Neutrino mass and Hubble constant, or Fermi constant

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ABSTRACT

Utilizing a dimensional analysis, Vavel found a formula for the neutrino mass depending on the Hubble constant *H*, yielding a mass value of 4.26 meV. Recently, a related formula for the electron neutrino was proposed by Mongan, depending on *H* and the dark energy parameter Ω_Λ. He assumed a spherical model for the electron neutrino and he proposed that its mass depends on the low mass density of the Universe. Furthermore, he assumed that its mass is constrained by the Compton wavelength. The obtained formula predicts a mass m_1 of 1.37 meV.

An alternative formula for the mass m_1 of the electron neutrino, also depending on *H* and Ω_Λ, can be obtained from a toroidal model of leptons, recently proposed by Biemond. In this model a toroidal shape is assumed for the electron neutrino with a radius r_1 of the torus and a radius r_2 of the tube. This torus is assumed to be filled with the low mass density of the Universe. A mass m_1 of 1.52 meV is obtained in this case.

By combination of the magnetic moment of a massive Dirac neutrino, deduced in the context of electroweak interactions at the one-loop level, and a magnetic moment for a neutrino arising from gravitational origin, a formula for the neutrino mass m_1 was obtained in 2015. This result depending on the Fermi constant forms a bridge between electroweak interactions and gravitation. In this case a more accurate value of 1.530 meV was obtained for mass m_1.

1. ELECTRON NEUTRINO MASS AND THE HUBBLE CONSTANT

In the Standard Model of elementary particles symmetry arguments imply that neutrinos are strictly massless. However, two independent squared mass differences have been extracted from neutrino oscillation observations. In the framework of three-neutrino oscillations three neutrinos are distinguished: the electron neutrino, the muon neutrino and tauon neutrino, with mass m_1, m_2 and m_3, respectively. Only normal hierarchy is considered in this work, so that m_1 is the smallest mass. In that case the following squared mass differences have approximately been deduced from observations: $\Delta m_{21}{}^2 \equiv m_2{}^2 - m_1{}^2 \approx 75$ meV^2 and $\Delta m_{31}{}^2 \equiv m_3{}^2 - m_1{}^2 \approx 2500$ meV^2. From these estimates a lower bound of $m_2 \approx 8.7$ meV and $m_3 \approx 50$ meV can, respectively, be calculated for mass $m_1 = 0$. An absolute value for the masses is not obtained by neutrino oscillation experiments.

Application of a dimensional analysis led Valev [1] to a formula for the mass of a neutrino. He considered a series of possible masses by combining the following constants: the reduced Planck constant $\hbar$, the speed of light *c*, the gravitational constant *G* and the Hubble constant *H*. Subsequently, he identified mass m_i (i = 1, 2, 3) as neutrino mass

$$m_i = \left(\frac{\hbar^3 H^2}{c^3 G}\right)^{1/4}. \tag{1.1}$$

Notice that the right-hand side of (1.1) yields the same result for m_1, m_2 and m_3. Substitution of a recent value for *H*, e.g., of 69.8 km s^{-1} Mpc^{-1} from ref. [2] and the values of the other constants into (1.1) yields a value of m_i = 4.26 meV. Since the result of m_i is much smaller than the minimal value of $m_2 \approx 8.7$ meV, the mass m_i must be identified as mass m_1.

Another formula for the mass of neutrinos, also depending on the constants $\hbar$, c, G, and H has been recently been proposed by Mongan [3, 4]. In order to obtain that formula, he assumed that the electron neutrino or neutrino 1 is spherical and that its mass m_1 is constrained by its Compton wavelength l_1

$$l_1 = \frac{\hbar}{m_1 c}. \tag{1.2}$$

In addition, he proposed the following relation for l_1

$$l_1 = \left(\frac{24\hbar}{\pi c \rho_{vac}} \right)^{1/4}. \tag{1.3}$$

Here ρ_{vac} is the low mass density of the Universe. It can be written as

$$\rho_{vac} = \Omega_\Lambda \rho_{crit} \quad \text{and} \quad \rho_{crit} = \frac{3H^2}{8\pi G}, \tag{1.4}$$

where ρ_{crit} is the critical mass density of the Universe and Ω_Λ the so-called dark energy parameter or Omega sub Lambda parameter. Combination of (1.2), (1.3) and (1.4) yields the following expression for mass m_1

$$m_1 = \left(\frac{\Omega_\Lambda}{64} \right)^{1/4} \left(\frac{\hbar^3 H^2}{c^3 G} \right)^{1/4}. \tag{1.5}$$

Substitution of all constants into (1.5), including a value of $\Omega_\Lambda = 0.6847$ from the Planck 2018 data [5], yields a value of 1.37 meV for the mass m_1 of the electron neutrino. This result corresponds to the value of 1.36 meV from Mongan [3] for slightly different values of H en Ω_Λ.

