The fine structure constant and its links to gauge invariance in lattice spacetime, Pythagorean prime 137 and the golden ratio

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Abstract

We present a model that links the fine structure constant α to 1/137, based on the quantization of Minkowski spacetime, gauge invariance and special relativity. We derive a set of integer equations that demand 137 be a Pythagorean prime in triple and quintuple. We derive a Dirac-type equation but in lattice spacetime with quantized internal energy, momentum, and the overall relativistic mass energy. We estimate an electron's radius to be $R_e = 1.43957 \times 10^{-15}\ m$. We also found the simple empirical formula for the ratio between two electrons' Coulomb and gravitational forces at the same distance as $F_C/F_G = 2\sqrt{2} \times (432)^{16} \times 1.0008$, the Planck length to R_e ratio as $L_{Planck}/R_e = (432)^{-8} \times \left(10^2/3\sqrt{6}\right) \times 1.0008$, also the ratio $m_{proton}/m_e = 3\sqrt{2} \times 432 \times 1.0018$ between a proton and an electron, where 432 ($\sim 137\pi$) is the volume of the 4D unit hyper-cell. . Our conjecture could provide a guideline for the theoretical development of quantum gravity. This work not only elucidates the mysterious relation of prime 137 and the fine structure constant in electromagnetism, but it also sheds some light on intricate but presently unknown links to gravity and other types of forces in nature.

Main Text:

One of the baffling mysteries in physics that has remained unsolved over the last century is the dimensionless fine structure constant introduced by Sommerfeld (1,2), $\alpha = e^2/\hbar c$ which happens to be ~1/137 (3,4). This ubiquitous constant plays a very important role in atomic and molecular physics, chemistry, the evolution of stars, and life on earth. Many pioneering physicists, including Pauli, Dirac, and Feynman, have commented on this mystery (1,2). There have been many attempts in the past to explain why α takes such a value, yet none have provided acceptable physical explanations. In this work, we present a physical model to explain the origin of this value by linking it to prime number theory (5,6), with the Pythagorean prime tuple of $137 = 11^2 + 4^2$ and prime quintuple with $137 = 2^2 + 6^2 + 9^2 + 4^2$. This link is a direct result of combining gauge theory for electromagnetism, Einstein's special relativity theory for the mass-energy relation, and spacetime quantization. We will show how the existence of an indivisible fundamental unit of spacetime, and quantized gauge function enables us to derive a Dirac-type equation in a discrete lattice, which leads to quantized momentum, internal energy, and relativistic mmass-energy and charge. We will also show how this model enables us to obtain an estimate of the electron's radius $R_e = 1.3055 \times 10^{-15} m$ and a simple empirical formula $L_{Planck}/R_e = \ 15 \times (432)^{-8}$ for the Planck length $\sqrt{\hbar G / c^3}$ (7,8). Moreover, we found a formula for the ratio between two electrons' Coulomb and gravitational forces $F_C / F_G = 2\sqrt{2} \times (432)^{16}$, where 432, approximately 137π, is the volume factor of a 4D hyper-cell with 4, 2, 6, and 9 as its axial lengths. We have also found out how 137 leads to a golden ratio for 4D discrete spacetime lattice, which is very close to the traditional golden angle value (9) on a 2D continuum plane. The power dependence factor of 16 in F_C / F_G appears to be related to the nuber of independent bilinear tensor products of four independent 4D vectors in the geometrical algebra formalism of the Minkowski spacetime.

Our model is based on three key physical principles: gauge invariance (10,11) of the Lorentz group (12), Einstein's mass-energy relation, and spacetime quantization. The postulate of quantized spacetime is a key crucial component in our model which has been used in the loop quantum gravity theory (13) to treat the quantization of the gravitational field. The spacetime lattice is not rigid and could be deformed due to excitation. An elementary particle can be regarded as a type of 4D topological spacetime deformation as schematically illustrated in Fig. 1. The gauge symmetry in electromagnetism leads to charge conservation and the coupling strength is related to the fine structure constant. For gauge invariance in the continuum to hold, the gauge function $\lambda(t, \mathbf{r})$ must satisfy

