Detection of the Gravitational Wave of the Crab Pulsar in the O3b series from LIGO

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After compensation for phase modulation and frequency drift, the pulsar's GW can be detected in the records of all three interferometers. The signatures agree with the known values measured with electromagnetic waves.

1 Introduction

All previous searches for continuous gravitational waves (GW) using different methods have been unsuccessful. This is amazing because the LIGO interferometers are extremely sensitive and have recorded large amounts of data. The present study has for the first time succeeded in detecting the gravitational waves of the crab pulsar using standard methods of communications engineering.

This is also due to the fact that the frequency is precisely known from electromagnetic wave observations. Astronomers at the Jodrell Bank Centre for Astrophysics have been observing the pulsar, which spins about 30 times per second. In communications, very weak signals are always received using the same principle: Interfering noise is removed with extremely narrowband filters. The bandwidth of the filter can only be selected to be particularly small if the signal frequency is constant. Unprepared, no GW meets this requirement, as some effects increase the minimum bandwidth:

- The Crab pulsar radiates energy and in 2019, the frequency $f_{GW} \approx 59.225$ Hz of the radiation decreases with the velocity $\dot{f}_{pulsar} = -7.37 \times 10^{-10} s^{-2}$. One must double the values given in [1] because for theoretical reasons $f_{GW} = 2 f_{spin}$ holds.
- Since the LIGO interferometers rotate once around the axis of the Earth in 24 hours, the Doppler effect produces a small periodic frequency shift of $\Delta f \approx \pm 85\ \mu$Hz (see equation (1)).
- Since the pulsar lies close to the plane of the ecliptic, the Doppler effect produces very large periodic frequency changes of about ± 6 mHz because of the high orbital velocity of the Earth around the Sun (about 30 km/s). This frequency uncertainty can be reduced by choosing special observation periods.
- Occasionally and at irregular intervals, the pulsar changes its rotation frequency by several microhertz. Such events are unlikely to interfere if the investigation is limited to a few days.

The prospects of detecting the GW of the Crab pulsar improve if all known modulations are identified and eliminated to reduce the interfering noise by minimizing the signal processing bandwidth.

2 The observation period

Each summer, the Sun passes through the constellation Taurus, which contains the explosion cloud M1 with the Crab pulsar as its center (ecliptic longitude $\lambda = 84.1^o$, ecliptic latitude $\beta = -1.3^o$). On June 16, Earth - Sun - M1 form almost exactly one line. On this day, the Doppler shift in frequency caused by the high velocity of the Earth as it orbits the Sun is close to zero. This is a good opportunity to identify the GW of the Crab pulsar because the exact value of the Doppler shift also depends on how fast the GW propagates. There is no experimental confirmation yet for the usual conjecture $v_{GW} = c$.

All of the following measurements are made in a second time window with similar properties: During a short period of a few days around December 16, the frequency change caused by the Earth's orbit is very small and changes proportionally to the time [2]. On this date no sun interferes between pulsar and earth and therefore the discussion if and how the sun influences the propagation of GW is omitted.

A measurement period of at least 96 hours ensures a sufficient frequency resolution for the following investigation. During this time, the Earth rotates four times around its axis and an interferometer at the prime meridian of the Earth receives twice a day – at midnight and noon – the "true" frequency of the GW generated by the Crab pulsar. At these times, the frequency shift caused by the Doppler effect due to the Earth's rotation disappears. If this instrument had an isotropic antenna sensitivity, it would measure the maximum redshift at 6h UTC and the maximum blueshift at 18h UTC.

There is no antenna at this reference point, and for the Hanford, Livingston and Virgo interferometers, the times of the extremes of daily redshift and blueshift are shifted according to their geographic positions. Table 1 shows the expected results for measurements repeating at 24-hour intervals in mid-December.

Position	Redshift μHz	at the time UTC	Blueshift μHz	at the time UTC	max. sensitivity UTC
$\lambda = 0^o, \varphi = 0^o$	-85	6.0 h	85	18.0 h	0 h
Virgo	-61.5	5.3 h	61.5	17.3 h	23.3 h
Livingston	-73.1	12.05 h	73.1	0.05 h	6.05 h **(good)**
Hanford	-58.5	13.96 h	58.5	1.96 h	7.96 h

Table 1): *Target values of the Doppler shift of the GW of the Crab pulsar on December 16. For other days, the given times shift by -237 s/day (sidereal system). The Livingston antenna provides the best signal daily at 6 o'clock (UTC) because its geographic latitude is about the same as the astronomical declination of the Crab pulsar.*

Each interferometer has a vertical main lobe and is insensitive when the source of the GW is close to the horizon (node of the antenna pattern). But this is the best time period to measure the maximum daily frequency shift (columns 2 and 4 of table 1). Correspondingly, the resulting declination of the GW source can be determined inaccurately.