In order to calculate mass m_1, an alternative method will be outlined below. Its starting point is a more detailed model for neutrino 1, recently developed by Biemond [6]. A toroidal shape for all charged leptons and neutrinos was proposed. For all leptons the torus is characterized by two radii: a radius r_1 of the torus and a radius r_2 of the tube of the torus. It appears that all charged leptons and the electron neutrino can be described by the same limiting case $r_1 >> r_2$. In case of the muon neutrino and the tauon neutrino, the limiting case $r_2 >> r_1$ applies.

As an illustration, the torus for the electron neutrino or neutrino 1 is given in figure 1. It is noticed that figure 1 in ref. [6] represent the analogous torus for the charged leptons (For convenience'sake, the charge e of the leptons is taken positive in the latter figure). The Cartesian coordinates x, y and z in the present figure 1 are again given by the set of equations (1.1) in ref. [6]. For the electron neutrino (and also for all charged leptons) a value of $N = 1$ was deduced. The numbers 1, 2, 3 and 4 denote the location of mass m_1 at time $t = 0$, $t = ¼\, T$, $t = ½\, T$ and $t = ¾\, T$, respectively, where T is defined by $T \equiv 2\pi/\omega$. Note that the speeds vary at these different times, e.g., at position 1: $\dot{\mathbf{x}} = \dot{\mathbf{x}}(t) = 0$, $\dot{\mathbf{y}} = \dot{\mathbf{y}}(t) = \mathbf{v}_1 + \mathbf{v}_2$ and $\dot{\mathbf{z}} = \dot{\mathbf{z}}(\mathrm{t}) = -\mathbf{v}_2$. It is noticed that the positions 1, 2, 3 and 4 are lying in the same plane, but the orbit of the mass m_1 of neutrino 1 (drawn in red) is not completely flat. Furthermore, for the positive mass m_1 the z- and y-component of the magnetic dipole moment $\boldsymbol{\mu}_z(1)$ and $\boldsymbol{\mu}_y(1)$ are lying along the positive z-axis and y-axis, respectively. The total dipole moment $\boldsymbol{\mu}(1)$ is also denoted in figure 1 (compare to comment below (3.5) in ref. [6]).

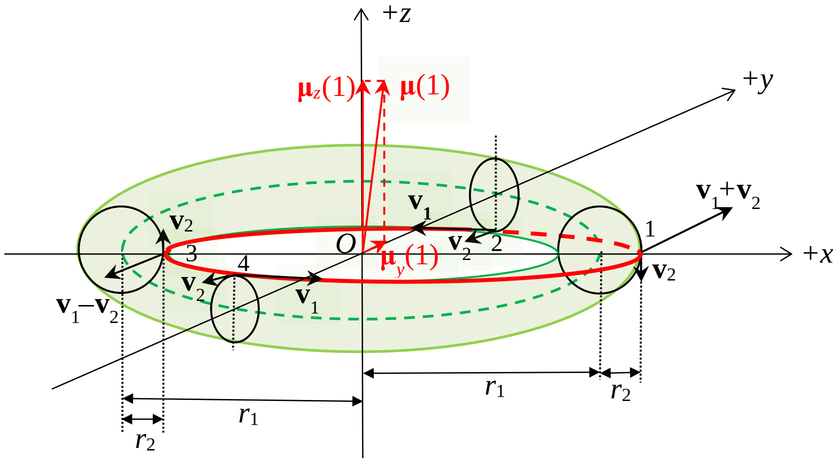

Figure 1. Toroidal model of the electron neutrino, according to eq. (1.1) of ref. [6] for $N = 1$ and $r_1 >> r_2$. When O is the origin of the coordinate system, the location of mass m_1 is fixed by the Cartesian coordinates $x = x(t)$, $y = y(t)$ and $z = z(t)$ of eq. (1.1) in ref. [6]. Mass m_1 moves with an average speed v_1 in a ring of radius r_1 and a speed v_2 ($v_1 >> v_2$) in a circle of radius r_2. The green blocked line is a circle with radius r_1 in the x-y plane and the orbit of mass m_1 is drawn in red. For clarity reasons the values of r_1, r_2, v_1 and v_2 are not drawn to scale. The vectors of the y- and z-component of the magnetic dipole moment $\mathbf{\mu}(1)$ of neutrino 1 are also shown. Additional comment is given in ref. [6].

