Anyhow, we will see just in a minute the reason why this is not important.

$$\nabla\boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}}\nabla - \frac{1}{\sqrt{2}\|g\|}\boldsymbol{g}\acute{\nabla}\left(\acute{\boldsymbol{g}}^{\frac{1}{2}}\acute{\boldsymbol{g}}^{\frac{1}{2}}\right)\acute{\nabla}\boldsymbol{g}\frac{1}{\sqrt{2}\|g\|} + \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}}\sqrt{\Lambda} = 2\sqrt{\frac{2\pi\mathrm{G}\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla 2\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\sqrt{\frac{2\pi\mathrm{G}\hbar}{c^3}} \quad (6.5)$$

To make this equation completely symmetrical we have to take the square root of -1. There are different solutions for this square root in Geometric Algebra. In Annex A3, I explain why we will use t/||t|| of all the possible possibilities. Anyhow, if you do not like this move, do not worry. You can still use the minus sign and not so much will really change. I just want to keep the most symmetrical as possible solution.

$$\nabla \boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}}\nabla + \frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\acute{\nabla}\left(\acute{\boldsymbol{g}}^{\frac{1}{2}}\acute{\boldsymbol{g}}^{\frac{1}{2}}\right)\acute{\nabla}\frac{\boldsymbol{g}}{\|g\|}\frac{\hat{t}}{\|\hat{t}\|}\frac{1}{\sqrt{2}} + \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}}\sqrt{\Lambda} = 2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\sqrt{\frac{\hbar G 2\pi}{c^3}}2 \quad (6.6)$$

And now, is when we make the Dirac move [13]. As the equation symmetric, if the equation is fulfilled for the left hand of the elements, it will be fulfilled and exactly the same (probably with sign changes) for the right side of every element. So, we can eliminate the right side of every element of the equation not losing information.

$$\nabla \boldsymbol{g}^{\frac{1}{2}} + \frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\acute{\nabla}\acute{\boldsymbol{g}}^{\frac{1}{2}} + \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}} = 2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi} \quad (6.7)$$

Keeping the convention that the del operator applies to the right we can remove the accents from the nomenclature:

$$\nabla \boldsymbol{g}^{\frac{1}{2}} + \frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\nabla\boldsymbol{g}^{\frac{1}{2}} + \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}} = 2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi} \quad (6.8)$$

Taking common factor the del operator being applied to $\mathbf{g}^{1/2}$:

$$\left(1 + \frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)\nabla\boldsymbol{g}^{\frac{1}{2}} + \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}} = 2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi} \quad (6.9)$$

The units of this equation are 1/m. In the first element the only dimensionless element is the del operator which is the derivative through (division by) length. For the second element, the cosmological constant Λ units are length^{-2} [19] and as it is in a square root of it, we are left with length^{-1} (1/m).

We can see it also in the right-hand side where (knowing that the wavefunction ψ is dimensionless):

$$Units\left(\sqrt{\frac{G\hbar}{c^3}}\nabla\nabla\right) = \sqrt{\frac{\left(\frac{m^3}{kg\, s^2}\right)\left(kg\frac{m^2}{s}\right)}{\left(\frac{m^3}{s^3}\right)}}\frac{1}{m}\frac{1}{m} = \sqrt{m^2}\frac{1}{m}\frac{1}{m} = \frac{1}{m} \quad (6.10)$$

Ok, let's go now with the Dirac Equation.

7. Dirac Equation

In [5] we arrived at the following equation (where we considered c=$\hbar$=1) expressed in Geometric Algebra:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t} - \hat{y}\hat{z}\frac{\partial}{\partial x} - \hat{z}\hat{x}\frac{\partial}{\partial y} - \hat{x}\hat{y}\frac{\partial}{\partial z} - m\right)\boldsymbol{\psi} = 0 \quad (7.1)$$

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t} - \hat{y}\hat{z}\frac{\partial}{\partial x} - \hat{z}\hat{x}\frac{\partial}{\partial y} - \hat{x}\hat{y}\frac{\partial}{\partial z}\right)\boldsymbol{\psi} - m\boldsymbol{\psi} = 0 \quad (7.2)$$

Putting explicitly the constant c and $\hbar$ again we have (see [5] and [13]):

$$\hbar\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{c\,\partial t} - \hat{y}\hat{z}\frac{\partial}{\partial x} - \hat{z}\hat{x}\frac{\partial}{\partial y} - \hat{x}\hat{y}\frac{\partial}{\partial z}\right)\boldsymbol{\psi} - mc\boldsymbol{\psi} = 0 \quad (7.3)$$

We can take $\hat{z}\hat{y}\hat{x}$ as common factor having:

$$\hat{z}\hat{y}\hat{x}\,\hbar\left(-\frac{\partial}{c\,\partial t}+\hat{x}^{-1}\frac{\partial}{\partial x}+\hat{y}^{-1}\frac{\partial}{\partial y}+\hat{z}^{-1}\frac{\partial}{\partial z}\right)\boldsymbol{\psi}-mc\boldsymbol{\psi}=0 \qquad (7.4)$$

To simplify this factor $\hat{z}\hat{y}\hat{x}$ we can use the definition we made in [5], where we called $\hat{z}\hat{y}\hat{x}=\hat{t}^{-1}$. It is not necessary that you consider if this is related to time or not, as we did in [5]. It is just a definition for an expression for the trivector. The important thing is that the square of $\hat{t}^{-1}$ -1 (as the square of *i* , the square of the trivector or the square of the bivectors).

