Hydrogen spectral series as Harmonic overtones of a single fundamental wavelength.

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Abstract

In this paper all lines of the six observed Hydrogen spectral series in and around the optical part of the electromagnetic spectrum are accurately and correctly modelled in a first order approximation using harmonic overtones and subharmonics (undertones) generated from a single fundamental wavelength. The fundamental wavelength predicted in this paper is the Lyman alpha wavelength of 121.57nm.

1. Introduction

Before the Bohr model was proposed, theorists of the day including Maxwell, Rydberg, Balmer, Ritz and others thought that spectral lines in the emission and absorption line spectra of elements were harmonic overtones and subharmonics produced from underlying fundamental wavelengths. However early attempts to accurately model these spectral lines as such were unsuccessful as no obvious harmonic structure in the observed Hydrogen spectral lines could be seen. In that no mathematical relationship using harmonics could be supplied for the whole integer progression of n1,2,3 etc seen in the Rydberg formula. An unfortunate failure by theorists of the day which the harmonic model presented here in this paper corrects. As a result, in the early 20th Century the Bohr model seemed to be the only successful model to explain the whole numbers seen in the n1, n2,...series of the Balmer and Rydberg equations. Bohr theorised that the n series found, but not understood, by Balmer and Rydberg, could be explained by an electron shell model. Where electrons jump between various fixed orbital shells in discreet quantised amounts.

Although it soon became apparent even the Bohr model was only successful in modelling the Hydrogen spectral series. As when it was applied to Helium or any of the other heavier elements it failed to predict most of the observed spectral lines. Despite this failure it is still used today as the basis for the current electron valence shell model. Although it only works if propped up with an ever increasing complex of various orbital shells, imaginary quantum effects from spin, and other interactions between the still ever growing list of assumed atomic particles that never quite manages to explain the spectral signatures of elements. My analysis in this paper shows that all the observed wavelengths of all 30 lines of the 6 Hydrogen series [1] can be correctly predicted using only harmonic overtones and sub harmonic wavelengths produced by just one fundamental wavelength f. That fundamental wavelength being the Hydrogen Lyman alpha spectral line at 121.57nm. Ironically the clues to the harmonic structure of the hydrogen spectral series can be found in the original Rydberg formula

$$\frac{1}{w} = R\left(\frac{1}{n1^2} - \frac{1}{n2^2}\right)$$

An unfortunate missed opportunity for theoretical physics as had the theorists of the day seen that the Rydberg formula does indeed show the harmonic structure in the spectral lines of the hydrogen atom, we would not have had to resort to the electron valence shell model. A model which incidentally still struggles to accurately predict many of the element's spectral lines. The intent is to show that a harmonic model where the lines are created by overlapping wavelengths of emr can explain the Hydrogen spectral series without having to resort to the Bohr shell model.

1. Analysis

The first step to understanding the harmonic structure of the hydrogen atom comes from the Rydberg formula itself. In the formula the numerators of 1/n1^2 and the numerator of the sum of 1/n1^2 -1/n2^2 always give us a whole number fraction >1. A fraction which also gives us

the harmonic relationship between the wavelength of any particular observed line and the corresponding shorter wavelength of its series limit.

(For instance; 1/n1 ^2 - n2^2 becomes 9/36 -4/36 = 5/36)

One can then assemble a table (table 1) of these harmonic fractions of all the lines of the six hydrogen series. At which point the harmonic structure and harmonic relationship becomes apparent. In particular note how the wavelength of the Lyman alpha line at 121.57nm is a 4/3 fraction of its Lyman limit wavelength of 91.175nm. Or, that multiples of the Lyman limit wavelength also give us the limiting wavelengths for each of the other five series. Also notice how multiples of the Lyman alpha line of 121.57nm also give us other lines in the series. These being the lines of Balmer b, Paschen c, etc, as seen in a diagonal line from top left to bottom right in table 1.