$$\begin{aligned} &\mathbf{A} \;\to\; \mathbf{A} - \nabla f(t,\mathbf{r}), \quad \phi \;\to\; \phi + \frac{\partial}{c\partial t} f(t,\mathbf{r}) \\ &\frac{1}{c^2}\frac{\partial^2}{\partial t^2} f(t,\mathbf{r}), -\nabla\cdot\nabla f(t,\mathbf{r}) = 0. \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\lambda(t,\mathbf{r}) \to \sqrt{\frac{c\hbar}{e^2}}\Lambda(t,\mathbf{r}) \\ &\Psi(t,\mathbf{r}) \;\to\; \exp\left(i\sqrt{c\hbar/e^2}\,\Lambda\right)\Psi(t,\mathbf{r}), \end{aligned} \tag{2}$$

where $\sqrt{c\hbar/e^2}$ and $\Lambda(t_n, x_i, y_j, z_k)$ are dimensionless and are related to electric potential ϕ and vector potential **A**. In quantized spacetime, the coordinate needs to be expressed as quantized in terms of a fundamental length unit L and angular frequency unit $\Omega_0 = c\pi / L_0$. Since the momentum component along each axis is quantized, it needs to be replaced by operators. Because the standing wave of the fundamental mode has a wavelength equal to twice the lattice length L, and the fundamental units for the wave vector and frequency in the Fourier domain are given by $K = \pi/L$ and $\Omega = c\pi/L$. Using a discrete Fourier transform, the gauge transformation $\exp\left(i\sqrt{c\hbar / e^2}\Lambda(t\ ,\mathbf{r}\ )\right)$ becomes $\exp\left(i\sqrt{\hbar c/e^2}\,\mathbf{\Lambda}\right)$, and the operator $\mathbf{\Lambda}$ also becomes dimensionless in the 4D wave vector-frequency domain. Following Eq. (1) with a quantized gauge function for the eigenstate $|\Psi\rangle$ in discrete frequency-wave vector domain, one can express the quantized $\sqrt{c\hbar/e^2}\mathbf{\Lambda}$ in terms of four anti-commutative operators as

$$\begin{aligned} &n_0\sqrt{\hbar c/e^2}\,c\hbar\Omega\mathbf{F}_0|\Psi\rangle = \hbar K\left(n_1\mathbf{F}_1 + n_2\mathbf{F}_2 + n_3\mathbf{F}_3 + n_4\mathbf{F}_4\right)|\Psi\rangle \\ &n_0\sqrt{\hbar c/e^2}\,c\hbar\Omega\mathbf{F}_0\ |\Psi\rangle = \hbar K\left(n_5\mathbf{F}_5 + n_4\mathbf{F}_4\right)|\Psi\rangle. \end{aligned} \tag{3A}$$

With the definition of the same lattice unit L_0 with $K = \pi / L_0$, Eq. (3A) becomes

$$\left(-n_0\pi\mathbf{F}_0 + \sqrt{e^2/\hbar c}\,\pi\left(n_1\mathbf{F}_1 + n_2\mathbf{F}_2 + n_3\mathbf{F}_3 + n_4\mathbf{F}_4\right)\right)|\Psi\rangle = 0 \tag{3B}$$

$$\left(-n_0\mathbf{F}_0+\sqrt{e^2/\hbar c}\,(n_5\mathbf{F}_5+n_4\mathbf{F}_4)\right)|\Psi\rangle=0, \tag{3C}$$

where $\{\mathbf{F}_\mu,\mathbf{F}_\nu\}=2\delta_{\mu\nu}\mathbf{I}$, $\mu,\nu=0,1,2,3,4$, and $\{\mathbf{F}_5,\mathbf{F}_4\}=0$. To represent these anti-commutative and orthonormal operators $\boldsymbol{F}_\mu$, one can define $\mathbf{F}_k=\boldsymbol{\alpha}_k$ and $\mathbf{F}_0=\boldsymbol{\beta}$, which were used by Dirac in his original paper (14) on the equation for an electron, $i\hbar\,\partial\Psi/\partial t=\left(c\boldsymbol{\alpha}\cdot\mathbf{P}+\boldsymbol{\beta}m_0c^2\right)\Psi$, i.e.,

$$\begin{aligned}&\mathbf{F}_k=\begin{pmatrix}-\sigma_k & 0\\ 0 & \sigma_k\end{pmatrix}, k=1,2,3,\ \mathbf{F}_0=\begin{pmatrix}0 & -\mathbf{I}_2\\ \mathbf{I}_2 & 0\end{pmatrix}, \mathbf{F}_5=\begin{pmatrix}0 & -i\sigma_1\\ i\sigma_1 & 0\end{pmatrix}, \mathbf{F}_4=\begin{pmatrix}0 & \mathbf{I}_2\\ \mathbf{I}_2 & 0\end{pmatrix},\\ &\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\ \sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\ \mathbf{I}_2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}.\end{aligned} \tag{3D}$$