$$\hat{t}^{-1}\hbar\nabla\boldsymbol{\psi}-mc\boldsymbol{\psi}=0 \qquad (7.5)$$

We know that the trivector (in this case, we have called it $\hat{t}^{-1}$) is the equivalent for the imaginary unit *i* in Geometric Algebra [3][5] and also that the product of the basis vectors anti-commutates. So, above equation is the equivalent for the known one in standard Algebra:

$$i\hbar\gamma^{\mu}\partial_{\mu}\psi-mc\psi=0 \qquad (7,6)$$

Coming back to equation (7.5), we divide it by $\hbar$:

$$\hat{t}^{-1}\nabla\boldsymbol{\psi}-\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.7)$$

Also, for simplicity we can pre-multiply both elements by $\hat{t}$, so we remove the power to -1 and also, we transfer the basis vector to the second element -the one without operators for simplicity-.

$$\nabla\boldsymbol{\psi}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.8)$$

The units of this equation are 1/m. We can see it in the first element (the wavefunction ψ is dimensionless and the del operator makes the derivative through length -so it divides by length-). Anyhow, you can see it also in the second element (again knowing that the wavefunction ψ and the trivector $\hat{t}$ are dimensionless):

$$units\left(\frac{mc}{\hbar}\right)\rightarrow\frac{kg\,\frac{m}{s}}{kg\,\frac{m^2}{s}}=\frac{1}{m}$$

Now, as we made in (3.3) instead of applying the del operator, we will apply the Covariant Derivative (as defined in [9]) to Dirac Equation as defined in (7.8):

$$D\boldsymbol{\psi}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.9)$$

As we made with general multivector **A** in (3,6), we will separate (in a conceptual way) the coefficients of the wavefunction from its vectors, leading to:

$$\boldsymbol{\psi}=\psi\boldsymbol{g}^{\frac{1}{2}} \qquad (7.10)$$

So, following the chain rule, equation (7.9) would read:

$$\nabla\psi\boldsymbol{g}^{\frac{1}{2}}+\psi\nabla\boldsymbol{g}^{\frac{1}{2}}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.10)$$

$$\nabla\boldsymbol{\psi}+\psi\nabla\boldsymbol{g}^{\frac{1}{2}}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.11)$$

Remember that $\hat{t}$ is just another way of calling the trivector or the imaginary unit i in other mathematical frameworks.

8. Special case: Orthogonal metric very near to Euclidean (square of the basis vectors near to 1) but not constant in time or position (the result of applying the del operator to the metric is not zero)

From this point on, our intention will be to play with these two equations:

$$\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)\nabla\boldsymbol{g}^{\frac{1}{2}}+\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}=2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi} \qquad (6.9)$$

$$\nabla\boldsymbol{\psi}+\psi\nabla\boldsymbol{g}^{\frac{1}{2}}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.11)$$

To try to find a solution. As it is very difficult, if not impossible, to find a general analytical solution for the general case, we will try to solve it for a special case/approximation.

In this approximation, we will consider the metric as a known value (in the end, we will substitute it by the Euclidean metric, as we say it is very near to it), but the variation of it (the element that has the del operator applying to it, is an unknown value).

$$\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)\nabla\boldsymbol{g}^{\frac{1}{2}}+\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}=2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi} \qquad (6.9)$$

We pass the cosmological element to the right-hand side:

$$\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)\nabla\boldsymbol{g}^{\frac{1}{2}}=2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}} \qquad (8.1)$$

We pre-multiply both sides by the inverse of the first parentheses:

$$\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)\nabla\boldsymbol{g}^{\frac{1}{2}}=\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right) \qquad (8.2)$$

$$\nabla\boldsymbol{g}^{\frac{1}{2}}=\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right) \qquad (8.3)$$

Now we substitute (8.3) in (7.11):

$$\nabla\boldsymbol{\psi}+\psi\nabla\boldsymbol{g}^{\frac{1}{2}}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.11)$$

$$\nabla\boldsymbol{\psi}+\psi\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right)-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (8.4)$$

Now, if we compare it with the Dirac equation:

$$\nabla\boldsymbol{\psi}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.8)$$

We can see that the difference is the second element in equation (8.4). To solve this equation in this form is a very complicated issue.

So, what we will do is to compare two scalar differential equations with an unknown function y of only one variable x (this is y(x)), that resemble equations (7.11) and (8.4) and see the difference in the results. As they are scalar, we will consider that the basis vectors that are divided by its norm ($\frac{\hat{t}}{\|\hat{t}\|}$ and $\frac{\boldsymbol{g}}{\|g\|}$) are unitary. So, $\frac{\boldsymbol{g}}{\|g\|}$ =1. In the case of $\frac{\hat{t}}{\|\hat{t}\|}$**,** as its square is -1, its scalar equivalent is *i*. Also, as we have commented that the metric is nearly Euclidean, we will consider also directly **g**≈1 anyhow. And equivalently we will consider $\hat{t}$ equal to *i* because the basis vector in a nearly Euclidean metric have norm equal to 1. The case for the square root of $\frac{\hat{t}}{\|\hat{t}\|}$ is similar, as its modulus is 1 (See Annex A3).

