A brief remark on modal realism

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Summary

The “introduction” concerns some sentiments which motivated the author to write this document, including an opinion about the logical consistency of points as limits. The rest concerns accessibility in modal logic, and chains of possibility. The article concludes with a theorem and a conjecture regarding the symbolic formalization of the “extended modal realism” thesis.

Introduction

I. For every component of a system, be it hierarchical, heterarchical, or otherwise horizontal, there are those associations by proximity to other components, and those arising by similarity of shape, color, or texture.

II. For the latter kind of association, requiring no context about wheresoever the site under consideration is situated, they shall be equally thought eternal, unwavering, and internal to each and every datum they are associable, or agree with.

III. Given that sites themselves, being those components of systems which are subordinate under the atomism and decomposition of vivid and dynamic pictures into constituent minimalist parts, are objects atop which internal properties emerge, they too shall be seen rightfully as systems, with regard to the multi-textural collections which inherit of them.

IV. Thus, in order to differentiate the callous and austere notion of a “point,” frenetically ideal and feverish in scope, we shall move from such a notion to its logical superior: the *super*-site.

V. Wherever sites are found, and sites are defined to be those neutral elements which are inferior to all types and spaces, it is to be recognized as a matter of fact that such neutrality either does not exist outside the domain of pure abstraction, or, and perhaps also, that such neutrality exists only as the harmonious juxtaposition of competing terms and further reducible parts which are fair in proportion to their intensity and distance from one another.

VI. A point is inconceivable as an irreducible representation of a position within spacetime.

VII. Points do not correspond to distinct locations, but to *states*, just as a Hilbert space with infinite dimensions does. To this extent, points too are infinite.

VIII. That which is infinitely small is not substantially differentiated from that which is infinitely large; they may differ in metric, but not in cardinality. Sharing a common metric, we may write $\infty\pm$.

IX. If a point be infinitely small, and an ∞-dimensional space infinitely large, then there it is contradictory to assume that points may be treated as limits, for the limit of the position of a point within infinitely scalable space does not exist, as it approaches zero from the perspective of largeness, and infinity from the perspective of smallness.

X. Either we are to reject that limit points exist, or plague ourselves with considerations of higher-dimensional spaces which perform similar functions.

XI. So, finally, we shall say in summary that a *super*-site is lacking or deficient in some ways, and provide richness in others; namely, they are absent of the surrealness, ridiculousness, and abject blithe of the zero-dimensional pointlike minimal model; where they miss the qualities of self-contradiction and vacuous acceptance, they reign in the logical extension of absolute smallness to the interiority therein found, and in the relationship between the exotic and mundane, and so also in the relationship between the absolutely minimal and the absolutely maximal, and thus, what can be seen as ideally small is indicative and projective of what is insurmountably large, and sharing in these same properties, they are below all that is quantifiable and may be used for fundamental reasoning: that thing which is median with respect to each.

Accessibility

I. Define the relationship of accessibility as follows: "for some possible state of affairs, there is a preceding possible state of affairs," and call such a notion *weak*, or "possibly possible," and write as $(t_1(\Diamond(t_2(\Diamond p))))$, or tersely: $\Diamond^2 p$. This is to say, that, from the vantage point of the moment **now**$=t_0$, there is a possible event in the subsequent moment which is intermediary to another which will not necessarily occur given t_1.

II. It is in accordance with basic logic of all kinds that t_0 be that which is *absolutely* (or, *strictly*) *necessary*, t_1 be necessarily possible, and t_2 be purely possible, but not necessarily so.

III. In order to elucidate the preceding sentence, write: $\mathbf{1} \equiv_2 t_0 \lesssim t_2$, and interpret this as follows: "the transaction of a single necessary moment, requiring two purely possible states, allows access from the necessarily necessary into the necessarily possibly necessary."

IV. Generalize this identity to the following form: $\mathbf{X} \equiv_n t_{X-1} \lesssim t_n$, where **X** is the number of terms required to permit or guarantee access from *now* into n, n is taken to be some allowable future, and X-1 is whichever assembly of qualities is taken to be strictly necessary, practical, active, or material, particularly with respect to the present moment.

V. Denote the collection of k-possible futures as $\Diamond^k p$, and write them as $(t_k, (t_k)', (t_k)'', \ldots)$, until one has b^k many choices; for example, 2^2 where two alternatives are considered for each subsequent term, and one is considering a possibly-possible moment.

