Using Saint-Venant equations for liquid flows

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Abstract. The method is based on the use of geometry rules for a continuous medium in the form of the Saint-Venant equations. The article transforms the compatibility equations in order to use them in fluid mechanics. The transformation is performed within the framework of the averaged turbulence model and does not use additional restrictions. The resulting compatibility equations for a liquid take into account the influence of the linear (u_x, u_y, u_z) and angular (ω_x, ω_y, ω_z) particle rotation speeds. This means that they describe the turbulent flow mode. Using the conditions $\omega = 0$ or $u = 0$ allows us to obtain two special cases that are designed for laminar flow and vortex flow with a fixed axis.

The Stokes equation is transformed and it is shown that its second form exists, taking into account the influence of u and ω on the forces of inertia and viscous friction. This form of the equation also breaks down into two special cases under the same conditions. As a result, three closed systems of equations were compiled for three flow regimes - turbulent, laminar, and rotation with a fixed axis. The application of this closure method does not require the use of the equations of the semi-empirical theory of turbulence.

The solutions of a particular problem are compared with the help of the equations of compatibility and motion. The considered method of closing the equations of fluid motion can be relevant for use in CFD, as well as in the dynamics of non-Newtonian fluids.
Keywords: Compatibility equations, fluid equation of motion, Newtonian fluid, angular rotation velocity of particles.

Introduction

The behaviour of a continuous medium is described by the equation of motion in stresses (Navier), which has nine unknowns and in the theory of elasticity is closed by six compatibility equations (Saint-Venant). These equations are derived using the rules of geometry and do not contain the properties of a specific continuous medium [1, 3]. Thus, there is a fundamental possibility to application them to close the equations of motion not only for solids, but also for other continuous media, such as liquids and gases.

In fluid mechanics, one of the exact equations of motion of a continuous medium is the Stokes equation, which is derived from the Navier equation [2, 7]. The Stokes equation contains four unknowns and is closed by the continuity equation, which expresses the mass conservation law in differential form and at a constant density has the form div u = 0 (where u is the flow velocity) [2, 7]. On fig. 1 shows a diagram of the derivation and Stokes equation closure.

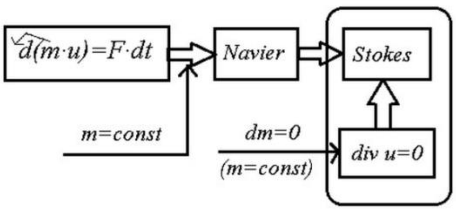

Fig. 1.

From fig. 1 follows that the m = const condition apply in the output of the Navier equation to simplify the pulse theorem. Thus, all variants of the Navier equation and its special cases contain this condition. The repetition of the same condition violates the requirement of independence of the closing equation, and it does not impose additional restrictions on solving any equation of motion.

This contradiction is also manifested in the implementation of numerical calculations, where the closing equations from various models of the semi-empirical theory of turbulence are used. Such a closure method is used in FlowVision, OpenFoam programs and others [4, 7]. The sharing of accurate equations of motion and approximate closing equations leads to satisfactory calculation results for a limited circle of problems. For part of the other problems, the solution has an unstable character, which limits computer modelling capabilities.

Thus, it is topical to find exact closing equations that should be suitable for various fluid flow regimes.

§1. Problem statement and transformations

This problem is considered within the framework of the averaged turbulence model, which takes into account two dynamic properties of the fluid medium - linear (u) and angular velocity (ω) of particle rotation.

Let us transform the compatibility equations for a solids, in order to determine the effect of two velocities u(x, y, z) and ω(x, y, z) on the fluid flow.

In full form, the compatibility equations for a solids consist of two groups (1.1) and (1.2) [1, 3]:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2}+\frac{\partial^2 \varepsilon_y}{\partial x^2}=\frac{\partial^2 \gamma_{xy}}{\partial x \partial y},\qquad \frac{\partial^2 \varepsilon_y}{\partial z^2}+\frac{\partial^2 \varepsilon_z}{\partial y^2}=\frac{\partial^2 \gamma_{yz}}{\partial y \partial z},\qquad \frac{\partial^2 \varepsilon_z}{\partial x^2}+\frac{\partial^2 \varepsilon_x}{\partial z^2}=\frac{\partial^2 \gamma_{zx}}{\partial z \partial x} \tag{1.1}$$