This leads to:

$$\nabla\boldsymbol{\psi}+\psi\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right)-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (8.4)$$

$$y'+y\left(1+i\frac{1}{\sqrt{2}}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}y''-\sqrt{\Lambda}\right)-i\frac{mc}{\hbar}y=0 \qquad (8.5)$$

$$y'+\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1+i\frac{1}{\sqrt{2}}\right)}y''y-\frac{\sqrt{\Lambda}}{\left(1+i\frac{1}{\sqrt{2}}\right)}y-i\frac{mc}{\hbar}y=0 \qquad (8.6)$$

$$y'+\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1+i\frac{1}{\sqrt{2}}\right)}y''y-\left(\frac{\sqrt{\Lambda}}{\left(1+i\frac{1}{\sqrt{2}}\right)}+i\frac{mc}{\hbar}\right)y=0 \qquad (8.7)$$

If we do the same with equation (7.11) -the standard Dirac Equation- we get:

$$\nabla\boldsymbol{\psi}-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (7.11)$$

$$y'-i\frac{mc}{\hbar}y=0 \qquad (8.8)$$

Solving this equation leads to:

$$y = e^{-i\frac{mc}{\hbar}x} \qquad (8.9)$$

Solving equation (8.7) analytically is very complex if not impossible. I have tried with Mathematica program, different webpages and even Chat GPT with no solution. But anyhow, we always have our tricks.

If we compare equations (8.7) and (8.8) -a form of the standard Dirac equation- we see that the only difference is the second and third element in (8.7). Are they big? Can they be approximated to something that it is easier to solve?

We can calculate their numerical value. We will use the following values for the constants (CODATA values [20]):

$$G = 6.6743E - 11\, m^3 kg^{-1} s^{-2}$$
$$\hbar = 1.054571817E - 34\, Js$$
$$c = 299792458\, m/s$$
$$\Lambda = 1.1056E - 52\, m^{-2}$$
$$m_{muon} = 1.8835327E - 28\, kg$$

The value of the cosmological constant Λ is not still confirmed but all measurements lead to an order of 1E-52 m^{-2} [19].

Now, we calculate the value of the coefficients appearing in (8.7):

$$2\sqrt{\frac{2\pi G\hbar}{c^3}} = 8.10270108398398742E - 35\, m$$
$$\sqrt{\Lambda} = 1.05147515424759E - 26\, m^{-1}$$
$$\frac{mc}{\hbar} = 5.35448500285853E14\, m^{-1}$$

So, we can see that the third element is neglectable compared to the fourth element in the sum in (8.7). And both affect the same way the unknown function y in its primitive (not derived) form. So, the third element can be neglected.

We cannot do the same with the second element. Even if the value of its coefficient is very small, we do not know the value of y'' so we cannot know if it is affecting or not the result of the equation.

So, at the moment we can approximate (8.7):

$$y' + \frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1 + i\frac{1}{\sqrt{2}}\right)} y''y - \frac{\sqrt{\Lambda}}{\left(1 + i\frac{1}{\sqrt{2}}\right)} y - i\frac{mc}{\hbar} y = 0 \qquad (8.7)$$

To this (8.10):

$$y' + \frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1+i\frac{1}{\sqrt{2}}\right)}y''y - i\frac{mc}{\hbar}y = 0 \qquad (8.10)$$

This cannot be solved analytically as commented. We can try to get a similar equation that can be solved. Of course, that equation would not be correct but would give the kind of results that we would expect from the correct equation. And anyhow, do not forget that this equation comes from the general equation (8.4) where even it is not clear if the correct equation would have exactly that form (see Annex A2).

The first thing to check about equation (8.10) is the units. The equation is in 1/m units (inverse of length). The unknown y represents the wavefunction that is dimensionless. When it is derived, it is derived with respect to x (unit of length) so the inverse of length is introduced.

So, y´ is 1/m. The factor $2\sqrt{\frac{2\pi G\hbar}{c^3}}$ is unit of lengths (m) as we have seen before. It is multiplied by y`` that is 1/m^2 so again we have 1/m. And the factor $\frac{mc}{\hbar}$ is 1/m as seen before.

Whatever change we make in the equation to make it solvable should keep this coherence of units. There are two possibilities.

One is to consider that in the original equation (8.4), opposite of what we have commented in Annex A2, we should have taken the square roots of the wavefunctions, leading to something like:

$$\nabla\boldsymbol{\psi} + \psi^{\frac{1}{2}}\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}^{\frac{1}{2}} - \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right) - \hat{t}\frac{mc}{\hbar}\boldsymbol{\psi} = 0 \qquad (8.11)$$

So, leading in our example to (again considering the cosmological constant neglectable):

$$y' + \frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1+i\frac{1}{\sqrt{2}}\right)}\left(y^{\frac{1}{2}}\right)''y^{\frac{1}{2}} - i\frac{mc}{\hbar}y = 0 \qquad (8.12)$$

We can see that the equation is coherent regarding measurement units. Because $y^{1/2}$ is dimensionless as y. So, if we derive it twice, we get 1/m^2 as before. Multiplying by $2\sqrt{\frac{2\pi G\hbar}{c^3}}$ (m) we get 1/m as expected.