The deduction of m_1 is started with the calculation of the volume V_{torus} of the torus in case of $r_1 >> r_2$ (see figure 1)

$$V_{torus} = 2\pi^2 r_1 r_2^2. \tag{1.6}$$

The mass of the electron neutrino is then assumed to be

$$m_1 = V_{torus}\rho_{vac} = 2\pi^2 r_1 r_2^2 \rho_{vac}. \tag{1.7}$$

In this equation it is assumed that the low energy density ρ_{vac} is homogenously distributed in the torus, whereas in the toroidal model of ref. [6] a point mass m_1 is assumed. Furthermore, the following approximate expression for radius r_1 of the electron neutrino is used (its deduction can be found from eqs. (1.4), (1.12) and (3.13) in ref. [6])

$$r_1 \approx \frac{\hbar}{m_1 c}. \tag{1.8}$$

Combining (1.7) and (1.8), radius r_1 can be written as

$$r_1 = \left(\frac{\hbar}{2\pi^2 c \rho_{vac}} \frac{r_1^2}{r_2^2} \right)^{1/4}. \tag{1.9}$$

Combination of (1.4), (1.8) and (1.9) then yields for m_1

$$m_1 = \left(\frac{3\pi}{4} \Omega_\Lambda \frac{r_2^2}{r_1^2} \right)^{1/4} \left(\frac{\hbar^3 H^2}{c^3 G} \right)^{1/4}. \tag{1.10}$$

An explicit value for the ratio of $r_2/r_1 \approx 1/10$ also follows from the toroidal model [6].

Introduction of the latter value and of all other constants into (1.10) yields a value of 1.52 meV for mass m_1. Note that the ratio m_1(eq. (1.10))/m_1(eq. (1.5)) = $(48\pi\, r_2^2/r_1^2)^{1/4}$ = 1.11.

2. ELECTRON NEUTRINO MASS AND THE FERMI CONSTANT

Within a minimal extension of the standard model with right-handed neutrinos a non-zero electromagnetic moment $\boldsymbol{\mu}_i$(em) for the left-handed Dirac neutrino has been deduced by Lee and Shrock [7] and Fujikawa and Shrock [8]. In the context of electroweak interactions at the one-loop level the following expression for $\boldsymbol{\mu}_i$(em) was obtained for the Dirac neutrino with a mass m_i (i = 1, 2, 3)

$$\boldsymbol{\mu}_i(\mathrm{em}) = \frac{3|e|G_\mathrm{F} m_i c^4 \hbar}{8\pi^2\sqrt{2}}\boldsymbol{\sigma} = \frac{3G_\mathrm{F} m_i m_e c^4 \mu_B}{4\pi^2\sqrt{2}}\boldsymbol{\sigma} = 3.2026\times10^{-22}\left(\frac{m_i}{\mathrm{meV}}\right)\mu_B\boldsymbol{\sigma}, \qquad (2.1)$$

where $G_\mathrm{F} = 1.16638\times10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi coupling constant, $\boldsymbol{\sigma}$ is the Pauli matrix and $\mu_B = |e|\hbar/2m_e$ is the Bohr magneton. Notice that $\boldsymbol{\mu}_i$(em) is proportional to mass m_i.

At present, no magnetic moment of any neutrino has been measured. So far, the tightest constraint on μ_i(em) comes from studies of a possible delay of helium ignition in the core of red giants in globular clusters. From the lack of observational evidence of this effect a limit of $\mu_i(\mathrm{em}) < 3\times10^{-12}\mu_B$ has been extracted [9]. Therefore, the value of m_i cannot yet be calculated from (2.1).

Since 1891 many authors have already investigated a gravitational origin of the magnetic field of celestial bodies and other rotating bodies, including neutrinos [10]. Particularly, the so-called Wilson-Blackett formula for the gravitomagnetic moment $\boldsymbol{\mu}$(gm) of a massive body with angular momentum $\mathbf{S}$ has often been considered

$$\boldsymbol{\mu}(\mathrm{gm}) = -\frac{\beta}{2}\left(\frac{G}{k}\right)^{1/2}\mathbf{S}, \qquad (2.2)$$

where G is the gravitational constant and $k = (4\pi\varepsilon_0)^{-1}$ is the Coulomb constant. The parameter β is assumed to be a dimensionless constant of order unity. It is emphasised that the gravitomagnetic moment $\boldsymbol{\mu}$(gm) is identified as an electromagnetic moment. For a large series of rotating bodies an averaged absolute value of $\beta = 0.13$ is obtained (see table 2 and discussion in ref. [10]).

