Entropy denial — How small are quantum objects

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Abstract

This paper makes three points: Entropy is real. Entropy is a good measure of how small quantum objects are. Entropy increase and the Born rule happen together.
Entropy was shown to be real before atoms were shown to be real. Yet many people persist in believing that entropy is an illusion. One reason is: in classical mechanics, entropy must be an illusion. Boltzmann showed how to create and measure the illusion of entropy. Poincare and others showed that it was an illusion.
In quantum mechanics entropy can be real. The quantum representation of an object is a probability distribution over wavefunctions (a density matrix NOT A WAVEFUNCTION). There is no reason that the probabilities are any less real than other reality.
Entropy is a dimension in which a quantum object must be small. Only objects with zero entropy have a wavefunction. Another way to ask *how small are quantum objects?* is *how small must objects be to exhibit amplitude interference?* For interference ((Signal + Noise) / Signal) is exponential in entropy, measured in natural units. Thus the natural unit of entropy provides a scale of how small quantum objects are.
Introducing the "Entropy Is Real" (EIR) interpretation of quantum mechanics. Whenever entropy can increase, the probability (or rate) that a given path is taken to increase entropy is given by the appropriate Born Rule (not time reversible). Only with the residual probability, the density matrix propagates by the appropriate time reversible (Hamiltonian) (Unitary) operator.
It is time to retire the idea that entropy is an illusion.
Yet many people persist in believing that entropy is an illusion.

I know, everything after the abstract is obvious. I don't know how to separate the obvious which needs to be said, from the obvious which should be left unsaid.

Entropy is real in quantum mechanics

Entropy was shown to be real with no reference to atoms. Macroscopic Thermodynamic observations work very well. However, in order to unify physics, entropy is shown be a function of the probability distribution of atomic states and motions. This is called statistical mechanics. In classical mechanics the probability distribution represents lack of knowledge of the states but only one state can be real. Thus the probability distribution is an illusion. Further, in classical statistical mechanics, but not in quantum statistical mechanics, entropy contains fudge constant for coarse graining.
In quantum mechanics entropy is function of the probability distribution of wave functions.
There is no reason that the probabilities are any less real than the wave function amplitudes.
In quantum mechanics entropy can be measured in bits, if one wants to.
In either case entropy is the sum of $P_i \log P_i$ where P_is are the individual probabilities. The difference is whether the probabilities are real, or whether there is a single real state and the probabilities simply represent lack of knowledge of that state.

Examples of Entropy Denial

An example of Entropy denial is the claim that everything has a quantum mechanical wave-function. Anything with non-zero entropy must be represented by a density matrix, not a wave function. A density matrix may be thought of as a probability distribution over wave-functions.
Another example of Entropy denial is the claim that the propagator in quantum mechanics is time reversible. This denies the Born Rule, which is the most obvious part of quantum mechanics.

Quantum objects are small

Entropy is a dimension in which a quantum object must be small. Only objects with zero entropy have a wavefunction. Another way to ask *how small are quantum objects?* is *how small must objects be to exhibit amplitude interference?* For interference ((Signal + Noise) / Signal) is exponential in entropy, measured in natural units. Thus the natural unit of entropy provides a scale of how small quantum objects are. Note that the natural unit of entropy is extremely small.
The statistics are required to see an interference effect. Statistics increase as the square of Noise to Signal ratio (or faster if the signal is negatively correlated with the noise.) Therefore statistics is exponential based on half a natural unit of entropy.
Entropy can be measured in Bits. (Use log base 2 instead of log base e in sum PlogP.) The size of Bit of entropy is between half and one natural unit of entropy.
Thus one Bit is a reasonable scale of how small a quantum object is.

The Entropy is Real (EIR) interpretation of quantum mechanics

Here is the **Entropy is Real** (EIR) interpretation of quantum mechanics.
The quantum representation of an object is a probability distribution over wavefunctions (a density matrix). As a probability distribution it includes entropy.
Whenever entropy can increase, the probability (or rate) that a given path is taken to increase entropy is given by the appropriate Born Rule (not time reversible). Only with the residual probability, the density matrix propagates by the appropriate time reversible (Hamiltonian) (Unitary) operator. For some problems Born Rule and Hamiltonian can be evaluated sequentially. For other problems they should be integrated together.
When propagation makes a mark: some probabilities become events and the rest of the probabilities get renormalized.