For simplification we will solve for the magnitudes of the complex numbers (their moduli) and we will call (remind that the modulus of the square root of *i* is 1, see Annex 3):

$$a = \left|2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}\right| = 8.10270108398398742E - 35\,m \qquad (8.13)$$

$$b = \left| i\frac{mc}{\hbar} \right| = \frac{m_{muon}c}{\hbar} = 5.35448500285853E14\ m^{-1} \qquad (8.14)$$

$$d = \left| 1 + i\frac{1}{\sqrt{2}} \right| = \sqrt{1^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1.22474487139 \qquad (8.15)$$

Where the units of a are meters, the units of b are 1/m and d is dimensionless.

So, now we have:

$$y' + \frac{a}{d}\left(y^{\frac{1}{2}}\right)'' y^{\frac{1}{2}} - b\,y = 0 \qquad (8.16)$$

This equation is solvable looking for a solution of the form:

$$y = e^{kx} \qquad (8.17)$$

Where k is a constant which units are 1/m so the exponential is dimensionless (because x has the unit of meters).

$$y = e^{kx} \quad (8.17)$$

$$y' = ke^{kx} \quad (8.18)$$

$$y^{\frac{1}{2}} = e^{\frac{kx}{2}} \quad (8.19)$$

$$\left(y^{\frac{1}{2}}\right)' = \frac{k}{2}e^{\frac{kx}{2}} \quad (8.20)$$

$$\left(y^{\frac{1}{2}}\right)'' = \frac{k^2}{4}e^{\frac{kx}{2}} \quad (8.21)$$

Substituting in the equation we have:

$$ke^{kx} + \frac{a}{d}\left(\frac{k^2}{4}e^{\frac{kx}{2}}\right)\left(e^{\frac{kx}{2}}\right) - b\,e^{kx} = 0 \qquad (8.22)$$

$$ke^{kx} + \frac{k^2}{4}\frac{a}{d}e^{kx} - b\,e^{kx} = 0 \qquad (8.23)$$

$$k + \frac{k^2}{4}\frac{a}{d} - b = 0 \qquad (8.24)$$

$$\frac{k^2}{4}\frac{a}{d} + k - b = 0 \qquad (8.25)$$

$$k^2 + \frac{4d}{a}k - \frac{4bd}{a} = 0 \qquad (8.26)$$

$$k = \frac{-\frac{4d}{a} \pm \sqrt{\left(\frac{4d}{a}\right)^2 - 4\left(-\frac{4bd}{a}\right)}}{2} = \frac{-\frac{4d}{a} \pm \sqrt{\frac{16d^2}{a^2} + \frac{16bd}{a}}}{2} = \frac{-\frac{4d}{a} \pm \frac{4d}{a}\sqrt{1 + \frac{ba}{d}}}{2}$$

$$= -\frac{2d}{a} \pm \frac{2d}{a}\sqrt{1 + \frac{ba}{d}} = \frac{2d}{a}\left(-1 \pm \sqrt{1 + \frac{ba}{d}}\right) \quad (8.27)$$

We will comment about these results in a moment.

As commented, there is another possibility for the equation to be solvable. That would be simply to remove the y (the wavefunction that multiply the second derivative of the wavefunction). This is possible as y is dimensionless. This would mean that the equation (8.4) would really be something like:

$$\nabla\boldsymbol{\psi}+\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{\|\hat{t}\|}\frac{\boldsymbol{g}}{\|g\|}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\nabla\nabla\boldsymbol{\psi}-\sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right)-\hat{t}\frac{mc}{\hbar}\boldsymbol{\psi}=0 \qquad (8.28)$$

Leading to:

$$y'+\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{i}}{\left(1+i\frac{1}{\sqrt{2}}\right)}y''-i\frac{mc}{\hbar}y=0 \qquad (8.29)$$

Again, this is solvable with:

$$y=e^{kx} \qquad (8.30)$$

$$y'=ke^{kx} \qquad (8.31)$$

$$y''=k^2e^{kx} \qquad (8.32)$$

Again, solving for the moduli of the complex numbers and using a,b and d we have:

$$y'+\frac{a}{d}y''-b\,y=0 \qquad (8.33)$$

Solving:

$$ke^{kx}+\frac{a}{d}k^2e^{kx}-b\,e^{kx}=0 \qquad (8.34)$$

$$k+\frac{a}{d}k^2-b=0 \qquad (8.35)$$

$$\frac{a}{d}k^2+k-b=0 \qquad (8.36)$$

$$k^2+\frac{d}{a}k-\frac{bd}{a}=0 \qquad (8.37)$$

$$k=\frac{-\frac{d}{a}\pm\sqrt{\left(\frac{d}{a}\right)^2-4\left(-\frac{bd}{a}\right)}}{2}=\frac{-\frac{d}{a}\pm\sqrt{\frac{d^2}{a^2}+\frac{4bd}{a}}}{2}=\frac{-\frac{d}{a}\pm\frac{d}{a}\sqrt{1+\frac{4ba}{d}}}{2}$$

$$=-\frac{d}{2a}\pm\frac{d}{2a}\sqrt{1+\frac{4ba}{d}}=\frac{d}{2a}\left(-1\pm\sqrt{1+\frac{4ba}{d}}\right) \qquad (8.38)$$

We can see that both equations give similar results in its "shape form" but the values are approximately different by a factor of 4. But we know that these equations could not be correct, they are just modifications of what was considered the "correct" (with a lot of doubts) equation (8.4). So, the important thing is not the values of the solutions themselves but the components that have. Both of them are a combination between a,b and d involving some square roots. Also, important is that the solution has the dimensions of inverse of length (as commented when we talked about k).