The gauge invariance in quantized spacetime leads to

$$\exp\left(-n_0\pi\,\mathbf{F}_0+\pi\sqrt{e^2/\hbar c}\,\mathbf{G}\right)|\Psi\rangle=|\Psi\rangle\ , \tag{3B}$$

where $\mathbf{G}\equiv n_{11}\mathbf{F}_1+n_2\mathbf{F}_2+n_3\mathbf{F}_3+n_4\mathbf{F}_4$. By matrix diagonalization of the matrix $\mathbf{G}$, the eigenvalues are $n_{11}, n_2, n_3,$ and n_4, and the determinant is $n_{11}n_2n_3n_4$, which is the volume of the 4D hyper-cell formed by four spacetime axes. One can show for $\mathbf{F}_0$

$$\begin{aligned}&\exp\left(n_0\pi\begin{pmatrix}0 & -\mathbf{I}_2\\ \mathbf{I}_2 & 0\end{pmatrix}\right)=\sum_{k=0}^{\infty}\begin{pmatrix}0 & -\mathbf{I}_2\\ \mathbf{I}_2 & 0\end{pmatrix}^k\Big/k!\\ &=\mathbf{I}\cos(n_0\pi)+\begin{pmatrix}0 & -\mathbf{I}_2\\ \mathbf{I}_2 & 0\end{pmatrix}\sin(n_0\pi).\end{aligned} \tag{4}$$

Because $\mathbf{F}_0{}^2=-\mathbf{I}$, this 4x4 matrix $\mathbf{F}_0$ behaves like an imaginary number i with $i^2=-1$. Eq. (4) resembles Euler's equation $\exp(i\theta)=\cos\theta+i\sin\theta$ with the arc $\theta=n_0\pi$, and for $n_0=1$ and the radius $\sqrt{\hbar c/e^2}$, one has the famous Euler identity equation of $e^{i\pi}+1=0$. Therefore, we obtained an interesting result for the quantized gauge potential in 4D spacetime, representing the generalization of Euler's equality equation from the complex plane to the 4D discrete spacetime lattice. The Euler's formula presented in this work for discrete spacetime lattice is a generalization of the usual Euler formula $exp(i\theta)=cos\,\theta+i\,sin\,\theta$ on the 2D complex plane to describe, they play an important role in describing periodical behavior in nature.

Based on Eq. (3B), for the primary fundamental model $n_0=1$, one can show

$$\exp\left(\pi \sqrt{e^2/\hbar c}\left(n_{11}\mathbf{F}_1 + n_2\,\mathbf{F}_2 \, + n_3\,\mathbf{F}_3 + n_4\mathbf{F}_4\right)\right)$$

$$= \mathbf{I}\cosh\left(\pi\sqrt{\Phi e^2/\hbar c}\ \right) + \ \frac{\sinh\left(\ \pi\sqrt{\Phi e^2/\hbar c}\right)}{\sqrt{\Phi e^2/\hbar c}}\left(n_{11}\mathbf{F}_1 + n_2\,\mathbf{F}_2 \, + n_3\,\mathbf{F}_3 + n_4\mathbf{F}_4\right), \tag{5}$$

where $\Phi = \sum_{j=1}^{4} n_j{}^2$. The invariance of the quantized gauge function leads to $\exp\left(-n_0\sqrt{\hbar c/e^2}\,(\pi/L)\mathbf{F}_0 + \mathbf{G}\,\pi/L\ \right) = \mathbf{I}$. Therefore, one has

$$\exp\left(-n_0\pi\,\mathbf{F}_0 + \pi\sqrt{e^2/\hbar c}\,\mathbf{G}\ \right) = \cosh\left(\ \pi\sqrt{\ n_0^2 + \Phi\, e^2/\hbar c}\right)$$

$$+\frac{\sinh\left(\ \pi\sqrt{n_0^2 + \Phi\, e^2/\hbar c}\right)}{\left(\sqrt{n_0^2 + \Phi\, e^2/\hbar c}\right)}\left(-n_0\,\pi\,\mathbf{F}_0 + \pi\sqrt{e^2/\hbar c}\mathbf{G}\ \right) = \ \mathbf{I}. \tag{6A}$$