VI. Write $\Diamond^k p \rightarrow 0$ for $c^x(\equiv_n)$, where c^x indexes the chosen path from some moment $t_{0-k} \rightarrow t_0$. It is clear that by t_{0-k} we do not refer to the collection of previously possible realms, $\Diamond^{-k}p$, but instead to an already determined choice among such realms. Thus, by writing $t_{X-1} \lesssim t_n$, one is referring by t_{X-1} to a sequence of the form $(t_{X-p} \rightarrow t_{X-(p-1)} \rightarrow t_{X-(p-2)} \rightarrow \ldots \rightarrow t_{X-(X-2)} \rightarrow t_{X-(X-1)})$, until one has exhausted all that has necessarily been possible, and arrived at the present from what has been possibly, but not necessarily, possible, until now.

VII. We shall allow for no "closed timelike curves," or backwards arrows, except from the future, only as temporary placeholders which signify a transitory closure of the possible as it escapes into the co-domain of the necessary; we therefore have, emerging from the partially ordered temporal set, a strictly ordered set, that which has been disclosed and enacted by virtue of restriction to necessity, and elimination of possibility.

VIII. Thus, the ordered sequence of real-world events is *acyclic* and *directed*; more precisely, it is a *monotone chain*.

Chain Complexes

I. Let T be the sequence of terms (chain) from $t_{x-\underline{\omega}}$ to t_x, set x=0, and identify $\underline{\omega}$ with the supremum of all strongly inaccessible cardinals, possibly $\aleph^\infty$. Then, we have that the number of possible moments generated by an eternal past must be $b^{\underline{\omega}}$; because $\underline{\omega}$ is super-maximal, or maximal over all maximal ideals, this is impossible; therefore, the past must be finite, or it must be impossible, as in a sequence of terms (possibly possibly possibly possibly … possible), there is nothing that is necessary, and so the limit: $\Diamond^{\infty}$, is impossible.

II. By similar logic, the future can be proved infinite. Allow $\forall k \, \exists t_k < t_{k+n}$; for all n, there is a necessary path $k \rightarrow k+n$. Assume that there exists a finite endpoint k+n. Then, either k is infinite, or n is infinite, and therefore, $\Box(k \vee n) \leftrightarrow (\exists(\Diamond^{\infty}))$, a contradiction. Therefore, there cannot be any finite term which is maximal in the set of possible moments. We have, for all accessible futures, $b^{<\omega}$ possibilities.

III. Write $T|_{\beta}$ to denote the necessary chain from some moment $\{\varnothing\}\pm$, which we will call the "generator" on t_0, and let β denote the class of choices from each p, i.e. $\{(t_k)', (t_{k-1})''', (t_{k-2})'')\}$, etc. We have the necessary identification $c^{x} \triangleq \beta$ for some $x \coloneqq (\mathrm{Sup}(\beta) \leftrightarrow \mathrm{Inf}(\beta))$

IV. The necessary chain complex of directed moments from generator to presence is monoidal, and unitary, and therefore unique; there exists no knowable necessary chain complex such that the present moment is the generator, or, simply: $\Diamond^{n} \neq \Box^{k}$ for any (n,k).

V. Those chain complexes which are not necessary consist in two varieties: the possible, and the impossible.

VI. Impossible chain complexes: $((\Diamond(T|_{\beta}) \wedge \neg(c^{x}=T|_{\beta})) \leftrightarrow \neg\Box(t_0)) \leftrightarrow \nexists T|\gamma$, where $(\gamma \cap \beta \neq \beta)$, and therefore $c^{x} \notin \gamma$, or $(c^{x} \in \beta) \neq (c^{x} \in \gamma) \rightarrow (c^{x} \neq c^{x}) \in \beta \cup \gamma$, which would be a contradiction.

VII. Possible chain complexes: $(t_1(\Diamond(t_2(\Diamond p)))) \cap \Diamond(\Box(t_2(\Diamond(t_3(\Diamond p)))))$

VIII. Possible and impossible chain complexes are both trivial, but in different senses: the former are trivial in that they exist merely as an outgrowth of possibility in general, and are unknowable unless restricted to presence by some future validation of necessity; the latter, having been mandated by the existence of now, have become so only after the fact.

IX. To be trivial is to have been uniquely trivializable.

X. The complex of all chains, past and future, is the intersection of all possible chains; in other words, the intersection of the necessary present with the union of all possibly necessary futures.

XI. $(T|_{\beta}) \cap (T|_{\Delta})$, $x \in \beta \;\; \forall \beta \subset \Delta \;\; \forall c^{x} \triangleq (t_{0-k} \rightarrow t_0)$

XII. Unique chains which are not necessary are not unique, and thus, they are abundant, and so-called "trivial" in the pedestrian sense; necessary chains are unique, however there is only one of "them", and so "they" are trivial.