The opposite is true for the times in columns 3 and 5, because these can be calculated from the zeros of the phase shift ϕ_{day} (see equation (5)). At certain times (column 6) the main lobe of the antenna pattern points approximately into the direction of Crab Pulsar and the corresponding interferometer receives a particularly strong signal during a short period of time. The Livingston antenna is well positioned with respect to Crab Pulsar, but was out of service on 2019-12-16.

When receiving a GW, the position of the Sun does not matter. Therefore, the sidereal daylength of 23.93447192 hours applies in all calculations.

3 Signal processing

The LIGO interferometers have not yet measured multi-year records, so one must look for the GW of the Crab pulsar in short data segments of only a few days duration. This time span is sufficient to measure the periodic frequency shift as a result of the rotation of the Earth. The small relative frequency change $\Delta f / f_{GW} \approx 10^{-6}$ (see equation (1)) is easier to measure if the signal frequency is greatly reduced. Therefore, one mixes f_{GW} with a locally generated frequency f_{Osz} (Figure 1) to produce a lower frequency f_{ZF} (heterodyning). For the frequencies, $f_{ZF} = |f_{GW} - f_{Osz}|$ is valid.

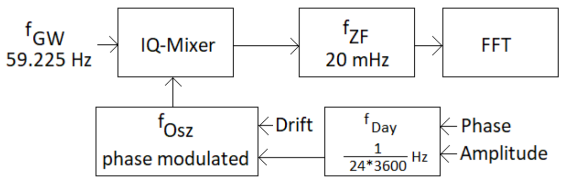

Figure 1): *Principle of the MSH method: The parameters for frequency drift and phase modulation of the auxiliary oscillator are iterated until the amplitude of the spectral line at f_{ZF} reaches a maximum.*

Usually, the value f_{Osz} is constant in order not to modify the modulation content of the signal. In the search for GW, the opposite is true: one must remove the known but unwanted phase modulation (PM) in order to reduce the bandwidth. This leads to the development of the "Modified SuperHet" (MSH): One modulates the frequency f_{Osz} with the goal of obtaining a *constant* diffence frequency f_{ZF}. When the modulation of the received signal and the oscillator coincide, the "picket fence-like" spectrum of a PM signal turns into a single high-amplitude spectral line. Illustratively speaking: The many spectral lines inside the bandwidth are rearranged so that they add up to a large total length. The MSH method does not require spectral decomposition of the GW signal as an intermediate step; it combines the sidebands of *one* signal to produce a single, strong spectral line. Neighboring signals are distorted.

An analysis shows the advantages of reducing the signal bandwidth by the MSH method: the daily rotation of the interferometers around the earth axis with $f_{day} = 11.6\ \mu$Hz produces a phase modulation with the peak frequency-deviation

$$\Delta f = f_{GW} \cdot \left(\sqrt{\frac{v_{GW} + v_{equator}}{v_{GW} - v_{equator}}} - 1 \right) \cdot \cos(\varphi) \cdot \cos(\delta) \leq 85\ \mu\text{Hz} \tag{1}$$

The variables mean:
v_{GW} is the propagation velocity of the GW, presumably c

$v_{equator} = 464$ m/s
φ is the geographic latitude of the interferometer
$\delta = 22^o$ is the declination of the GW source Crab pulsar

The aim of this study is to confirm the GW of the Crab pulsar with all properties (table 1). This requires that the phase modulated signal is processed with the Carson bandwidth BW to avoid distortions.

$$BW \geq 2(\Delta f + f_{day}) = 193\ \mu\text{Hz} \tag{2}$$

The intermediate frequency f_{ZF} must be sufficiently high to allow this bandwidth. The wide range BW contains the 17 major spectral lines that represent the spectrum of the Crab pulsar, unwanted perturbations, and additional spectral lines produced by other, previously undetected pulsars. The MSH procedure compensates for the phase modulation and the frequency drift of *one* signal frequency. The 17 spectral lines recombine to a single one and the bandwidth may be reduced from 200 μHz to about 3 μHz. The consequences:

- The amplitude of the noise is reduced by the factor $a_1 = \sqrt{200/3} \approx 8$
- The MSH process ensures that the energy content of the GW, which was previously distributed over 17 spectral lines, is now concentrated in the *unmodulated* carrier frequency f_{ZF}. Therefore, the amplitude increases by a factor of $a_2 \approx 3$.
- It is not necessary to identify, measure and recombine of the phases and amplitudes of about 17 spectral lines in the noise.