From Eq. (6A), one can obtain

$$\cosh\left(\pi\sqrt{\left(-n_0{}^2 + \left(e^2/\hbar c\right)\sum_{j=1}^{4} n_j{}^2\right)}\ \right) = 1. \tag{6B}$$

The above equation can be satisfied if

$$\left(\hbar c/e^2\right)n_0^2 = \ \sum_{j=1}^{4} n_j{}^2. \tag{6C}$$

Similarly, based on Eq. (3C) one can also obtain

$$\cosh\left(\pi\sqrt{\left(-n_0{}^2 + e^2/\hbar c^2\left(n_5^2 + n_4^2\ \right)\right)}\right) = 1 \tag{6D}$$

or

$$\left(\hbar c/e^2\right)n_0^2 \ = \left(n_5^2 + n_4^2\ \right). \tag{6E}$$

Finally, from Eqs. (6C) and (6E) or Eqs. (3B) and (3C) we reach the same result for the key equations involving a set of integers as

$$\begin{aligned}&\left(\hbar c/e^2\right)n_0{}^2 = n_1{}^2+n_2{}^2+n_3{}^2+n_4{}^2\\&\left(\hbar c/e^2\right)n_0{}^2 = n_4{}^2+n_5{}^2\\&n_5{}^2 = n_1{}^2+n_2{}^2+n_3{}^2,\end{aligned} \tag{7}$$

where $\hbar c/e^2$ and n_5 must be a prime number for the solution to represent a fundamental mode, instead of a higher harmonic mode. Consequently, we must have $n_0=1$. We have found one solution with $\hbar c/e^2 = 137$ and $\{n_1, n_2, n_3, n_4, n_5\} = \{2, 6, 9, 4, 11\}$. It indicates that 137 is not just a prime but also part of a Pythagorean prime quintuple with $137 = 2^2+6^2+9^2+4^2$, and a Pythagorean prime triple with $137 = 11^2+4^2$. In addition, one has 11 as a Pythagorean quadruple. The 432 factor is the volume of the hyper-cell with side lengths of 2, 4, 6, and 9, and equals to the determinant of the matrix $\mathbf{G}$ in Eq. (3B), an invariant under unitary diagonalization transformation. In addition, the ratio $432/\pi \approx 137.50987083$ is close to the prime 137 and the golden angle $360/\varphi^2 \approx 137.50776405$ (9). Therefore, we also use this invariant volume as a constraint to check if the hyper-cell volume is close to an integer multiple of 432.

Eq. (7) represents an equation involving only integers without a unit, it can be restored to the original Eq. (3A) with $\Omega = c\pi/L$ and $K=\pi/L$, and one has

$$\alpha_0 = \frac{e^2}{\hbar c} = \frac{1}{137}, \tag{8A}$$

$$137\left(\frac{n_0\pi\hbar c}{L}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/4}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2, \tag{8B}$$

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$$\left(\frac{n_0\pi\hbar c}{L/11}\right)^2 = \left(\frac{n_0\pi\hbar c}{L/2}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/6}\right)^2 + \left(\frac{n_0\pi\hbar c}{L/9}\right)^2, \tag{8C}$$

For the primary mode with $n_0=1$, Eq. (8B) is analogous to Einstein's relation for a relativistic particle with the quantized over mass energy, internal mass energy, and momentum components as

$$\begin{aligned}&E^2 = m_0^2c^4 + c^2P_1^2 + c^2P_2^2 + c^2P_3^2\\&E = mc^2 = \sqrt{137}\,\pi\,\hbar c/L\\&m_0 c = \pi\hbar/(L/4)\\&P_1 = \pi\,\hbar/(L/2),\ P_2 = \pi\,\hbar/(L/6),\ P_3 = \pi\hbar\, c/(L/9),.\end{aligned} \tag{9}$$

The time unit length in Eq. (8B) is an unknown but scalable variable $\Omega = c\pi/L$. One might consider assigning this fundamental lattice length L to the Planck length, but the corresponding energy of ~10[18] GeV is the scale for the grand unification theory (15), unsuitable for the known elementary particles that involve electromagnetic, weak or strong interactions. Because an electron is the lightest and most accurately determined mass $m_e c^2 = 0.510998910(13)\ MeV$) (16) among the elementary particles of the Standard Model (17), we choose it as a reference for comparison.