9. Muon g-2

The aim of this paper was first to make a combination of the Einstein equations and the Dirac equation using Geometric Algebra. This aim has been "somehow" fulfilled even not completely proved. What takes us to the second aim. The idea was to solve these equations and obtain an explanation for the muon g-2 discrepancy. This has not been fulfilled as such but as we will see now, the numbers tell us that we are in some way nearby.

The g-2 discrepancy tells us that the magnitude a_μ of a muon [21] should be according to theoretical physics [21]:

$$a_{\mu\_theor} = \frac{g-2}{2} = 0.001165918 \quad (9.1)$$

But the measurements tell us:

$$a_{\mu\_measured} = \frac{g-2}{2} = 0.00116592089 \quad (9.2)$$

This magnitude a_μ (as also g) is dimensionless (it does not have measurement units).
The discrepancy between both could be related to the ratio between them:

$$ratio_{a_\mu} = \frac{a_{\mu_{measured}}}{a_{\mu_{theor}}} = \frac{0.00116592089}{0.001165918} = 1.0000024787335 \quad (9.3)$$

Another way to measure the discrepancy is to measure the difference:

$$Difference_{a_\mu} = a_{\mu_{measured}} - a_{\mu_{theor}} = 0.00116592089 - 0.001165918 = 2.89E-9 \quad (9.4)$$

In the previous chapter even trying to adapt the equations so the can be solvable we did not get a number like that in any of the solutions.
Anyhow, it is normally said [22] that the gravitational effects are not the ones causing this distortion as their effects are so small. As we will see now, this is not like that.
We can get these numbers just with simple combinations of the coefficients of the previous equations (8.4) and (8.12) for example. This means, the solution of those equation will be for sure depending on those coefficients. And they could lead to these results, as we will see later.

When you solve a differential equation, you need some initial or boundary conditions. In this case, as the integration is considering length units, after the solving you will need to introduce some information regarding the muon that has length units. It is the same as when you have to integrate a magnitude from some point to another (that you have to introduce these points that are in the same units as the integral).

In [23] I had to integrate a magnitude related to the electron from infinity to a finite point that could not be zero. This point had to be the classical electron radius for everything to work.

For the muon, we can guess we will need something similar. We can start with the muon radius for example. Using the same definition as electron radius [24] we would have:

$$r_{muon} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_{muon}c^2} = 1.36284863E-17m \quad (9.5)$$

Where:

$$m_{muon} = 1.883531627E-28kg$$
$$e = 1.60217663E-19C$$
$$\varepsilon_0 = 8.8541878176E-12\frac{C^2}{Nm^2}$$

Another characteristic of the muon that has length units and it is related to its mass (in the meaning that it is related to gravitational effects) is the mass of the muon expressed in length units (this is done taking half the Schwarzschild radius corresponding to that mass [25]), this is:

$$m_{muon(length\ units)} = \frac{Gm_{muon}}{c^2} = 1.3987415E-55m \qquad (9.6)$$

We can see that this value is very low, and this is the reason that it is thought that gravitation effects are too small to affect the g-2 value [22]. But we can see that the muon radius above (9.5) is a quantity in the order of the values we need, and it is also related mainly with the muon mass. In fact, as commented, the electron radius, as an example, was used in [23] as a parameter related to gravitational effects.

In the previous chapter 8 we have seen that in the solution of the equations appear:

- Square roots
- The factor a = 8.10270108398398742E-35 m
- The factor b = 5.35448500285853E14 m^{-1}
- The factor d = 1.22474487139

We can add here (as a candidate to enter as boundary condition):

- r_{muon} = 1.36284863E-17m

The goal is to obtain one of the following dimensionless values using above data:

- Ratio_a_μ = 1.0000024787335 (9.3)
- Difference_a_μ = 2.89E-9 (9.4)

We can see that something as simple as an hypothetical solution like:

$$Sol_1 = \sqrt{\frac{ad}{r_{muon}}} = \sqrt{\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{1^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{r_{muon}}} = \sqrt{\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{1^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{m_{muon}c^2}}}$$
$$= 2.78E-9 \qquad (9.7)$$

If we compare with Difference_a_μ (9.4) we see that:

$$\frac{Difference_{a_\mu} - Sol_1}{Difference_{a_\mu}} = \frac{2.89E-9-2.78E-9}{2.89E-9} = 3.95\%\ Error \qquad (9.8)$$

We have found an expression just using elements obtained with an equation merging Einstein Equations and Dirac Equation (this is, Dirac Equation with gravitational effects) that can give results in the order that we need to explain g-2 discrepancy.

If you do not like the idea of including the muon radius there, you can see other possibilities as:

$$Sol_2 = 4\pi\sqrt{ab} = 4\pi\sqrt{2\sqrt{\frac{2\pi G\hbar}{c^3}}\frac{m_{muon}c}{\hbar}} = 2.617E-9 \quad (9.9)$$

With this error:

$$\frac{Difference_{a_\mu} - Sol_2}{Difference_{a_\mu}} = \frac{2.89E-9 - 2.617E-9}{2.89E-9} = 9.44\%\ Error \quad (9.10)$$

A much higher error but in the order of the values we need.