As has been discussed previously [11, 12], the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) for an elementary particle like a neutrino with mass m_i (i = 1, 2, 3) and angular momentum $\mathbf{S}$ = $(\hbar/2)\boldsymbol{\sigma}$ may be written as

$$\boldsymbol{\mu}_i(\mathrm{gm}) = -\frac{g_i\beta}{4}\left(\frac{G}{k}\right)^{1/2}\left(\frac{m_i}{m_1}\right)\hbar\boldsymbol{\sigma}, \qquad (2.3)$$

where the parameter g_i (i = 1, 2, 3) is a dimensionless quantity of order unity, related to the g_l-factor for charged leptons l ($l = e, \mu, \tau$). Starting from the Dirac equation, however, in first order the same factor $g_i = +2$ is deduced (see refs. [11, 12]) for all neutrinos m_i, analogously to the factor $g_l = +2$ for charged leptons l. In order to get a linear dependence on mass m_i, a factor m_i/m_1 has been added to the right-hand side of (2.3). If the electromagnetic moment $\boldsymbol{\mu}_i$(em) of (2.1) may be put equal to the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (2.3), one obtains the following formula for the electron neutrino mass m_1

$$m_1 = \frac{4\pi^2\sqrt{2}}{3|e|G_F c^4}\left(\frac{G}{k}\right)^{1/2}. \qquad (2.4)$$

Note that the reduced Planck constant drops out in this result. In order to obtain a positive result for mass m_1, a negative value of $\beta = -1$ has been chosen. Insertion of all constants in (2.4) then yields a value of 1.530 meV for mass m_1. The accuracy of this result mainly depends on the value of the least accurate constant in (2.4), i.e., on the gravitational constant $G = 6.674\times 10^{-11}$ kg^{-1} m^3 s^{-2}. Therefore, the latter value is more accurate than the value m_1 = 1.52 meV from (1.10) that depends on more uncertain parameters like H and Ω_Λ. It is striking, however, that both results for mass m_1, due to totally different origins, nearly coincide.

3. DISCUSSION OF THE RESULTS

In this work a number of different formulas is discussed for the electron neutrino or neutrino 1 with mass m_1: eq. (1.1) from Valev [1], eq. (1.5) from Mongan [3] and eq. (1.10) from Biemond. All these formulas connect the mass m_1 of neutrino 1 to the Hubble constant H, a typical cosmological parameter. The last two formulas also connect mass m_1 to the dark energy parameter Ω_Λ, another cosmological parameter. So, a bridge is found between the microscopic electron neutrino and the large-scale parameters H and Ω_Λ.

In order to be able to calculate the last two formulas for mass m_1, details of the basic structure of neutrino 1 are necessary. For the calculation of (1.5) Mongan assumed that the neutrino is spherical and that its mass is constrained by its Compton wavelength.

In the calculation of mass m_1 of (1.10) results are utilized from a recently developed toroidal model for leptons from Biemond [6]. This model implies a radius r_1 for the torus and a radius r_2 for the tube of the torus. As a consequence, a value for the ratio r_2/r_1 can be obtained. In addition, the radius r_1 is approximately given by the Compton wavelength, both in case of the electron and the electron neutrino. Furthermore, the expressions for the magnetic moments of both elementary particles are of the same form, the leading term of the so-called magnetic moment anomaly included. According to ref. [6], the ratio r_2/r_1 of the electron and of neutrino 1 can, respectively, be written as

$$\text{electron: } \frac{r_2}{r_1} \approx \sqrt{\frac{\alpha}{\pi}} = \frac{1}{20.75}, \qquad \text{electron neutrino: } \frac{r_2}{r_1} \approx \sqrt{\frac{\alpha_W}{\pi}} \approx \frac{1}{10}, \tag{3.1}$$

where α is the fine-structure constant and α_W is the electroweak coupling constant at low energy.

Combination of the electromagnetic moment $\boldsymbol{\mu}_i$(em) of (2.1) from refs. [7, 8] and the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (2.3) from refs. [11, 12] leads to the fourth formula for the electron neutrino mass m_1. The first magnetic moment $\boldsymbol{\mu}_i$(em) is a consequence of the theory of the weak interactions, working at microscopic level and characterised by the Fermi constant G_F. The gravitational constant G connects the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (2.3) to the theory of gravitation. The combined result of (2.4) forms a bridge between electroweak interactions and gravitation.

Concerning the gravitomagnetic moment $\boldsymbol{\mu}_i$(gm) of (2.2), many authors have already considered a gravitational origin of the magnetic field of celestial bodies from Moon up to Galaxy. In this approach the Schuster-Wilson-Blackett hypothesis [10] has been playing an important role for more than a century. A recent review of relation (2.2) and the extra-polation to the neutrinos of (2.3) has also recently been given in ref. [10].

Summing up, four different formulas for the neutrino mass m_1 are discussed. The first mass, given in eq. (1.1) from Valev [1] depends on the Hubble constant H, whereas eq. (1.5) from Mongan [3, 4], and eq. (1.10) from Biemond depend on H and the dark energy parameter Ω_Λ. The fourth formula for the neutrino mass m_1 depends on the Fermi constant G_F, a constant characteristic for weak interactions. Surprisingly, the obtained masses m_1 from (1.10) and (2.4), m_1 = 1.52 meV and m_1 = 1.530 meV, respectively, nearly coincide, although they follow from totally different starting points.

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