Counting different types of relations

Israel Andersen Yañez

Abstract

This paper is about sequences. The author uses the new theorems to approximate better the values on the sequence, the bigger the number, the better the approximation.

1 Introduction

I use the difference theorems and formula with n factors n $\in$ A-Values to solve different sequences.

2 Theorems of sequence and difference

Objects in a sequence theorem (1): Let $[N]_n$ a sequence, define by
$[N]_n$= $([R]_0, [R]_1, [R]_2, ..., [R]_n)$
$[R]_n$ is define by the last term in the sequence.
Example (1):
The sequence $[N]_n$ its the natural numbers sequence so, $[N]_n$=(1,2,3,4,5,...,n) such that $[R]_0$= 1 and $[R]_1$= 2

Difference theorem between terms of a sequence (2): Let $[D]_n$ define by $[D]_n$=$[R]_{n+1}$−$[R]_n$ for $[R]_n \in [N]_n$

Set of difference theorem (3): Let C(n) the set of the difference such that C(n)=($[D]_0$,$[D]_1$,..., $[D]_n$)

Order difference theorem (4): Let $N[D]_n$, N represent the order to sequence to solve this sequence, example the natural numbers sequence its 1-order sequence, because the difference between terms its equals to formula in the first difference. $N[D]_n$= $(N-1)[D]_{n+1} - (N-1)[D]_n$, the $0[D]_n$ its the theorem (2)
Example (2): I use the first example to show this, let $[N]_n$ and $[N]_n$ its the natural sequence $[N]_n$=(1,2,3,4,5,...,n) then the
C(n)=($[D]_0$= $2-1=1$,$[D]_1$= $3-2=1$,..,$[D]_n$= 1), then implied that $[D]_n$ for $[D]_n$ for natural sequence or $[N]_n$=(1,2,3,4,5,...,n) its equals to 1 which implies that $[D]_n$= $f(n)$ and f(n)=1 for all $n \in f$ so that the order to this difference its 1.

3 A-Values components and theorems

Sub-order of $[N]_n$ (5) : Let $[N]_n$ a sequence with $[R]_n$ terms then. $[N]_n = ([R]_0, [R]_1, [R]_2, [R]_3, ..., [R]_n)$, the sub order "L", its called $[N]_n^L$ such that $[N]_n^L = ([R]_L, [R]_{L+1}, [R]_{L+2}, [R]_{L+3}, ..., [R]_n)$ with $L \neq 0$ and positive integer.
A_n **set theorem (6) :** Let A_n a set define by $A_n = [N]_n - [N]_n^L$, then $A_n = ([R]_0, [R]_1, [R]_2, [R]_3, ..., [R]_L)$
Yañez-Set theorem (7): Let $Y - set = (a, b, c, ..., n)$ such that the elements into $Y - set$ its equals to $Y - set = ([Y]_0, [Y]_1, [Y]_2, ..., [Y]_n,)$
$[A-Y]_n$ **set theorem (8) :** Let $[A-Y]_n$ define by $[A-Y]_n = ([Y]_{A-A}, [Y]_{A-1}, [Y]_{A-2}, ..., [Y]_{A-n})$. only for $[Y]_{A-n} \in Y - set$ for n integer positive and A positive integer.
F operator theorem (9) : Let F a operator define by $F = [Y]_{n-1} \cdot [Y]_n$.
$[F]_n$ **sequence corollary (1) :** Let F operator such that define the sequence with $[F]_n = ([Y]_0 \cdot [Y]_1, [Y]_1 \cdot [Y]_2, ..., [Y]_{n-1} \cdot [Y]_n)$.
$[F]_n$ **order set corollary (2) :** Let $[F]_n$ sequence such that the order in this sequence is given by form $N[F]_n$, and that is $N[F]_n = (((N-1)[Y]_0 \cdot [Y]_1), (N-2)([Y]_1 \cdot [Y]_2), ..., (N-N)([Y]_{n-1} \cdot [Y]_n)$.
Andersen-order-Cycle sequence theorem(10): A A-order-Cycle sequence correspond a sequence with combinations of the elements in order, its called $[A-C]_n = ([A-Y]_n \cup$ ([A-Y]$_n \cdot$[A-Y]$_n$)$\cup$ A[F]$_n$).
2-order-Cycle sequence theorem (11): The 2-order-cycle sequence its called $[2-C]_n$, the "2" represent the order of this cycle and represent by $[2-C]_n = (a, b, a^2, b^2, ab)$
Introduction to 2-cycle and A-values theorem: This theorem involve the what is the "candidate" for this solution type $A^n + B^n = y$, have different types of this "candidates" or only $A^n = y$ or only $B^n = y$ everything will depend but mostly pertain to this values $(2, 3, 6, 9)$, and using this values write every sequence in approach to formula.
A-Values theorem (12) : This values in A-Order-Cycle its define by a set such that this set use a 2-order-cycle sequence and designated the values A-values set its $A - values = (2, 3, 4, 6, 9)$, correspond if a=2, and b=3.
2-Cycle with A-values formula theorem (13): Let $A^n + B^n + C^n + D^n = y$ $A, B, C, D \in$ A-values, values its a combination in the form 2-order-cycle with A-values
Odd or pair sequence theorem (14): Let a sequence $[N]_n = ([R]_0, [R]_1, [R]_2, [R]_3, ..., [R]_n)$ such that if $([R]_0, [R]_1, [R]_2, ..., [R]_n)$ have form by 2k (except 1) in all terms is a pair sequence if is $2k + 1$ form its odd, and if contain the two forms its a pair-odd-sequence.