XIII. To conclude: the complex consists of all that which may be made simple under a common generator, the simple being defined as that monad consisting dually of the generator itself, and the uniquely presented moment fixed and determined by that which it makes possible, and by that which has necessitated its occurrence.

Some Notation

I. Allow the concave diamonds (⟢ and ⟣) to respectively mean “was possible” and “will be possible.”

II. ⟣(t_x) means that there is a chain from t_0 into t_x, and ⟣□□(t_x) means that, of the (possibly knowable) collection of futures, one moment in particular, t_x, will inevitably be realized as the proper, true t_0, and become knowable. We will call such a moment an *unfalsifiable promise*, and the equivalence □($t_x \rightarrow t_0$) the “witness” to such a promise.

III. Promises are uniquely determined by their arising necessity, so, trivially, each individual present moment is promised within all possible chains.

IV. A promise which has not yet been trivialized will be called a “secret,” |k|, and the act of revelation will be called “realization,” ||k||.

V. Such quasi-promises, or open promises, are locally determined by their immediately successive pasts and possible futures, but globally are directed by {∅}±.

VI. All that has been necessarily possible has been possibly necessary; therefore, necessity is the strongest connective; further, “to have been possible” is associative and commutative with necessity: □ > (⟢□=□⟣)

VII. If some possible moment may be made so by another, then it is necessary that it will be possible; therefore, ⟣=$◊^n$, and $◊^n◊^n$=$◊^{2n}$=$⟣^2$.

VIII. If something will be possible by following a single path from the present moment towards its realization, but such a path is deviated from in a manner that makes the previously possible future inaccessible from a successive present moment, then that which once would be possible will no longer be possible.

IX. ⟣(◊(□)) ≠(⟣□◊=⟣□◊□)

X. To have been, or the ability to become, assume priority under the constitution which prefers their consideration: ⟣⟢=⟣ and ⟢⟡=⟢

Universes

I. For any chain $T|_*$, let there be a corresponding universe **V** in which every *-element is embedded in. Let $\mathbf{V}^*_{*\cup,}$ denote a universe in which two simultaneous present moments are accessible, such that $t_0 \in \{\beta|\gamma\}$ and $(\beta \cap \gamma \notin T|_*)$ for any proper chain of promises.

II. Let $\mathbf{V}^*_\omega$ be the proper superset of all such universes.

III. Any union or intersection of any number of promises are all contained within $\mathbf{V}^*_\omega$.

Theorem $\mathbf{V}^*_\omega$ represents a model of “extended modal realism”

Proof Let $\Diamond = \Box$ represent the belief that all possible worlds are necessary. We have that, for all $\gamma = \neg(T|_*)$, there is a corresponding open subset of $\mathbf{V}^*_\omega$ which includes the γ-universe $\mathbf{V}{:}T|_\gamma$ and its extension $\mathbf{V}^*$, such that the intersection: $\gamma \cap \beta$ is contained in $\mathbf{V}^*_\omega$, and is open. We require that it be open so that we do not exhaust the entire set of possible universes in order to include the simultaneous cases $((\Box(*)) \in \beta$ and $(\Box(\neg *)) \in \gamma)$. We drop our restriction $T|_*$, allowing us to simply write $\mathbf{T} \subseteq \mathbf{V}^*_\omega$, where **T** is the powerset of all chain complexes and $\mathbf{V}^*_\omega$ is the ultra-universe for all $\mathbf{V}^*_*$. It is trivial to show that they are equiconsistent.

Proposal The universal generator with a metric is homogenous across all promises in $\mathbf{V}^*_\omega$

Sketch of proof In order to illustrate this proposal, one would select a suitable choice of t_{-k} for some known promise t_0. Assuming that the kernel of $T|_*$ consists of some promise chain of the form $\{\varnothing\}\pm\coloneqq(\Diamond(\Box(\Diamond(\Box(...)))))$, one has that any adjustment of the operators in parenthesis will suffice as a description for a generator outside the universe **V** of $T|_*$, and therefore unequal chains result from distinct generators. Allowing for simultaneity of promises, all such unequal chains which would be generated on distinct kernels become equivalent under the relationship $\mathbf{V}_{\{\varnothing\}\pm} \subsetneqq \mathbf{V}^*_\omega$. Because this equivalence holds for all possible chains, it holds for *impossible* chains, because they are possible in $\mathbf{V}^*_\omega$.