I have not provided illustrative examples because I don’t want to get bogged down with Hilbert spaces or their relativistic equivalent, or spin-statistics. EIR should work on any quantum representation.

How I realized that Entropy increase triggered the Born Rule

I first realized that the Born Rule was connected with entropy when I observed that any measurement involves making a mark. Making a mark always involves settling to a local minimum in some free energy space. Settling involves dissipation, therefore increases entropy.
When I say “makes a mark” I considered saying “makes an enduring mark which could have been different”, but I decided that that was already implied by the word mark.
Enduring does not imply eternal, just not short on the timescale under consideration.
If the would-be mark’s duration is short on the timescale under consideration, the result just goes back to entropy.
Then I realized that when entropy increases without making a mark, there is a lot of overlap of wavefunctions, the ideal conditions for the Born Rule.

Embedded quantum objects

Some zero entropy quantum objects are embedded in objects having non-zero entropy. Examples of this are super fluid liquid helium and cooper pair condensates.

EIR interpretation is compatible with quantum mechanics.

Wavefunctions over whatever the appropriate space is: Relativistic spaces, Hilbert spaces, and or matrix components, present no problem for EIR. EIR does not specify the detailed form of the Born rule or the Unitary propagation operator as these do depend on the appropriate space
Quantum Field Theory which can calculate very precise differences between (zero entropy) wavefunctions for such properties as energy, presents no problem for EIR.
The rest of quantum mechanics of zero entropy objects, presents no problems for EIR.

What the EIR is NOT compatible with.

1. Conservation of information.
2. Holographic principle.
   The mathematical demonstration of the AdS/CFT duality uses an extreme system which has zero entropy.
   The holographic principle basically states that the state of a (mathematical) space is given by the state of its boundary. For a four-dimensional space-time everything in the interior is determined by initial conditions and boundary conditions. This is true for classical mechanics. It is true in any deterministic system. It is not true if you admit probabilities, or the Born Rule, or a boundary which splits an entangled state.
3. The claim that: every object has a wavefunction. A claim made by every elementary book about quantum mechanics (except a few before 1950). Every object with non-zero entropy requires a density matrix.
4. Schrodinger’s cat: (I don’t know if one can get a cat to zero entropy. But I know one thing about a cat with zero entropy: it’s dead.) Only the nucleus which may or may not decay has an amplitude. After the decay is detected (or not) what remains is just the probability of the unknown. Nothing quantum there.

Density Matrix as probability distribution

I have referred to a probability distribution over wavefunctions as a density matrix.
If you think a (quantum) density matrix is something other than a probability distribution over wavefunctions: Take an example of your density matrix. Project it onto the object of interest. Diagonalize it. You have now exhibited the density matrix as a probability distribution over wavefunctions.

One might imagine a density matrix where the basis wavefunctions evolved only by the unitary operator, and the probabilities evolved only by the Born rule.
However, the intricacies of the unitary operator, with enough interactions to increase entropy greatly, usually make this impractical to calculate. Note that each wavefunction would involve going from states more kinetic energy to states with additional particles (photon, phonon, meson, etc) and back to different states while preserving amplitude. While this can be made to work with carefully controlled beams and cavities, any interaction with entropy or escape of a particle destroys the coherence of amplitude. It is easier to calculate with collision-less wavefunctions and change the density matrix basis after any collision.

Misguided objections to the Born Rule

Let me dispel what seem to be four objections to including the Born Rule in quantum mechanics.

1. Born Rule requires an external trigger.
   Entropy is not external. It is in the probability distribution over wave functions.
2. Born Rule does not conserve energy because some forms of the Born Rule integrate over different energies of a final component (typically a photon).
   When we want to conserve energy (microcanonical view) we include all initial and final components.
3. Born Rule does not conserve energy because Noether's theorem does not apply.
   The fact that Noether's theorem does not prove that the Born Rule conserves energy in no way means that the Born rule does not conserve energy.
4. Born Rule does not conserve probability.
   This refers to when probability becomes an event, when propagation makes a mark. The remaining probabilities must be renormalized. This is just the way probabilities work, it has nothing to do with quantumness. (Bayes may understand this part of probability better than Kolmogorov.)

Quantum experiments

A typical quantum experiment has three phases: preparation propagation and measurement.