4 Symmetric, Any, Reflexive, Transitive, Pre-order and Parital order relation

4.1 Any Relation

This refer to A002416 OEIS sequence

Elements	Any	Type of relation
0	1	not apply
1	2	2k
2	16	2k
3	512	2k
4	65536	2k
5	33554432	2k
6	68719476736	2k
7	562949953421312	2k
n	???	2k

If you note its easy to conclude that sequence its a pair sequence define by theorem (14), that means that using A-values how its only pairs results a=2, and b=2 using 2-order cycle (11) then $[2-C]_n$= (a, b, a^2, b^2, ab) with a=2 and b=2, its $(2, 2, 4, 4, 4)$ therefore this value correspond to A-Values for this sequence, now use theorem (13) obtain then formula $2^n + 2^n + 4^n + 4^n + 4^n$ but simplify because have commonly values and obtain this $2^{n+1} + 3 * 4^n$

n	k	Any	$2^{k+1} + 3 * 4^k$
0	-1.584963	1	1
1	-0.866216	2	2
2	1	16	16
3	3.6707116	512	512
4	7.2042651	65536	65536
5	11.707375	33554432	33554434.03557
6	17.207516	68719476736	$6.87195 * 10^{10}$
7	23.707519	562949953421312	$5.6295 * 10^{14}$
n	???	???	$2^{k+1} + 3 * 4^k$

Now i have a new sequence called "k" its define by $k = (-1.584963, -0.866216, 1, 3.6707116, ..., [R]_n)$ use the theorem (2) to show a pattern in this sequence, using the theorem (3) i have new set called C this set its a set of difference in k sequence, and use the theorem (4) to assign order in the differences.

n	k	1[D]$_n$	2[D]$_n$
0	-1.584963	0.718747	1.147469
1	-0.866216	1.866216	0.8044956
2	1	2.6707116	0.8628419
3	3.6707116	3.5335535	0.9695564
4	7.2042651	4.5031099	0.9970311
5	11.707375	5.500141	0.999862
6	17.207516	6.500003	0.999997
7	23.707519	7.5	1
n	$x^2 * 0.5 - 0.8$	$x + 0.5$	$x^{\frac{1}{x\pi}}$

If you note in the $2[D]_n$ the difference converges to 1 for specific result. But with this comes another formula, have this form $x^{\frac{1}{x\pi}}$, the formulas that obtain in k, $1[D]_n$ and $2[D]_n$ its a approximation to real formula. With more differences its less certain because the formulas to approach its most difficult to calculate in more differences.