The idea is not to obtain an exact expression or a final equation (this would be only possible finding a solution to equation (8.4) or a similar to it. The idea is to demonstrate that the g-2 discrepancy could be merely related to gravitational effects (once an equation joining Einstein Equation and Dirac Equation is found (something like (8.4)).

10. Conclusions

In this paper, we have used Geometric Algebra to combine the Einstein Field Equations with the Dirac Equation, arriving to the following equation:

$$\nabla\boldsymbol{\psi} + \psi\left(1+\frac{1}{\sqrt{2}}\frac{\hat{t}}{||\hat{t}||}\frac{\boldsymbol{g}}{||g||}\right)^{-1}\left(2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{\frac{\hat{t}}{||\hat{t}||}}\nabla\nabla\boldsymbol{\psi} - \sqrt{\Lambda}\boldsymbol{g}^{\frac{1}{2}}\right) - \hat{t}\frac{mc}{\hbar}\boldsymbol{\psi} = 0 \quad (8.4)$$

Where ψ is the wavefunction o the particle and $\mathbf{g}^{1/2}$ is the collection of the basis vectors which square is the collection of the product of the basis vectors by themselves (the metric **g** in geometric Algebra representation. And the **t** represents the trivector in Geometric Algebra Cl3,0.

The original aim of the paper was to demonstrate that the muon g-2 discrepancy could be explained by gravitational effects created by its own muon mass (not even necessary to consider Earth's or Sun's gravity.

The difference of muon g-2 between measured value and the theoretical value is:

$$Difference_{a_\mu} = a_{\mu_{measured}} - a_{\mu_{theor}} = 2.89E-9 \quad (9.4)$$

Solving equation (8.4) is very complicated and depends on the boundary conditions. But we know that the set of the possible solutions will depend only in its coefficients and the boundary conditions.

We can see that applying only the coefficients of equation (8.4) and as a possible boundary condition the classical radius of the muon, we can get solutions as:

$$Sol_1 = \sqrt{\frac{2\sqrt{\frac{2\pi G\hbar}{c^3}}\sqrt{1^2+\left(\frac{1}{\sqrt{2}}\right)^2}}{r_{muon}}} = 2.78E-9 \qquad (9.7)$$

That has less than 4% error compared to the expected value.

$$\frac{Difference_{a_\mu} - Sol_1}{Difference_{a_\mu}} = \ 3.95\%\ Error \qquad (9.8)$$

There are other possibilities, as:

$$Sol_2 = 4\pi\sqrt{2\sqrt{\frac{2\pi G\hbar}{c^3}}\frac{m_{muon}c}{\hbar}} = 2.617E-9 \qquad (9.9)$$

With a higher error:

$$\frac{Difference_{a_\mu} - Sol_2}{Difference_{a_\mu}} = \ 9.44\%\ Error \qquad (9.10)$$

None of these solutions gives the exact value, but it has been demonstrated that yes, the muon g-2 discrepancy could be due to gravitational effects (they are in the order of the difference between the measured value and the theoretical value). It is important to remark that the gravitational effects in this regard are due the mass of the muon itself, it is not necessary to include higher gravitational potentials as the Earth or the Sun gravity. Just the muon mass is capable of creating this discrepancy.

Also, we have seen how using Geometric algebra it is possible to combine gravitational equations (generally used in tensor formalism) with the Dirac Equation (normally used in Matrix formalism).

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AAAAÁBCCCDEEIIILLLLLMMMOOOPSTU

If you consider this helpful, do not hesitate to drop your BTC here:

bc1q0qce9tqykrm6gzzhemn836cnkp6hmel5lmz36f

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A1. Annex A1. Meaning of $g^{1/2}$

The meaning of $\mathbf{g}^{1/2}$ is more conceptual than an explicit expression. But the most applicable one would be something like:

$$\boldsymbol{g}^{\frac{1}{2}} = \hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z} \qquad (A1.1)$$

And **g** in geometric algebra would be the square of that function:

$$\boldsymbol{g} = \boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}} = (\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})(\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})$$
(A1.2)

We will get a lot of cross products that in orthogonal metric will cancel with each other. In non-orthogonal metric, we will have to use the cross relations (that we already saw in [4][5][6] for example:

$$\hat{x}\hat{y} = 2g_{xy} - \hat{y}\hat{x} \qquad (A1.3)$$
$$\hat{y}\hat{z} = 2g_{yz} - \hat{z}\hat{y} \qquad (A1.4)$$
$$\hat{z}\hat{x} = 2g_{zx} - \hat{x}\hat{z} \qquad (A1.5)$$

Also, the inner products for square of basis vector will apply [4][5][6]:

$$\hat{x}^2 = \|\hat{x}\|^2 = g_{xx} \quad (A1.6)$$
$$\hat{y}^2 = \|\hat{y}\|^2 = g_{yy} \quad (A1.7)$$
$$\hat{z}^2 = \|\hat{z}\|^2 = g_{zz} \quad (A1.8)$$

Other possibilities for the **g** is instead of being the square of $\mathbf{g}^{1/2}$ being the product of $\mathbf{g}^{1/2}$ by a reverse version of it as:

$$\boldsymbol{g} = \boldsymbol{g}^{\frac{1}{2}}\widetilde{\boldsymbol{g}^{\frac{1}{2}}} = (\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})(\hat{x} + \hat{y} + \hat{z} + \hat{y}\hat{x} + \hat{z}\hat{y} + \hat{x}\hat{z} + \hat{x}\hat{y}\hat{z})$$
(A1.9)

Where sometimes the reverse version could include a reverse also in the trivector (for the example above, the trivector has not been reversed, this would be another issue to be checked).