Eq. (3A) resembles a Dirac-type equation for an electron. But, unlike the conventional Dirac equation with variable and continuous values for its mass and momentum, Eq. (3A) demands the relativistic mass energy, momentum, internal effective mass energy due to its internal structure to be quantized. Based on Eq. (8) we assign $m_e c^2$ to $L_e \equiv \pi\hbar c / \left(m_{es} c^2 \sqrt{137} \times 36\right)$ and $R_e = L_e / 2 = 1.43957 \times 10^{-15}\ m$, where the factor 36 is the least common multiple of four axial length 4, 2, 6 and 9 for constructing a perfect 4D hyper-cube from hyper-cuboids. We consider a 4D hyper-cell structure with different axial lengths to describe a building block. It can be regarded as one of the degenerate eigenstates among three possible axial orientations with the same total eigenenergy. An electron can be regarded as in coherent superposition of these degenerate eigenstate, and because of the couplings of the 4D coordinates to those anti-commutative operators, an electron with a ½ spin can be regarded to possess a hyper-dimensional **Möbius**-type structure.

An electron as a coherent superposition state of three degenerate eigenstates, and thus becomes symmetric in shape. Suppose one equates the mass energy of an electron to the electrostatic energy of two point-particles with the same electron's charge. In that case, one obtains a distance of $1.4090 \times 10^{-15} m$, very close to R_e. Both values are smaller than the theoretical classical radius $2.818 \times 10^{-15} m$, and three orders of magnitude smaller than electron's Compton wavelength $2.43 \times 10^{-12}\ m$. In comparison, the experimental proton's radius is $\sim 0.842 \times 10^{-15} m$, and the radius of a quark $\sim 0.43 \times 10^{-18} m$.

We have found simple relation for the ratio between two electrons' Coulomb and gravitational forces with $F_C / F_G = 2\sqrt{2} \times (432)^{16}$ and $L_{Planck}/R_e = (432)^{-8} \times \left(10^2 / 3\sqrt{6}\right) \times 1.0008$ for the ratio of the Planck length and R_e. We also found a simple mass ratio formula for a proton and an electron with $m_{proton} / m_e = 1836.1527 = 3\sqrt{2} \times 432 \times 1.0018$. Because of the presence of the scaling factor 432, which is the volume of the hyper-cuboid in all these formulae, therefore, in addition to the constraints in Eq. (7), we also include a constraint for the search of the prime value for $\hbar c / e^2$ with the ratio of the hyper-cell volume to the suitable prime be sufficiently close to an integer multiple of π. Because $432/137 \approx 1.0037\pi, \sim\pi$ in our screening procedure for the prime value for $\hbar c/e^2$, we impose that the remain of the quotient must be less than 0.5%. After using a screening algorithm to search all combinations of integers below10000, we have only found one primary solution with $\{n_1, n_2, n_3, n_4, n_5\} = \{2, 6, 9, 4, 11\}$ that meets the constraints. The above set of constraints demonstrates that the dimensionless fine structure constant is a geometric constant in lattice spacetime imposed by quantized gauge symmetry in the Lorentz group and special relativity.

In Eq. (8B) the time and spatial axial length units are scalable. or an electron, we chose a scaling factor 432 which we used in determining the electron's radius. We found that the ratio $432/\pi \approx 137.50987083$ is extremely close to the golden angle defined with a discrepancy of one part per hundred thousand, where $\varphi = (1+\sqrt{5})/2 \approx 1.61803399$ is the golden ratio. The golden ratio is related to the Fibonacci sequence of 1, 1, 2, 3, 5, 8, 13, 21, 34 …, etc., with each new integer being the sum of the two previous integers. As the sequence gets longer, the ratio of two consecutive integers approaches φ. The golden ratio plays an essential role in nature and the arts, and the Fibonacci sequence is also known to play a role in biological growth such as sunflowers, pine cones, etc. This work shows the intricate links of 137 and 432 to the fine structure constant for electromagnetism, and possibly other types of forces in nature.