Table 1

	Ly	Ba	Pa	Br	Pf	Hu
a	121.57 121.57/∞ =1.33 4/3 =1.33 4/3	656.3 656.3/∞ =1.8 9/5 = 1.8 9/5	1875 1875/∞ =2.24 16/7 = 2.24 16/7	4051 4051/∞ =2.777 25/9 = 2.777 25/9	7460 7460/∞ =3.2727 36/11=3.2727 36/11	12370 12370/∞=3.769 49/13 = 3.769 49/13
b	102.57 102.57/∞=1.125 9/8 = 1.125 9/8	486.1 486.1/∞=1.33 4/3 = 1.33 16/12	1282 1282/∞=1.5625 25/16=1.5625 25/16	2625 2625/∞ = 1.8 9/5 = 1.8 36/20	4654 4654/∞=2.0416 49/24=2.0416 49/24	7503 7503/∞=2.2857 64/28=2.2857 64/28
c	97.254 97.254/∞ = 1.066 16/15 = 1.066 16/15	434 434/∞=1.190 25/21=1.190 25/21	1094 1094/∞ =1.33 4/3 =1.33 36/27	2166 2166/∞=1.4848 49/33=1.4848 49/33	3741 3741/∞=1.6410 64/39=1.6410 64/39	5908 5908/∞=1.8 9/5 = 1.8 81/45
d	94.974 94.974/∞=1.041 25/24=1.0416 25/24	410.2 410.2/∞=1.125 9/8 = 1.125 36/32	1005 1005/∞=1.225 49/40=1.225 49/40	1944 1944/∞=1.33 4/3 =1.33 64/48	3297 3297/∞=1.4464 81/56=1.4464 81/ 56	5129 5129/∞=1.5625 25/16=1.5625 100/64
e	93.780 93.78/∞=1.028 36/35=1.028 36/35	397 397/∞=1.0888 49/45=1.0888 49/45	954.6 954.6/∞=1.1636 64/55=1.1636 64/55	1817 1817/∞=1.2416 81/65=1.2416 81/65	3039 3039/∞ =1.33 4/3 =1.33 100/75	4673 4673/∞=1.4235 121/85=1.4235 121/85
limit ∞	91.175	364.6	820.4	1458	2279	3282

∞=91.175 ∞ x 4=364.6 ∞ x 9=820.4 ∞ x 16=1458 ∞ x 25=2279 ∞ x 36=3282
(Lyman limit)

121.57	—— nm
121.57/∞ =1.33	—— wavelength / limit
4/3 =1.33	—— harmonic fraction as decimal
4/3	—— normalised fraction

2.1. An interesting pattern that also becomes apparent in table 1 is how much more each successive subharmonic fraction increases as one moves either down or across the table. For instance in column 1 at the top left of table 1, the difference between the numerator or denominator of L-alpha (4/3) and L- beta (9/8) is 5.

And then going down in column 1 the difference between L-beta (9/8) and L-c (16/15), for both numerator or denominator is 7. This is increase of 2 (from 5-7). The pattern continues down with the difference between each two successive lines increasing by 2 each step down. The same pattern is seen duplicated going across. As seen below in table 2.

Lyman	Balmer	Paschen	Brackett	Pfund	Humphrey
5	7	9	11	13	15
7	9	11	13	15	17
9	11	13	15	17	19
11	13	15	17	19	21

What is most notable about table 2 is how presenting the data as harmonic fractions allows one to easily and accurately predict each successive shorter wavelength beyond those usually listed in any reference for each series. Take for example the Balmer series. The shortest wavelength Balmer-e, listed in the available reference [1] is 397nm. Its wavelength is 49/45 of the Balmer limit of 91.175 as listed in Table 1. If one continues with this pattern then the fraction for sixth line of the Balmer series would have its denominator and numerator increase by 15 to 64/60.

Which when multiplied by the Balmer limit of 364.6 gives a predicted line of 388.9.