With above definitions of **g** is clear that you get the geometric products of the vectors by themselves (the inner/scalar products when you multiply a vector buy itself) as it would be expected from a metric.

The problem with above definitions of **g** is that sometimes you loose the sequence (the order) of these products and you just get a sum of all them, not an ordered sequence (as it would be expected in a matrix).

There would be other possibilities that solve this issue as:

$$\boldsymbol{g} = \boldsymbol{g}^{\frac{1}{2}}\boldsymbol{g}^{\frac{1}{2}}\frac{\boldsymbol{g}^{\frac{1}{2}}}{\left\|g^{\frac{1}{2}}\right\|} = (\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})(\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})\frac{(\hat{x} + \hat{y} + \hat{z} + \hat{x}\hat{y} + \hat{y}\hat{z} + \hat{z}\hat{x} + \hat{x}\hat{y}\hat{z})}{\left\|g^{\frac{1}{2}}\right\|} \qquad (A1.10)$$

This would keep all the products ordered (each product would be multiplied by a basis vector, making not possible to sum them but to keep each one in one position). For the last factor not to affect the result of the products, it is divided by its own norm, so its norm is not affecting the result, it is just ordering the results (putting them “in boxes” represented by the unit basis vectors).

The problem with this, is that it is a clear an ad-hoc solution not really justified. Just to add, that this could be used even if the transverse product is needed:

$$\boldsymbol{g}=\boldsymbol{g}^{\frac{1}{2}}\widetilde{\boldsymbol{g}^{\frac{1}{2}}}\frac{\boldsymbol{g}^{\frac{1}{2}}}{\left\|g^{\frac{1}{2}}\right\|}=(\hat{x}+\hat{y}+\hat{z}+\hat{x}\hat{y}+\hat{y}\hat{z}+\hat{z}\hat{x}+\hat{x}\hat{y}\hat{z})(\hat{x}+\hat{y}+\hat{z}+\hat{y}\hat{x}+\hat{z}\hat{y}+\hat{x}\hat{z}+\hat{x}\hat{y}\hat{z})\frac{(\hat{x}+\hat{y}+\hat{z}+\hat{x}\hat{y}+\hat{y}\hat{z}+\hat{z}\hat{x}+\hat{x}\hat{y}\hat{z})}{\left\|g^{\frac{1}{2}}\right\|}\quad (A1.11)$$

A2. Annex A2. A defined event (an observable) occurs when the wavefunction is squared (multiplied by its conjugate/reverse). The wavefunction is the square root of a defined observable event.

The meaning of this mysterious title is the following. When we have a magnitude like the square of the momentum, or the square of the energy very well defined, means that its wavefunction has collapsed (has been multiplied by itself (or its conjugate or reverse) to get this defined value).

You can check in [5] that coming form the following Einstein equation (of real magnitudes, observables):

$$E^2=p^2+m^2\quad (A2.1)$$

We get to the following equation (that is a king of Klein-Gordon equation in GA.

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)\psi\left(-\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}+\hat{y}\hat{z}\frac{\partial}{\partial x}+\hat{z}\hat{x}\frac{\partial}{\partial y}+\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)=0\quad (A2.2)$$

What we did was to follow Dirac steps and take only one side of the equation:

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)\psi=0\quad (A2.3)$$

We know that this last equation (A2.3) is correct. But how do we know that (A2.2) is correct? In fact, using Klein-Gordon equation has been the origin of different issues.

It seems more plausible, the next equation for the Klein-Gordon equation (A2.2)

$$\left(\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}-\hat{y}\hat{z}\frac{\partial}{\partial x}-\hat{z}\hat{x}\frac{\partial}{\partial y}-\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)\psi\tilde{\psi}\left(-\hat{x}\hat{y}\hat{z}\frac{\partial}{\partial t}+\hat{y}\hat{z}\frac{\partial}{\partial x}+\hat{z}\hat{x}\frac{\partial}{\partial y}+\hat{x}\hat{y}\frac{\partial}{\partial z}-m\right)=0\quad (A2.4)$$

An in fact, equation (A2.3) (Dirac equation) that we know is correct, would keep exactly the same.

But (A2.4) seems more correct than (A2.2) because, as it has to represent a magnitude already defined as collapsed (A2.1) (energy and momentum are not wavefunctions any more in (A2.1), they are values), it seems logic that wavefunction has already collapsed (this means we need that it has been squared -or multiplied by its conjugate or reverse-).

As commented, it is difficult to find more examples, but the philosophy is the following. When an equation is classical (in the meaning that some magnitudes have a completely defined value) means that the wavefunction has already collapsed in that equation. So this implies that it has been squared already in that equation and it is therefore already hidden -it has disappeared leaving its value defined, not as wavefunction anymore-.

It is like the world has some events that are observable and defined and have to occur. But in general, we do not live in that world, but in “its square root”. In this “square root world” the wavefunction gives freedom for the events to occur as long as when it collapses fulfills the defined ones.