$$\begin{gathered}
\frac{\partial}{\partial x}\left(\frac{\partial \gamma_{zx}}{\partial y}+\frac{\partial \gamma_{xy}}{\partial z}-\frac{\partial \gamma_{yz}}{\partial x}\right)=2\frac{\partial^2 \varepsilon_x}{\partial y \partial z},\\
\frac{\partial}{\partial y}\left(\frac{\partial \gamma_{xy}}{\partial z}+\frac{\partial \gamma_{yz}}{\partial x}-\frac{\partial \gamma_{zx}}{\partial y}\right)=2\frac{\partial^2 \varepsilon_y}{\partial z \partial x},\\
\frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x}+\frac{\partial \gamma_{zx}}{\partial y}-\frac{\partial \gamma_{xy}}{\partial z}\right)=2\frac{\partial^2 \varepsilon_z}{\partial x \partial y},
\end{gathered} \tag{1.2}$$

where $\varepsilon_x, \varepsilon_y, \varepsilon_z$, $\gamma_{xy}, \gamma_{yz}, \gamma_{xz}$ - relative linear and angular deformations, which are expressed by the following equations.

$$\varepsilon_x=\frac{\partial u}{\partial x},\quad \varepsilon_y=\frac{\partial v}{\partial y},\quad \varepsilon_z=\frac{\partial w}{\partial z},\quad \gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x},\quad \gamma_{yz}=\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y},\quad \gamma_{zx}=\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}, \tag{1.3}$$

where u_x, u_y, u_z, - are linear displacements along the coordinates.

For a Newtonian fluid, the obtained equations for the rates of angular deformities can be expressed in terms of shear stresses [2]:

$$\gamma_{xy}=\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}=\frac{\tau_{xy}}{\mu}\qquad \gamma_{yz}=\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}=\frac{\tau_{yz}}{\mu}\qquad \gamma_{zx}=\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}=\frac{\tau_{zx}}{\mu}, \tag{1.4}$$

where $\tau_{xy}, \tau_{yz}, \tau_{zx}$ - are shear stresses in the corresponding planes, μ is the dynamic viscosity of the fluid.

Let's use the equations (1.4) and find the speed curl by adding to each equation two identical terms with different signs. Then for γ_{xy} we get:

$$\gamma_{xy} = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} + \frac{\partial u_x}{\partial y} - \frac{\partial u_x}{\partial y}\right) = \left[(curl\ u)_z + 2\frac{\partial u_x}{\partial y}\right] = 2\left(\omega_z + \frac{\partial u_x}{\partial y}\right),$$

where $2\omega_x = (curl\ u)_x = \left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right)$, $2\omega_y = (curl\ u)_y = \left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right)$, $2\omega_z = (curl\ u)_z = \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right)$ - angular velocities along the normal to the plane of rotation.

The results of similar transformations for others are shown in the table.

Table 1. Angular strain rates γ_{ij}

$\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} = 2\left(\omega_z + \frac{\partial u_x}{\partial y}\right)$	$\gamma_{yz} = \frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y} = 2\left(\omega_x + \frac{\partial u_y}{\partial z}\right)$	$\gamma_{zx} = \frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z} = 2\left(\omega_y + \frac{\partial u_z}{\partial x}\right)$

Taking into account Table 1, we transform the first group of compatibility conditions (1.1).

Then for the first equation we get:

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

$$\frac{\partial^2}{\partial y^2}\left(\frac{\partial u_x}{\partial x}\right) + \frac{\partial^2}{\partial x^2}\left(\frac{\partial u_y}{\partial y}\right) = 2\frac{\partial^2}{\partial x \partial y}\left(\omega_z + \frac{\partial u_x}{\partial y}\right).$$

After expanding the brackets and abbreviations, we get:

$$\frac{\partial^3 u_y}{\partial y \partial x^2} - \frac{\partial^3 u_x}{\partial x \partial y^2} = 2\frac{\partial^2 \omega_z}{\partial x \partial y}. \quad (1.5)$$

We transform equation (1.5):

$$\frac{\partial^2}{\partial x \partial y}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) = 2\frac{\partial^2 \omega_z}{\partial x \partial y}$$

Since the left and right sides of this equation are the same, we get an identity.

Let us perform similar transformations for the other compatibility equations of the first group (1.1) and obtain the remaining relations in Table 2.

Let us transform the second group of compatibility equations (1.2).