Overall, the amplitude of the single spectral line at f_{ZF} can be $a_1 \cdot a_2 = 24$ times higher than the amplitude of the surrounding noise. Thus, MSH enables the detection and analysis of signals below the noise floor. To my knowledge, no comparable method has ever been used to remove phase modulation from a signal.

4 The sensitivity of the LIGO interferometers

The S/N determines the quality of the signal reception. The average noise amplitude A_{noise} depends on the inherent noise of the receiver, described by the PSD value, and the bandwidth BW of the receive channel.

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{3}$$

The LIGO interferometers have PSD values of around $3 \times 10^{-46}\ s^{-1}$ [3,4]. One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because then the necessary recording period T_{min}, which the filter needs to settle down, increases. This relationship was first formulated by Küpfmüller and is reminiscent of the Heisenberg uncertainty principle.

$$T_{min} \cdot BW \geq 0.5 \tag{4}$$

Each record of LIGO [5] lasts 4096 seconds and limits the spectral resolution to 122 μHz. Comparing this value with the result of equation (1) shows that one-hour records are too short to detect phase modulation in the diurnal rhythm. A minimum recording duration 96 hours improves the frequency resolution to 1.5 μHz. Filtering the received data with this bandwidth, the noise floor has the dimensionless value 2×10^{-26} according to formula (3). Since rotating neutron stars are expected to have strains around 10^{-26} [6], the GW of strong sources should be detectable in the records of the LIGO interferometers.

For unknown reasons, the interferometers interrupt the data recording very frequently. This corresponds to digital modulation and generates many and strong sidebands, which degrades the S/N of all received signals.

5 Mathematical Modeling

The goal of the investigation is to identify a signal in the records of the LIGO antennas which has the properties of the GW of the Crab pulsar. For this purpose, one generates an auxiliary frequency f_{Osz} and iterates its modulation until it agrees in all properties with the suspected GW (lower part of the figure 1). Then, f_{ZF} is constant and has especially large amplitude because the total energy of the GW is concentrated in a narrow frequency range. The auxiliary oscillation f_{Osz} is generated by the following approach:

$$f_{Osz} = \sin(2\pi t(f_{GW} + f_{ZF} + \dot{f}t + A_{day} \cdot \sin(2\pi t f_{day} + \phi_{day}))) \tag{5}$$

This equation contains a second oscillator of frequency f_{day} to generate the PM in daily rhythm. The parameters have the following meaning:

5.1 Frequency of GW (f_{GW})

We take the initial value of the frequency for 2019-12-15 from the table [1] and calculate the initial value for other dates using the result of section 5.3. This value is corrected in steps of 3 μHz until we find a high amplitude signal at f_{ZF} that has the expected properties of the GW of the pulsar.

5.2 Intermediate frequency of the MSH method (f_{ZF})

This value is arbitrary, but should be as small as possible so that the daily frequency changes $\Delta f/f_{ZF}$ can be easily seen. Minimum value is the necessary Carson bandwidth BW (equation (2)).

5.3 Frequency drift of GW ($\dot{f}$)

In addition to the inherent frequency drift of the Crab pulsar of $\dot{f}_{pulsar} = -7.365 \times 10^{-10}$ Hz/s [1], the Doppler effect produces a time-proportional frequency shift of $\dot{f}_{orbit} = -12.0 \times 10^{-10}$ Hz/s [2] because of the Earth's orbit. Since these two components cannot be separated experimentally, column 7 of the table 2 contains the measured sum. The nominal value is $= -19.365 \times 10^{-10}$ Hz/s.

5.4 Modulation index (A_{day})

Because of the Earth's rotation, the frequency of the pulsar changes daily by a maximum of ± 85 μHz (equation (1)). One models this variation by a sinusoidal PM. A good starting value for the modulation index of the Crab pulsar is $A_{day} = \Delta f / f_{day} \approx 5$. The geographic latitude of the antenna determines the exact value. Any strong deviation from the expected value indicates that the GW is *not* coming from the direction of the Crab pulsar.

5.5 Phase (ϕ_{day})

The narrow frequency range around f_{GW} seems to contain so many different GWs that the GW of the Crab pulsar can only be detected with a directed search. From the known RA of the pulsar follows when the maximum frequency shift of the GW occurs (see table 1). These times have to be kept during the iteration in order not to lose sight of the target pulsar. The initial value for ϕ_{day} is calculated from the start time of the analyzed records.