4.2 Transitive Relation

This refer to A006905 OEIS sequence

Elements	Transitive	Type of relation
0	1	not apply
1	2	2k
2	13	2k+1
3	171	2k+1
4	3994	2k
5	154303	2k+1
6	9415189	2k+1
7	878222530	2k
n	???	2k or 2k+1

This sequence its interesting sequence because contains pair elements and odd elements therefore its a combination with this terms this indicate that in A-Values (12) replace with the minor value pair and minor value odd for this case a=2 and b=3, obtain this A-Values (2,3,4,9,6) and with (13) theorem obtain this formula $2^n + 3^n + 9^n$ because its 2k,2k+1,2k+1 pattern.

Elements	k	Transitive	$2^k + 3^k + 9^k$
0	-0.94393554	1	1
1	-0.31987781	2	2
2	0.957380111	13	13
3	2.291547275	171	171
4	3.765335621	3994	3994
5	5.435878945	154303	154303
6	7.308079577	9415189	9415189.0064
7	9.372449355	878222530	878222530.8366
n	???	???	$2^k + 3^k + 9^k$

Now use the difference theorems and sequence theorems to approach formula to calculate k.

n	k	$1[D]_n$	$2[D]_n$	$3[D]_n$
0	-0.94393554	0.62405773	0.653200191	-0.596290948
1	-0.31987781	1.277257921	0.056909243	0.082711939
2	0.957380111	1.334167164	0.139621182	0.057133796
3	2.291547275	1.473788346	0.196754978	0.00490233
4	3.765335621	1.670543324	0.201657308	-0.009488162
5	5.435878945	1.872200632	0.192169146	-0.010246481
6	7.308079577	2.064369778	0.181922665	-0.0085571
7	9.372449355	2.246292443	0.173365565	-0.006963975
8	11.61874180	2.41965801	0.16640159	-0.00574314
9	14.03839981	2.5860596	0.16065845	-0.0048146
10	16.62445941	2.74671805	0.15584385	-0.00409216
11	19.37117746	2.9025619	0.15175169	-0.00351685
12	22.27373936	3.05431359	0.14823484	-0.00304946
13	25.32805295	3.20254843	0.14518538	-0.00266309
14	28.53060138	3.34773381	0.14252229	-0.00233876
15	31.87833519	3.4902561	0.14018353	-0.002062434
n	$0.4019x^{1.615}$	$x^{0.4616}$	$\frac{x^{\frac{1}{x}}}{8.483}$	its most difficult to calculate

Now have formulas to calculate this functions, in case need more accuracy use the formulas in the difference theorems to obtain better approach to formula. Therefore $2^{0.4019x^{1.615}} + 3^{0.4019x^{1.615}} + 9^{0.4019x^{1.615}}$ better approach.

Elements	$2^{0.4019x^{1.615}} + 3^{0.4019x^{1.615}} + 9^{0.4019x^{1.615}}$	Percent of error
0	3	66.66 %
1	5.29463	62.2259 %
2	21.1676	38.5854 %
3	201.124	22.6678 %
4	4042.62	1.20269 %
5	144823	6.54592 %
6	$8.43905 * 10^{6}$	11.5668 %
7	$7.65681 * 10^{8}$	14.6982 %
8	$1.05238 * 10^{11}$	16.1254 %
9	$2.14527 * 10^{13}$	16.0262 %
10	$6.37316 * 10^{15}$	14.6598 %
11	$2.71813 * 10^{18}$	12.339 %
12	$1.6427 * 10^{21}$	9.3931 %
13	$1.39067 * 10^{24}$	6.1402 %
14	$1.63237 * 10^{27}$	2.8710 %
15	$2.63236 * 10^{30}$	0.1572 %
16	$5.78342 * 10^{33}$	2.7188 %
17	$1.71805 * 10^{37}$	4.6111 %
18	$6.8529 * 10^{40}$	5.6418 %
n	Formula	less percent of error

But with using table of differences obtain better approximation because its a sequence and have minor percent of error if you take the first and second term as taked.