Preparation is the phase where a zero-entropy object or objects are isolated. If a decay, interaction or other trigger the Born rule occurs during what would otherwise be the propagation phase: that event is placed in the preparation phase and the event is excluded. Certain decays, with in resonant chambers where phase is maintained can be part of the propagation phase.
The atomic/particle/quantum nature of quantum mechanics is necessary for preparation to isolate a zero-entropy object or objects. I will not deal with further details.

Propagation is the phase where the unitary operator, amplitude, and phase, are employed.

Measurement everyone agrees is misnamed. Making a Mark is a better name. Here the Born rule applies probabilities become events, and residual probabilities must be renormalized.

"Weak Measurement" is a procedure where only the undesired elements make a mark, and are removed from the propagation phase. It is better classified as part of the preparation process, even though it continues for much of the propagation phase.

Entanglement entropy

I realize that entanglement, when projected onto an object which is of part of the entangled ensemble, looks like entropy.
Such an object requires a density matrix, not a wavefunction, as its quantum representation.
As soon as the object interacts with something with entropy, the entanglement disappears and entanglement entropy becomes entropy.
Entropy is extensive (the entropy of two objects is the sum of the entropy of each object).
Entanglement entropy is not extensive, the entropy of the of the entangled ensemble is zero, potentially much less than the sum of the entanglement entropy of the parts.
Note that both Entropy and entanglement entropy limit the size of quantum objects in the same way. If you choose to claim that all entropy is entanglement entropy, the limit to the size of quantum objects remains. Just to be clear, I do not think all entropy is entanglement entropy.

Interpretations of quantum mechanics

1. *Entropy Is Real (EIR)* is a theory of the PRESENT PRESENT. It does not make claims about an ABSENT PAST or an ABSENT PRESENT (see 4 to 7 below).
2. *Shut up and calculate*. This is compatible with EIR as long as it is restricted to what can be observed.
3. *Copenhagen interpretation*. (The Born rule is usually triggered by a classical measurement). Classical is external to quantum mechanics and is only an approximation. If we change the Born rule trigger to making a mark, this is compatible with EIR, except it leaves out entropy.
4. *Everett's multiple worlds*. This is a philosophical view, which does not answer how does probability convert to frequency (or an event) in OUR world. EIR answers that question. **Everett's multiple worlds** is a theory of the ABSENT PRESENT because the you in the present present is not the you in the other presents.
5. *Hidden Histories (HH)*. See **An Introduction to Consistent Quantum Theory** (Hohenberg 2010) and (Friedberg and Hohenberg 2018) and references therein, for examples of HH. Read the original article for a favorable view of HH. I have an unfavorable view. Since (Friedberg and Hohenberg 2018) proves the Born Rule, only probabilities exist in the present. Since a measurement (which makes a mark) is required to prune the hoard of hidden histories, there are more hidden histories after the measurement than before the measurement. Thus it is not possible to zero in on a single history. I have always found it absurd that when the set of histories needs pruning, there are more hidden histories after the pruning then before the pruning was needed. However, we all know that, in quantum mechanics, absurd does not necessarily mean wrong. These interpretations may be complementary to EIR.
   HH says: Un-seeable histories are eternal, entropy is an illusion.
   EIR says: Histories which leave no mark may cease to exist, present entropy is real.
   Hidden histories is a theory of the ABSENT PAST because the past can not be recovered from measurements.
6. A description of an entangled ensemble where you control measurement of every element, is a theory of a PRESENT PAST, but is not broad enough to be a quantum interpretation.
7. Recorded history is an interpretation of a PRESENT PAST, but is not quantum.

The arrow of time

Increase of Entropy, and the Born rule, have both been suggested as what gives time its direction. In EIR these are one and the same.

Future Possibilities

The calculation of certain relaxation times may be helped by the realization that the Born rule is what increases entropy. I suspect that this has already happened in some cases.

Aside on quantum gravity

Gravity includes acceleration.
Acceleration implies entropy (as shown by Bekenstein, Hawking, and Unruh). Since a wavefunction has zero entropy, it seems reasonable to expect than quantum gravity does not have wavefunctions. Perhaps a density matrix can be created using timeless wavefunctions (consistent with Wheeler's timeless Hamiltonian) and time varying probabilities with *Verlindes' Rules* in addition to the Born Rule.

References

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