64/60 = 1.06666666667

1.06666666667 x 364.6 = 388.9nm

Which is confirmed as the next shortest wavelength line in the Balmer series [2].

Using the same method once again one gets a prediction of a fraction for the next Balmer line of a fraction 81/77. Which in turn correctly gives the wavelength for the next observed line at 383.5nm. And once again confirmed in the NIST Hydrogen series catalogue. Etc.

There are also other lines in the hydrogen spectra not listed in the original 6 series. Yet they can still be shown to be generated as harmonics of other shorter wavelengths in the Hydrogen series. This is an interesting point to follow up in any future study as it indicates that the six series of hydrogen spectral lines are only just part of an overall harmonic structure in the hydrogen atomic spectra.

The fractions in table 1 only show the harmonic relationship as whole number fractions between all the 30 Hydrogen spectral lines in the 6 series to each of their limiting wavelengths. However it becomes apparent from table 1 that the common wavelength to all of the series limits and spectral lines is the Lyman alpha line at 121.567nm and not any of the six Lyman limit wavelengths. If one then designates this Lyman alpha line of 121.57nm as the fundamental wavelength f for all the Hydrogen series it is possible to then generate accurate harmonic based predictions for all the observed lines in the 6 series. This is probably the reason why in Rydbergs time the harmonic structure of the hydrogen series was not noticed. Today we have six series of Hydrogen lines to study and help us see the harmonic patterns in the data when laid out in table form. In Rydbergs day only the Balmer series was available for analysis.

Once Lyman alpha 121.567nm has been designated as f, the fundamental wavelength for the Hydrogen series can be used to generate the remaining 4 Lyman series lines from a set of harmonic fractions of f. And from this all the lines in the other 5 series, and others not usually cited in reference, can be predicted using this harmonic model of the atom. These harmonic fractions are listed in a separate table 3.

2.2. Harmonic overtones and Subharmonic undertones

At this point it is important to explain how the harmonic model calculates each of the 30 harmonics and 'sub-harmonic' overtone wavelengths in the six series. In table 3 the "harmonic" lines, or sometimes called overtones, are found in the first column. These are the Lyman series. Which are direct harmonic fractions of the fundamental wavelength of Lyman alpha 121.567nm. These fractions are listed as whole number fractions <1 in column 1 in table 3.

The subharmonic lines constitute the next five series, Balmer to Humphrey. These are created when the 'beats' of two or more different shorter wavelengths lines or existing harmonics of 121.57 occur at the same point in time and constructively interfere with each other. So for instance the Balmer line of 486 is created when both f and the 2/3 harmonic (40.5nm) of the fundamental wavelength f (121.5) overlap and interferes with the Lyman line of 97.2nm. As shown in the figure 1 where time t is the horizontal axis.

Figure 1
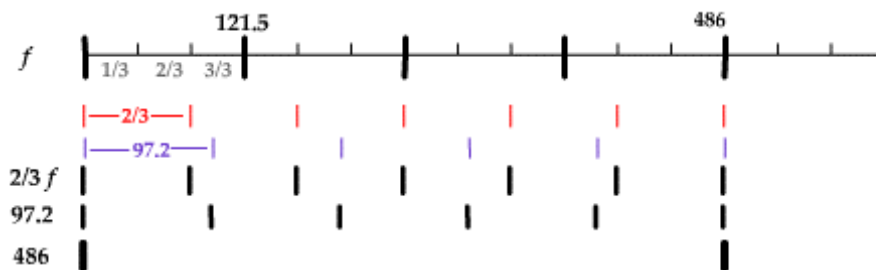
Notice that 2/3 harmonic and 97.2nm only have their 'peaks' or 'beats' overlap once every 486nm. In other words they constructively interfere with each other only every 486 nm. And subsequently create a new line where they are both in sync at each successive 486nm interval. Using this method and the fractions shown in table 3, one can correctly predict all of the observed 30 lines in the Hydrogen series. And others not usually listed in the reference.