6 First measurements

Our Galaxy probably hosts at least 10^6 pulsars in the frequency range 10 Hz to 200 Hz. The PM in the diurnal rhythm splits each GW into about 30 equidistant spectral lines. It follows that this region is filled with spectral lines whose average spacing is 6 μHz. Sometimes overlaps occur. Figure 2 shows that a raw, untreated spectrum is hardly decipherable, even if it contains only a few phase-modulated signals. The primary reason: each FFT produces a large set of complex numbers that are difficult to interpret. Usually, one forms the magnitude of these numbers to represent a spectrum. Because this operation destroys all phase relationships, PM cannot be undone.

The MSH method avoids any magnitude formation and reconstructs a single monochromatic line from related spectral lines without the intermediate step of spectral decomposition. Can it be used to identify the Crab pulsar signal from electromagnetic wave data obtained by the Jodrell Observatory on 2019-12-15? The well-positioned Livingston interferometer was offline on that date, so the test is done with data recorded by the Hanford and Virgo antennas.

Figure 3 shows the increase in amplitude when the entire energy of the Crab pulsar GW is concentrated in a single spectral line. In the spectrum, only the narrow red area

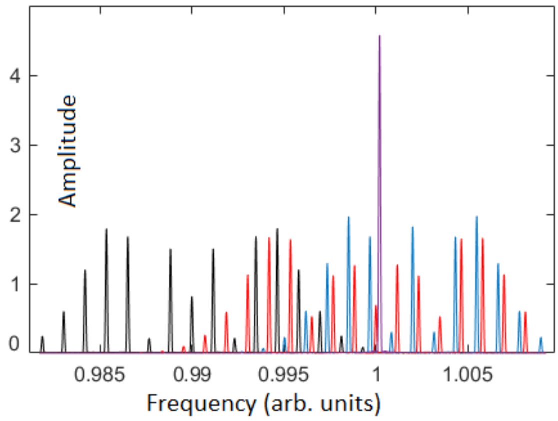

Figure 2): *Spectrum of a synthetic data set containing four GW with similar frequencies and the same energy. Three GWs are phase modulated in a 24-hour rhythm, one GW is unmodulated. The amplitude of this GW is higher by a factor of three because of the lack of splitting into 17 sideband frequencies. Noise and drift are missing because they would cause additional lines.*

at 20 mHz is of interest, which represents the modulation-free GW and corresponds to a filter with a bandwidth of 3 μHz. The surrounding area is a mixture of noise and distorted spectra of other GWs, which the interferometer cannot reject because of its low directivity. Some sidebands are probably caused by the frequent interruptions in the interferometer measurements, which act like digital modulation.

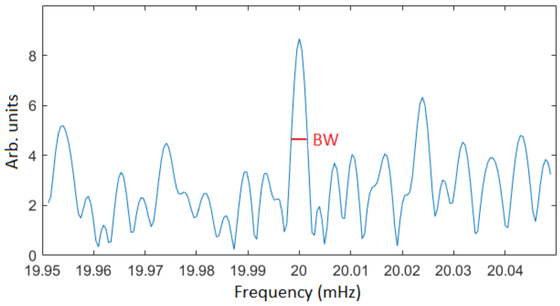

Figure 3): *The spectrum of the environment of the GW of the Crab pulsar after removal of the phase modulation by the MSH procedure and reduction of the frequency to 20 mHz. Source file = V1260445696, recording duration = 97 h. Only the narrow range BW is evaluated.*

Result of this measurement: the MSH method shown in figure 1 can clear the PM and the drift of a GW, reduce the bandwidth from 200 μHz (equation (2)) of a signal mixture to 3 μHz, and raise the signal amplitude of the GW by a factor of three.

7 Detailed measurements in December 2019

The interferometers receive a signal with twice the rotation frequency of the Crab pulsar. To determine the drift and PM properties of this signal, 19 data chains with different start dates between 2019-12-11 and 2019-12-19 were formed. Each begins at a precisely defined time and consists of 85 chronologically ordered files, each 4096 seconds long. Gaps and spikes are replaced by zeros. The intial frequency of a data chain is calculated by extrapolation with the nominal value of the drift (see section 5.3) and corrected iteratively with the MSH method.