4.3 Symmetric Relation

This refer to A006125 OEIS sequence

Elements	Symmetric	Type of relation
0	1	not apply
1	1	not apply
2	2	2k
3	8	2k
4	64	2k
5	1024	2k
6	32768	2k
7	2097152	2k
n	???	2k

How is 2k pattern have A-Values (2,2,4,4,4) with theorem (13) obtain $2^n + 2^n + 4^n + 4^n + 4^n$ but Any>Symmetric therefore obtain $2^n + 2^n + 4^n$ simplify and obtain this $2^{n+1} + 4^n$

Elements	k	Symmetric	$2^{n+1} + 4^n$
0	-1.27155330	1	1
1	-1.27155330	1	1
2	-0.44998431	2	2
3	1	8	8
4	2.820129476	64	64
5	4.954923115	1024	1024
6	7.492030201	32768	32768
7	10.49900377	2097152	2097151.9996
n	???	???	$2^k + 3^k + 9^k$

Now use the difference theorems and sequences to obtain a pattern.

Elements	k	$1[D]_n$	$2[D]_n$
0	-1.27155330	0	0.82156899
1	-1.27155330	0.82156899	-0.2715533
2	-0.44998431	0.55001569	1.270113786
3	1	1.820129476	0.314664163
4	2.820129476	2.134793639	0.402313447
5	4.954923115	2.537107086	0.469866483
6	7.492030201	3.006973569	0.493934601
7	10.49900377	3.50090817	0.49917439
8	13.99991194	4.00008256	0.49992274
9	17.99999450	4.5000053	0.4999948
10	22.4999998	5.0000001	0.4999999
11	27.4999999	5.5	0.5
12	32.9999999	6	0.5
13	38.9999999	6.5	0.5
n	$\frac{x(x-1)}{4}$	$\frac{x}{2}$ for infinity	$0.5 * (x^{\frac{1}{x}})$

And final formula for Symmetric Relation its $2^{\frac{x(x-1)}{4}+1} + 4^{\frac{x(x-1)}{4}}$ for better approximation to formula use differences.

4.4 Reflexive Relation

This refer to A053763 OEIS sequence

Elements	Reflexive	Type of relation
0	1	not apply
1	1	not apply
2	4	2k
3	64	2k
4	4096	2k
5	1048576	2k
6	1073741824	2k
7	4398046511104	2k
n	???	2k

How is 2k pattern have A-Values (2,2,4,4,4) with theorem (13) obtain $2^n + 2^n + 4^n + 4^n + 4^n$ but Any>Reflexive>Symmetric therefore obtain $2^n + 2^n + 4^n + 4^n$ simplify and obtain this $2^{1+n} + 2^{1+2n}$.

Elements	k	Reflexive	$2^{1+k} + 2^{1+2k}$
0	-1.44998431	1	1
1	-1.44998431	1	2
2	0	4	13
3	2.372648026	64	171
4	5.484060645	4096	3994
5	9.499003770	1048576	1048575.9997
6	14.49996887	1073741824	1073741827.2527
7	20.49999951	4398046511104	4398046489401.131
n	???	???	$2^{1+k} + 2^{1+2k}$

Now use the difference and sequence theorems to obtain the table of differences.

Elements	k	$1[D]_n$	$2[D]_n$
0	-1.44998431	0	1.44998431
1	-1.44998431	1.44998431	0.922663716
2	0	2.372648026	0.738764593
3	2.372648026	3.111412619	0.903530506
4	5.484060645	4.014943125	0.986021975
5	9.499003770	5.0009651	0.99906554
6	14.49996887	6.00003064	0.99996984
7	20.49999951	7.00000048	0.99999952
8	27.49999999	8	1
9	35.49999999	9	1
10	44.5	10	1
11	54.5	11	1
12	65.5	12	1
13	77.5	13	1
n	$\frac{x(x-1)}{2} - 0.5$	x	$x^{\frac{1}{x}}$

Obtain final formula $2^{\frac{x(x-1)}{2}+0.5} + 2^{x(x-1)}$

Elements	$2^{\frac{x(x-1)}{2}+0.5} + 2^{x(x-1)}$
0	2.41421
1	2.41421
2	6.82843
3	75.3137
4	4186.51
5	$1.05002 * 10^{6}$
6	$1.07379 * 10^{9}$
7	$4.39805 * 10^{12}$
8	$7.20576 * 10^{16}$
9	$4.72237 * 10^{21}$
10	$1.23794 * 10^{27}$
11	$1.29807 * 10^{33}$
12	$5.44452 * 10^{39}$
13	$9.13439 * 10^{46}$
n	$2^{\frac{x(x-1)}{2}+0.5} + 2^{x(x-1)}$

4.5 Preorder Relation

The same process that previously and obtain the first table. Use the A000798 OEIS sequence.