2.3. The next step is to make a table (table 3) of the 30 lines of the 6 Hydrogen series showing which harmonic fractions of the fundamental wavelength f (121.57) can be divided into each of the 30 Hydrogen spectral lines. Notice from the table that at least 1 of the harmonics of f can be divided into each spectral line by a whole number amount. In some cases multiples of that harmonic are also divisible into that particular line. For example the Pfund alpha line at 7460nm is divisible by not just the harmonic of 1/11. But also by the multiples 3/11, 5/11, and 9/11.

In addition to these harmonics of f, the table includes examples of which of the other harmonics or any shorter wavelength line from the 6 series can also be divided into that particular line. For example the Brackett d line of 1944nm is divisible by whole integers of not just the 1/3 and 2/3 and 3/3 harmonics of f, but also by whole amounts of the Balmer b line of 486.1 nm. This is fundamental to the success of the harmonic model. As for all the subharmonic lines in the series to be visible in the spectra, they must be generated by the "beats" of at least one harmonic of f, or a shorter wavelength lines from table1. In addition to one other whole number harmonic of f. In other words a minimum of two seperate wavelengths are needed to generate a new spectral line.

Table 3

For additional details and description for table 3 see section **3. Supplementary information**

	Ly	Ba	Pa	Br	Pf	Hu
a	121.57 1/1	656.3 1/5 93.7 x 7	1875 1/7 93.7 x 20	4051 1/3 1/9	7460 1/11 92.09 x 81	12370 1/4 1546.25x15
b	102.57 27/32 5/8	486.1 1/4 97.2 x 5 121.5 x 4	1282 1/14 91.6 x 14	2625 1/5 656.3 x 4 97.2 x 27	4654 1/7 94.97 x 49	7503 1/7 1875 x 4
c	97.254 4/5 4/5	434 1/7 5/7	1094 1/19 121.5 x 9	2166 1/5 1/6,1/11	3741 1/9 1/31	5908 1/5 656.3 x 9
d	94.974 25/32 7/8	410.2 1/8 102.5 x 4	1005 1/11 1/19	1944 1/3 486.1 x 4 121.5 x 16	3297 1/17 824 x 4	5129 1/11 1282 x 4 102 5 x 50
e	93.780 27/35 32/35	397 1/11 49/81	954.6 1/7 1/13	1817 1/19 1/37	3039 1/5 94.17 x 32 121.5 x 25	4673 1/9 97.3 x 48
limit ∞	91.175 3/4	364.6	820.4	1458	2279	3282

2.4 This paper focuses primarily on the 30 well known spectral lines in the Hydrogen spectral series. However there are many more observed Hydrogen lines and they too can be modelled as harmonics of the single fundamental wavelength f. One particular line to note that is not included in the usual list of Hydrogen spectral series is the line at 1736nm seen in the NIST catalogue. It is the next in the Brackett series below 1817 nm. Being a combination of 1/7 and 1/11 harmonics of f, and the Balmer-c line of 434nm constructively interfering with each other to produce a new sub harmonic at every 1736nm interval. Separately it is worth noting how the harmonic fraction 100/84 or 1.19 calculated for 1736 is also part of a diagonal pattern in table 1 going from upper left to lower right. As it is also seen just below the 4/3 harmonic fraction line of 486.1 in the Balmer series. It seems an underlying pattern of harmonic fractions (4/3, 9/8 etc) are radiating out from the fundamental wavelength line of Lyman alpha at 121.57 if presented in table form as is done in table 1.