The actual value of the drift of f_{GW} can be determined in two ways:

	GPS time s	f_{GW} Hz	A_{day}	$\dot{f}_O + \dot{f}_P$ $\cdot 10^{-10}$ s^{-2}	Redshift UTC	measured UTC	Δt Minutes
L	1260085248	59.2251827	2.70	-20.08	12.58	12.61	2.09
L	1260195840	59.2249566	6.83	-17.77	12.45	12.444	-0.12
V	1260269568	59.2248331	4.58	-18.93	5.61	5.605	-0.15
L	1260269568	59.2248295	4.40	-18.43	12.36	12.302	-3.44
V	1260281856	59.2247836	4.71	-17.58	5.59	5.302	-17.49
H	1260314624	59.2247312	4.71	-17.28	14.22	14.185	-1.85
V	1260355584	59.2246634	4.10	-19.37	5.51	5.691	11.08
H	1260433408	59.2245028	4.50	-17.81	14.08	14.366	17.35
V	1260433408	59.2244855	4.20	-17.68	5.42	5.930	30.85
V	1260445696	59.2244452	7.30	-18.03	5.40	4.795	-36.36
V	1260462080	59.2244286	5.18	-18.61	5.38	5.488	6.40
V	1260486656	59.2244024	3.58	-17.65	5.35	5.465	6.71
H	1260486656	59.2243974	5.19	-17.55	14.01	13.778	-14.15
H	1260531712	59.2243040	5.35	-19.49	13.96	14.248	17.19
V	1260531712	59.2243099	2.71	-19.12	5.30	5.268	-1.94
V	1260617728	59.2241339	6.83	-17.26	5.20	4.959	-14.41
H	1260658688	59.2240619	3.59	-20.01	13.81	13.910	5.90
V	1260736512	59.2239060	5.20	-18.73	5.06	5.117	3.45
L	1260752896	59.2238810	7.11	-19.72	11.79	12.081	17.36

Table 2): *Single measurements of the GW of the Crab pulsar. The first column gives the location of the antenna; A_{day} is the modulation index of the PM. Column 6 gives the time at which the largest value of the daily frequency shift of the GW is expected; column 7 shows the actual time and column 8 the deviation (in minutes).*

- One determines the drift of each 97-hour data string (column 5 in table 2) and averages the individual results $\dot{f}_O + \dot{f}_P = (-18.48 \pm 0.22) \times 10^{-10}\ s^{-2}$.
- One plots the initial frequencies of each data string (column 3 in table 2) as a function of the associated start time (figure 4). The proportionality factor is $\dot{f}_{Orbit} + \dot{f}_{Pulsar} = -19.56 \times 10^{-10}\ s^{-2}$.

The nominal value of the modulation index $A_{day} = \Delta f / f_{day}$ of the PM depends on the geographical latitude of the antenna (see equation (1). The obtained values (column 4 in table 2) are inaccurate, but close to the nominal values $A_{Livingston} = 6.3$ to $A_{Hanford} = 5.05$. The cause is the unfavorable antenna pattern of the interferometers: Whenever the frequency shift is particularly large because of the Doppler effect, the source of the GW is approximately in the plane defined by the two arms of the interferometer. Then these antennas are particularly insensitive. Interferometers of the present design are not very suitable to determine the declination of the GW source.

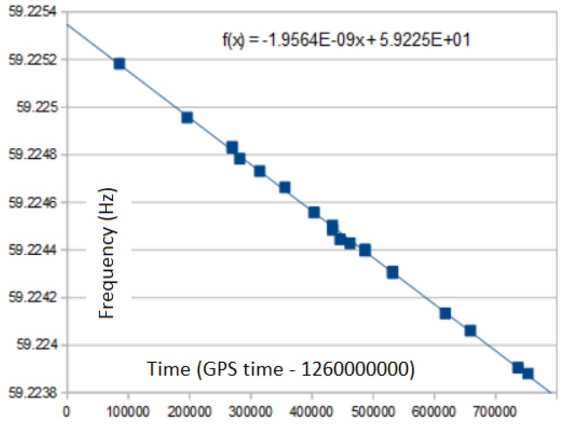

Figure 4): *Frequency f_{GW} of the Crab pulsar as a function of time. The dots are the initial frequencies of the 19 data strings (see table 2). Each stretches over a total duration of 97 hours. The start times were chosen so that the total duration contains as few data gaps as possible.*

8 Summary

The GW of the crab pulsar can be clearly identified in all records and the readings correspond to data measured with electromagnetic waves. When working with the MSH method, one gets the impression that the interferometer records are not unstructured noise, but consist of many GWs with closely spaced frequencies and numerous sidebands. For those GWs, the parameters A_{day}, $\dot{f}_O + \dot{f}_P$ and drift can be determined with surprising accuracy, but not (yet) assigned to any known astronomical object.

9 Data availability

The data underlying this article are available in the Gravitational Wave Open Science Center, O3b Data Release, https://www.gw-openscience.org/data/

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