Elements	Preorder	Type of relation
0	1	not apply
1	1	not apply
2	4	2k
3	29	2k+1
4	355	2k+1
5	6942	2k
6	209527	2k+1
7	9535241	2k+1
8	642779354	2k
9	63260289423	2k+1
10	8977053873043	2k+1
11	1816846038736192	2k
12	519355571065774021	2k+1
13	207881393656668953041	2k+1
n	???	2k,2k+1,2k+1 cycle

How is a particular function because have odd and pair values, only the transitive function have this form and $Transitive > Preorder$ this involves that the formula obtain from A-values its $2^k + 9^k$ and create the table.

Elements	k	Preorder	$2^k + 9^k$
0	-0.5339794	1	1
1	-0.5339794	1	1
2	0.44207004	4	4
3	1.48628231	29	29
4	2.66431543	355	355
5	4.02462181	6942	6942
6	5.57629851	209527	209527
7	7.31399446	9535241	$9.53524 * 10^6$
8	9.23042248	642779354	$6.42779 * 10^8$
9	11.3190631	63260289423	$6.32603 * 10^{10}$
10	13.574258	8977053873043	$8.97706 * 10^{12}$
11	15.991027	1816846038736192	$1.81684 * 10^{15}$
12	18.56495 -0	519355571065774021	5.1935510^{17}
13	21.292088	207881393656668953041	2.0788110^{20}
n	???	$2^k + 9^k$	

This another table its for differences, with this have better approach to formula in Preorder Relation

Elements	k	$1[D]_n$	$2[D]_n$
0	-0.5339794	0	0.97604944
1	-0.5339794	0.97604944	0.06816283
2	0.44207004	1.04421227	0.13382085
3	1.48628231	1.17803312	0.18227326
4	2.66431543	1.36030638	0.19137032
5	4.02462181	1.5516767	0.18601925
6	5.57629851	1.73769595	0.17873207
7	7.31399446	1.91642802	0.1722126
8	9.23042248	2.08864062	0.16655428
9	11.3190631	2.2551949	0.1615741
10	13.574258	2.416769	0.157154
11	15.991027	2.573923	0.153215
12	18.564950	2.727138	0.149705
13	21.292088	2.876843	0.14658
n	$0.2355x^{1.7525}$	$\frac{x^{0.876}+2}{4}$	$0.121x^{0.1}$

Its only approach to real formula, value comparative and percent of error in this another table.

Elements	k	$1[D]_n$	$2[D]_n$
0	-0.5339794	0	0.97604944
1	-0.5339794	0.97604944	0.06816283
2	0.44207004	1.04421227	0.13382085
3	1.48628231	1.17803312	0.18227326
4	2.66431543	1.36030638	0.19137032
5	4.02462181	1.5516767	0.18601925
6	5.57629851	1.73769595	0.17873207
7	7.31399446	1.91642802	0.1722126
8	9.23042248	2.08864062	0.16655428
9	11.3190631	2.2551949	0.1615741
10	13.574258	2.416769	0.157154
11	15.991027	2.573923	0.153215
12	18.564950	2.727138	0.149705
13	21.292088	2.876843	0.14658
n	$0.2501x^{1.734}$	$\frac{x^{0.876}+2}{4}$	$0.121x^{0.1}$

The final formula is $2^{0.2501x^{1.734}} + 9^{0.2501x^{1.734}}$

Elements	$2^{0.2501x^{1.734}} + 9^{0.2501x^{1.734}}$	Preorder	Percent of error
0	2	1	0.5
1	2.92172	1	0.342264
2	8.00147	4	0.499908
3	43.3491	29	0.668987
4	444.185	355	0.799217
5	7753.25	6942	0.895366
6	216028	209527	0.96991
7	$9.31192 * 10^{6}$	9535241	1.02398
8	$6.09217 * 10^{8}$	642779354	1.05509
9	$5.96059 * 10^{10}$	63260289423	1.06131
10	$8.6135 * 10^{12}$	8977053873043	1.04221
11	$1.8186 * 10^{15}$	1816846038736192	0.999036
12	$5.5573 * 10^{17}$	519355571065774021	0.934547
13	$2.4369 * 10^{20}$	207881393656668953041	0.853057
n	$0.2501x^{1.734}$	Final Formula	

The final formula to error in this case correspond to Sinusoidal and is this $0.35 \cdot \sin(0.3076x + 5.1) + 0.74$ with more differences have most certain formula and results.

