On the mass-energy formula $E = mc^2$, the measurement of kinetic energy and temperature

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Kinetic energy $E_k = \frac{1}{2}mv^2$, energy acting on pressure and energy $E = mc^2$ of the mass-energy equivalence principle are both correlated to $E_k = mc^2 \times (1 - \sqrt{1 - v^2/c^2})$.

I. FORCE AND ENERGY

In classical physics, Newton's three laws are very important laws in physics. If we correctly understand the relationship between Newton's three laws and the theory of relativity, we find that the two theories are complementary.

Newton's first law, the law of inertia, describes the momentum of a moving body. Momentum p is expressed as the product of an object's rest mass (m_0) and its moving speed (v). Since v is a vector, it does not constrain direction.

$$p = m_0 v. \tag{1.1}$$

An object moving in uniform linear motion indicates that no additional energy is required. In order for this object to change its speed, it must either accelerate or decelerate. Likewise, since light travels at the speed of light (c), no additional energy is required, too.
By differentiating the above equation (1.1) with Newtonian time (t_0), Newton's second law of acceleration can be obtained.

$$F = \frac{dp}{dt_0} = \frac{m_0 dv}{dt_0} = m_0 a, \tag{1.2}$$

where F is the force and $a = dv/dt_0$ is the acceleration.

In the equation (1.2), when an object with mass (m_0) accelerates from an initial motion velocity interval v_1 and v_2, the kinetic energy E_k between the two points is given as:

$$E_k = \int_{v_1}^{v_2} Fs = \int_{v_1}^{v_2} \frac{m_0 s dv}{dt_0}, \tag{1.3}$$

where the displacement s is

$$s = v dt_0. \tag{1.4}$$

Therefore, the total energy used between the interval v_1 and v_2 is

$$\begin{aligned} E_k &= \int_{v_1}^{v_2} m_0 v dv \\ &= \frac{1}{2} m_0 ({v_2}^2 - {v_1}^2). \end{aligned} \tag{1.5}$$

When an object with mass m_0 accelerates from $v_1 = 0$ and reaches a speed $v_2 = v$, the total kinetic energy E_k requires

$$E_k = \frac{1}{2} m_0 v^2. \tag{1.6}$$

The time scale according to the theory of relativity is defined as t instead of Newtonian time t_0 as follows.

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{1.7}$$

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Substituting this into (1.4) gives

$$s = vd\left(\frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}\right). \tag{1.8}$$

From the equation (1.3) gives

$$E_k = \int_{v_1}^{v_2} \frac{m_0 dv}{dt_0}\left(\frac{v dt_0}{\sqrt{1-\frac{v^2}{c^2}}}\right) \tag{1.9}$$

Substituting $v/c = \sin(\theta)$ for integration gives:

$$v = c\sin(\theta), \quad \text{and,} \quad dv = c\cos(\theta)d\theta. \tag{1.10}$$

If we rearrange using the above expression

$$E_k = m_0 c^2 \int_{\theta_1=\arcsin(v_1/c)}^{\theta_2=\arcsin(v_2/c)} \sin(\theta)d\theta. \tag{1.11}$$

From this

$$E_k = m_0 c^2 \left(\cos(\theta_1) - \cos(\theta_2)\right). \tag{1.12}$$

The interval $(v_1 = 0, v_2 = v)$ provides

$$E_k = m_0 c^2 \left(1 - \sqrt{1-\frac{v^2}{c^2}}\right). \tag{1.13}$$

If $v = c$ then

$$E = m_0 c^2. \tag{1.14}$$

For $v \ll c$ in the equation (1.13),

$$\begin{aligned} E_k &= m_0 c^2 \left(1 - \sqrt{1-\frac{v^2}{c^2}}\right) \\ &\sim m_0 c^2 \left(1 - \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right)\right) \\ &= \frac{1}{2} m_0 v^2. \end{aligned} \tag{1.15}$$

In other words, when the speed of motion v is much smaller than the speed of light c, the energy value calculated using the theory of relativity agrees with the value predicted by Newton's second law.

II. PRESSURE AND TEMPERATURE

Pressure can be obtained from Newton's third law, the law of action-reaction. Pressure is the force that opposes the pressing force. The magnitude of the force acting on the point of action or the plane of action is equal to the magnitude of the force reacting.

$$P = \frac{F}{A}, \tag{2.1}$$

where P is the pressure, F is the magnitude of the normal force, and A is the contact area.
A change in the internal pressure indicates a change in the force acting on the outside, which means that the internal

material under pressure is accelerated and the resistance speed is increased.
There are many equations for the pressure of a gas, but the universal law is the ideal gas law[6], written as:

$$PV = nRT = nk_BN_AT = Nk_BT, \tag{2.2}$$

where P is the pressure of the gas, V the volume of the gas, n the amount of substance of gas in the gas, R the ideal gas constant, T the temperature of the gas, n the amount of substance of gas, k_B the Boltzmann constant, N_A the Avogadro constant, and N the number of gas particles.

In the equations (1.2), (2.1), (1.4) and (1.7), the pressure P is given as

$$P = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{sA}\left(\frac{m_0 v dv}{\sqrt{1-\frac{v^2}{c^2}}}\right) = \frac{m_0c^2}{V}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right), \tag{2.3}$$

where sA represents the volume. From the equations (2.1), (2.3) and the ideal gas law (2.2), we have

$$PV = E = m_0c^2\left(1-\sqrt{1-\frac{v^2}{c^2}}\right). \tag{2.4}$$

Here, E represents both unit energy and total energy, so substituting $N = 1$ from the equation (2.2) gives

$$E = k_BT, \tag{2.5}$$

and,

$$T = \frac{E}{k_B}. \tag{2.6}$$

The amount of energy divided by the Boltzmann constant gives the temperature.
This equation is still applicable even after all matter has been converted to energy. In other words, all are applicable according to the matter-energy equivalence principle. Since the speed of light is invariant, energy is also converted to frequency when it is subjected to pressure. In other words, the higher the energy, the higher the frequency, and this is confirmed by the known Planck-Einstein relation,

$$E = h\nu, \tag{2.7}$$

where, h is the Planck constant and ν is the frequency of a light ray.
So,

$$T = \frac{h\nu}{k_B}. \tag{2.8}$$

By substituting $T = T_P = 1.416784 \times 10^{32} K$, $h = 6.62607015 \times 10^{-34} Js$, and $k = 1.3806504 \times 10^{-23} J/K$ of the above equation (2.8), we get ν,

$$\nu = 2.9521 \times 10^{42}/s. \tag{2.9}$$

It tells us that no light can be compressed more than this frequency.

[1] https://en.wikipedia.org/wiki/Newton%27s_laws_of_motion
[2] https://en.wikipedia.org/wiki/Boltzmann_constant
[3] https://en.wikipedia.org/wiki/Kinetic_theory_of_gases
[4] https://en.wikipedia.org/wiki/Pressure
[5] https://en.wikipedia.org/wiki/Time_dilation
[6] https://en.wikipedia.org/wiki/Ideal_gas_law
[7] https://en.wikipedia.org/wiki/Momentum
[8] https://en.wikipedia.org/wiki/Energy
[9] https://en.wikipedia.org/wiki/Mass%E2%80%93energy_equivalence
[10] https://en.wikipedia.org/wiki/Work_(physics)
[11] https://en.wikipedia.org/wiki/Kinetic_energy
[12] https://en.wikipedia.org/wiki/Reaction_(physics)
[13] https://en.wikipedia.org/wiki/Power_(physics)
[14] https://en.wikipedia.org/wiki/Planck_relation