Too much philosophy to explain that as the Energy stress tensor is one with defined magnitudes. With defined values according “classical” formulation, this means that it cannot have a single wavefunction inside (that would give freedom to its values) as happened in (5.24)

$$\mathrm{T} = ic\hbar\nabla\nabla\boldsymbol{\psi}\nabla\nabla \quad ?? \qquad (5.24)$$

Therefore, it has to have collapsed, this is, squared (multiplied by its conjugate or inverse) to get a fixed defined value as expected for a classical magnitude as the stress-energy tensor. As it is in (5.25)

$$\mathrm{T} = ic\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (5.25)$$

And as commented in Annex A3, the imaginary unit in Geometric Algebra can be exchanged by the trivector (when no preferred direction is implied in the equation, see Annex A3). In the general case (non-Euclidean metric), we have to divide it by its norm to keep square equal to 1 and not distorting the result (Annex A3).

$$\mathrm{T} = \frac{\hat{t}}{||\hat{t}||} c\hbar\nabla\nabla\boldsymbol{\psi}\widetilde{\boldsymbol{\psi}}\nabla\nabla \qquad (5.26)$$

Anyhow, as commented, this is only speculative and nothing -but intuition- tells us that the correct form of T could not be (5.24) -single wavefunction- instead.

A3. Annex A3. Using t/||t|| as imaginary unit

As we commented in [4][5][6] the square of the trivector and of the bivectors in $Cl_{3,0}$ (in Euclidean metric) is -1. This means, the imaginary unit *i* that is an unknown number with strange properties in the world of the scalars, can be substituted by the trivector and bivectors in Geometric Algebra.

Instead of putting a name (*i*) to an entity that we do not know, but that we need, that have to fulfill certain properties, we can use “real” entities that do exist in Geometric Algebra and have these properties (mainly that its square is -1) and are the trivector and the bivectors.

In Euclidean Algebra [4][5][6]:

$$(\hat{x}\hat{y})(\hat{x}\hat{y}) = -1 \qquad (A3.1)$$
$$(\hat{y}\hat{z})(\hat{y}\hat{z}) = -1 \qquad (A3.2)$$
$$(\hat{z}\hat{x})(\hat{z}\hat{x}) = -1 \qquad (A3.3)$$
$$t^{-1}t^{-1} = (\hat{x}\hat{y}\hat{z})(\hat{x}\hat{y}\hat{z}) = -1 \qquad (A3.4)$$
$$tt = (\hat{z}\hat{y}\hat{x})(\hat{z}\hat{y}\hat{x}) = -1 \qquad (A3.5)$$

In non-Euclidean metric, we have to divide by the norms, to be able to get -1 (instead of a number related to the metric):

$$\left(\frac{\hat{x}\hat{y}}{||\hat{x}\hat{y}||}\right)\left(\frac{\hat{x}\hat{y}}{||\hat{x}\hat{y}||}\right) = -1 \qquad (A3.6)$$
$$\left(\frac{\hat{y}\hat{z}}{||\hat{y}\hat{z}||}\right)\left(\frac{\hat{y}\hat{z}}{||\hat{y}\hat{z}||}\right) = -1 \qquad (A3.7)$$
$$\left(\frac{\hat{z}\hat{x}}{||\hat{z}\hat{x}||}\right)\left(\frac{\hat{z}\hat{x}}{||\hat{z}\hat{x}||}\right) = -1 \qquad (A3.8)$$
$$\frac{t^{-1}}{||t^{-1}||}\frac{t^{-1}}{||t^{-1}||} = \left(\frac{\hat{x}\hat{y}\hat{z}}{||\hat{x}\hat{y}\hat{z}||}\right)\left(\frac{\hat{x}\hat{y}\hat{z}}{||\hat{x}\hat{y}\hat{z}||}\right) = -1 \qquad (A3.9)$$
$$\frac{t}{||t||}\frac{t}{||t||} = \left(\frac{\hat{z}\hat{y}\hat{x}}{||\hat{z}\hat{y}\hat{x}||}\right)\left(\frac{\hat{z}\hat{y}\hat{x}}{||\hat{z}\hat{y}\hat{x}||}\right) = -1 \qquad (A3.10)$$

As you can check above the square of t/||t|| is -1 so it is a candidate to be used as imaginary unit *i* in these regards. As we commented in [4][5][6] when we have to choose an imaginary unit that does not have any direction of preference, we can use the trivector in any of its forms (one of them is t/||t||). And when the imaginary unit has a direction of preference (this happens for example in momentum operator) the bivectors are better candidates.

Anyhow, in this paper we have taken the absolute values of the coefficients, so really substituting the imaginary unit *i* by t/||t|| or directly removing it (removing the *i*, as done in the stress-energy tensor T) would not affect the spirit and the result of the paper.

In some parts of the paper appears this element related to the trivector:

$$\sqrt{\frac{\hat{t}}{||\hat{t}||}} \qquad (A3.11)$$

This would be like the square root of the imaginary unit in standard algebra:

$$\sqrt{i} \qquad (A3.12)$$

This has different solutions, taking the most straight forward one:

$$\sqrt{i} = \sqrt{e^{i\frac{\pi}{2}}} = e^{i\frac{\pi}{4}} \qquad (A3.13)$$

The important thing of this for this paper is that it modulus keeps being 1.

$$\left|\sqrt{i}\right| = \left|e^{i\frac{\pi}{4}}\right| = 1 \qquad (A3.14)$$

And therefore:

$$\left|\sqrt{\frac{\hat{t}}{\|\hat{t}\|}}\right| = 1 \qquad (A3.15)$$