4.6 Partial Order Relation

Refer to A001035 OEIS sequence, that this similarly process obtain the formula.

Elements	Partial Order	Type of relation
0	1	not apply
1	1	not apply
2	3	2k+1
3	19	2k+1
4	219	2k+1
5	4231	2k+1
6	130023	2k+1
7	6129859	2k+1
8	431723379	2k+1
9	44511042511	2k+1
10	6611065248783	2k+1
11	1396281677105899	2k+1
12	414864951055853499	2k+1
13	171850728381587059351	2k+1
n	???	2k+1

This show that this sequence is a odd sequence, and its the first only odd sequence therefore implied that the A-Values for this is only the odd values, such that select the minor values on the A-values and obtain a=3 b=5, and obtain this (3,5,9,25,15), but not its "bigger" that implies obtain this $3^n + 5^n$. And obtain this table.

Elements	k	Partial Order	$3^k + 5^k$
0	-0.5183703	1	1
1	-0.5183703	1	1
2	0.29732418	3	3
3	1.60246079	19	19
4	3.23976062	219	219
5	5.14494608	4231	4231
6	7.30177628	130023	130023
7	9.70629644	6129859	6.1298610^{6}
8	12.3530579	431723379	4.3172310^{8}
9	15.2342541	44511042511	4.451110^{10}
10	18.3416088	6611065248783	6.6110610^{12}
11	21.6675482	1396281677105899	1.3962810^{15}
12	25.2055246	414864951055853499	4.1486510^{17}
13	28.9499565	171850728381587059351	1.7185110^{20}
n	??	Formula to obtain	$3^k + 5^k$

Right now use the differences theorem to obtain the next table

Elements	k	$1[D]_n$	$2[D]_n$
0	-0.5183703	0	0.8156945
1	-0.5183703	0.8156945	0.4894421
2	0.29732418	1.3051366	0.3321632
3	1.60246079	1.6372998	0.2678857
4	3.23976062	1.9051855	0.2516447
5	5.14494608	2.1568302	0.24769
6	7.30177628	2.4045202	0.2422413
7	9.70629644	2.6467615	0.2344347
8	12.3530579	2.8811962	0.2261585
9	15.2342541	3.1073547	0.2185847
10	18.3416088	3.3259394	0.212037
11	21.6675482	3.5379764	0.2064555
12	25.2055246	3.7444319	0.2016829
13	28.9499565	3.9461148	0.1975693
n	$0.2844 * x^{1.806}$	$0.8173 * x^{0.6103}$	$0.454 * x^{-0.3527}$

Therefore the final formula is $3^{0.2844*x^{1.806}} + 5^{0.2844*x^{1.806}}$, and add another table for the percent of error.

Elements	k	Partial Order	Percent of error
0	-0.5183703	1	0
1	-0.5183703	1	0
2	0.29732418	3	0
3	1.60246079	19	0
4	3.23976062	219	0
5	5.14494608	4231	0
6	7.30177628	130023	0
7	9.70629644	6129859	1.00000000191221
8	12.3530579	431723379	0.999999982413612
9	15.2342541	44511042511	0.999999955209749
10	18.3416088	6611065248783	1.0000000422024304
11	21.6675482	1396281677105899	1.0000000124238580
12	25.2055246	414864951055853499	0.9999999893302439
13	28.9499565	171850728381587059351	0.99999998323775640
n	??	The final formula	1

And the max error on this case is 1 percent.

5 Conclutions of this formulas

Conclution about this formulas obtain from the theorems of sequence in combinations with A-Values to calculate approach to formulas, is more precissely with more differences, there is still a lot to create to be able to solve all types of sequence where the next term is greater than the previous term, and all terms positive. This manuscript explain how work this values and theorems to obtain differents patterns in formulas.