In this work, we present a model to explain the physical origin of the fine structure constant. The link pf the inverse fine structure constant to 137 is a direct consequence of gauge invariance of the Lorentz group, special relativity, and spacetime quantization. Quantized spacetime with an indivisible minimum unit can avoid the singularity divergence (18) and vacuum catastrophe (19) inherent in conventional quantum field theory in a continuum. Based on this model with such a postulate, we obtained Einstein's relation for quantized relativistic energy, quantized rest mass energy due to an internal electric interaction, and quantized momentum. For the electron, we obtain a radius of s $1.4093\times10^{-15}\,m$. In comparison the Planck length $L_{Planck} \equiv 1.616255 \times 10^{-35} m$, we obtained a surprisingly simple ratio formula of $L_{Planck}/R_e = 8\sqrt{3}\times(432)^{-8}\times 1.004$ between the Planck length and the radius of an electron. In addition, we have also found a formula for the ratio between electrons' Coulomb and gravitational forces $F_C/F_G = 2\sqrt{2}\times(432)^{16}\times 1.0008$, where 432, approximately 137π, is the volume factor of a 4D hyper-cell with 4, 2, 6, and 9 as its axial lengths. These simple relations imply an intricate link between electromagnetism and gravity, which dictates the Planck length and the gravitational constant. Although the ideal value of 1/137 deviates slightly from the experimental value of $1/\alpha_{\exp} = 137.035999206(11)$, we believe this slight deviation of about 0.026% is likely caused by the weak interaction that breaks the symmetry of the Lorentz group, or an electron's interaction with its own field, analogous to how a "bare" electron's gyromagnetic ratio of Dirac's theory needs to be refined by quantum electrodynamics. We obtain an empirical fit with $1/\alpha = 137.035999207 = 137 + 3(\zeta + \zeta^2)/4 + \zeta^3/4 + 5\zeta^5/17$, where $\zeta \equiv 2\pi/137$, which can be used as a guideline for developing a refinement theory. A slight increase in the effective fine structure constant observed at high energy over 90 GeV is not surprising, because at such a high energy regime hadron can be generated and strong interaction would occur on top of the electromagnetic interaction. In this work, we consider only the invariance of the quantized gauge of the Lorentz group for electromagnetism. Extension of spacetime quantization to treat weak, strong and gravitational interactions, and how to quantize these gauges await further investigation. As an end note, this magic prime 137 might join π, the circle's circumference to diameter ratio, and the Euler's constant e, as crucial dimensionless constants in physics, mathematics, and the fundamental laws in nature.

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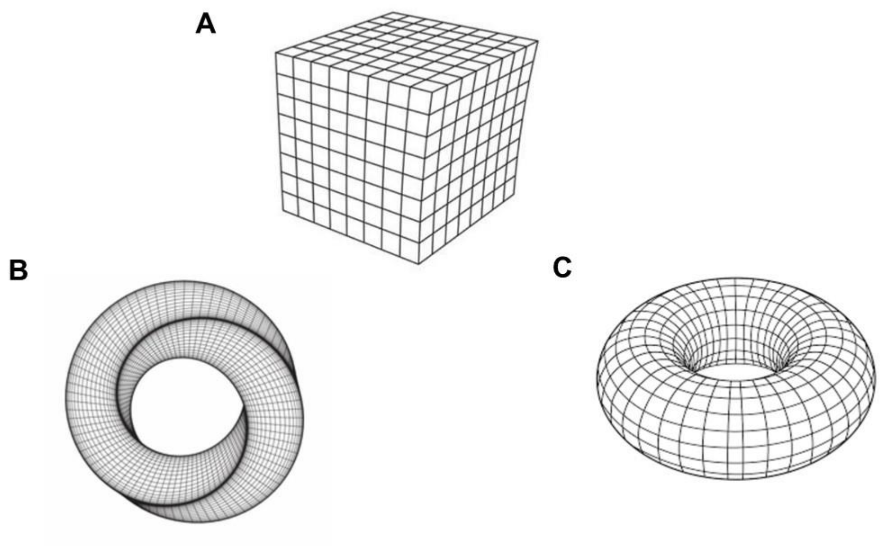

Fig. 1 .(A) Schematic illustrations of quantized spacetime but shown in 3D. The spacetime lattice is not strictly rigid but can be subject to structural deformations, such as twist, stretch, or compression can occur by external excitation. An elementary particle can be regarded as a specific type of 4D topological deformation, such as a hyper-dimensional Möbius structure similar to what is illustrated in (B) or a torus in (C) in 3D.