This also hi-lites the necessity in some cases to use a Hydrogen spectral line of a shorter wavelength than the usual listed in the 6 series in common reference [1] to make a successful prediction based on harmonics. For instance in table 3 the Pfund-alpha 7460 can only be

successfully predicted using the harmonic model with a 1/11 multiple of f, but one must then use it in conjunction with the Lyman line at 92.0947nm [2]. A Hydrogen line not normally listed in most reference but available from the NIST catalogue. The same occurs with the line at 1282nm where a Lyman line not usually listed gives 91.58 x 14 =1282nm

3. Supplementary information for Table 3

Column 2-6 shows, in red the harmonic fraction of 121.57 which can be divided by a whole amount into that specific line. And the numbers in blue show the common shorter wavelength line of another of the Hydrogen series which can also be divided into it. As outlined in section **2.3** of this paper.

Column 1 shows two fractions for each Lyman line from b-e. The red fraction is derived using the following formula: wavelength/f. So for example the Lyman b line divided by the Lyman alpha line is 102.57/121.57=27/32. These red fractions illustrate the harmonic relationship between the alpha line 121.57 denoted as the fundamental wavelength f in this paper and the other shorter wavelength lines in the Lyman series.

The blue fractions also show the harmonic relationship between the fundamental wavelength f and the other Lyman series lines w using the following formula: f-w/f-Lyman limit. So for example the blue fraction for the Lyman b line of 102.57 can be calculated as follows: 121.57-102.57/121.57-91.175=5/8. The blue fraction is essentially a new harmonic of the 1/4 harmonic of 121.57 indicating that the Lyman series are in fact new harmonic undertone fractions of the 1/4 harmonic of 121.57. To confirm this the "blue fraction" method can correctly predict further lines beyond those usually shown in the Lyman series. For example, the next predicted line for the Lyman series will be a 15/16 harmonic of the 1/4 harmonic of 121.57. Which works out, using the above 'blue fraction' formula, to 93.07468nm. As confirmed by the NIST database.

This is interesting to note as the 1/4 harmonic of f, 30.395nm , is also a common fraction to many other lines in the Lyman Hydrogen spectral series. Including the nm wavelengths of 364, 486, 820, 1094, 1458, 1945, etc. And 40.5233, the 1/3 harmonic of 121.57 is also a common fraction of many other of the Hydrogen lines. Including 364, 486, 1094, 1458, 3039, 3282, 4052, etc. Supplying additional evidence to support the veracity of the harmonic model of the atom as presented here in this paper.

4.Conclusions.

The harmonic atomic model outlined in this paper is successful in accurately predicting all the 30 lines in the hydrogen series. And also the other hydrogen lines not listed but seen in the NIST catalogue. The intent is to supply an alternative model to the electron shell model. Removing the necessity of electron shells where electrons jump by discreet energy levels and replacing this with a model based on harmonics in a model of a resonating wave based atom where oscillators at fundamental frequencies replace electron shells.

The harmonic resonating atom and its fundamental wavelength(s) can explain how these hydrogen spectral lines and there parent series are created. Lines created by constructive interference between two or more overlapping wavelengths. And not by an electron jumping between quantised electron shell levels. The assumption is that in the optical part of the spectrum at least, where Hydrogen has one fundamental frequency in relationship to its atomic number of one, then helium will have two fundamental frequencies , lithium three, etc. Preliminary investigations into this relationship between the number of fundamental wavelengths f, and the atomic number show just this relationship and will be published at a later date.

In this model the atom is considered to be a nodal point in space consisting of one or more overlapping resonant wavelengths, or as described in this paper, one or more overlapping "fundamental" wavelengths. Rather than a central nucleus surrounded by a quantum cloud of electrons. The absorption and emission properties seen in the optical part of the spectrum are created by these resonant wavelengths. Which respond to input energy and re-emit (or absorbs) specific harmonic wavelengths of radiation in response. The assumption is that in

and around the visible part of the spectrum one of these fundamental wavelengths of the Hydrogen atom, and denoted as f in this paper, is the Lyman alpha line at 121.567nm. And that it is this one line which generates all the other spectral lines seen in Hydrogen emission and absorption optical and NIR spectra.

Reference
[1] Wikipedia: Hydrogen Spectral series
[2] NIST Atomic Spectra Database. Lines, Data Hydrogen