Then for the first equation, taking into account Table 1, we get:

$$\frac{\partial}{\partial x}\left(\frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z} - \frac{\partial \gamma_{yz}}{\partial x}\right) = 2\frac{\partial^2 \varepsilon_x}{\partial y \partial z}.$$

$$\frac{\partial}{\partial x}\left[\frac{\partial}{\partial y}\left(\omega_y+\frac{\partial u_z}{\partial x}\right)+\frac{\partial}{\partial z}\left(\omega_z+\frac{\partial u_x}{\partial y}\right)-\frac{\partial}{\partial x}\left(\omega_x+\frac{\partial u_y}{\partial z}\right)\right]=\frac{\partial^2}{\partial y\partial z}\left(\frac{\partial u_x}{\partial x}\right).$$

After expanding the brackets and abbreviations, we get the following equation:

$$\frac{\partial^3 u_z}{\partial x^2\partial y}-\frac{\partial^3 u_y}{\partial x^2\partial z}=\frac{\partial^2\omega_x}{\partial x^2}-\frac{\partial^2\omega_y}{\partial x\partial y}-\frac{\partial^2\omega_z}{\partial x\partial z}$$

Performing similar transformations for the other two equations of the second group, we obtain two more equations.

Table 2 shows six transformed compatibility equations that characterize a continuous fluid flow, taking into account the influence of the linear and angular velocity of particle rotation. According to the definition, these equations characterize the turbulent flow regime within the averaged model [2, 7].

Table 2. Compatibility equations for liquid

N	Turbulent regime ($u\neq 0,\ \omega\neq 0$)	Number
1	$\frac{\partial^3 u_y}{\partial x^2\partial y}-\frac{\partial^3 u_x}{\partial x\partial y^2}=2\frac{\partial^2\omega_z}{\partial x\partial y}$	(1.5)
2	$\frac{\partial^3 u_z}{\partial y^2\partial z}-\frac{\partial^3 u_y}{\partial y\partial z^2}=2\frac{\partial^2\omega_x}{\partial y\partial z}$	(1.6)
3	$\frac{\partial^3 u_x}{\partial z^2\partial x}-\frac{\partial^3 u_z}{\partial z\partial x^2}=2\frac{\partial^2\omega_y}{\partial z\partial x}$	(1.7)
4	$\frac{\partial^3 u_z}{\partial x^2\partial y}-\frac{\partial^3 u_y}{\partial x^2\partial z}=\frac{\partial^2\omega_x}{\partial x^2}-\frac{\partial^2\omega_y}{\partial x\partial y}-\frac{\partial^2\omega_z}{\partial x\partial z}.$	(1.8)
5	$\frac{\partial^3 u_x}{\partial y^2\partial z}-\frac{\partial^3 u_z}{\partial y^2\partial x}=\frac{\partial^2\omega_y}{\partial y^2}-\frac{\partial^2\omega_z}{\partial y\partial z}-\frac{\partial^2\omega_x}{\partial x\partial y}$	(1.9)
6	$\frac{\partial^3 u_y}{\partial z^2\partial x}-\frac{\partial^3 u_x}{\partial z^2\partial y}=\frac{\partial^2\omega_z}{\partial z^2}-\frac{\partial^2\omega_x}{\partial z\partial x}-\frac{\partial^2\omega_y}{\partial z\partial y}$	(1.10)

The compatibility equations for liquids are third order in linear velocity and second order in angular velocity. The equations of motion have second and first order derivatives, respectively [5, 6]. Thus, when integrating the compatibility equations, additional constants arise, which can be determined using additional equations.

For a laminar flow regime, the angular velocity of rotation of particles $\omega=0$ and the application of this condition to (1.5 – 1.10) allows us to find six equations, three of which are used to close the equations of motion. A similar property also takes place in the theory of deformations [3].

For a vortex with a fixed axis liner velocity $u = 0$ and applying this condition to (1.5 – 1.10) also allows us to find six equations with the same purpose.

Table 3 shows the compatibility equations for these two special cases.

Table 3. Compatibility equations for a laminar regime and a vortex with a fixed axis

N	laminar regime ($\omega=0$)	Number	Vortex ($u=0$)	Number
1	$\frac{\partial^3 u_y}{\partial x^2 \partial y} - \frac{\partial^3 u_x}{\partial x \partial y^2} = 0$	(1.11)	$\frac{\partial^2 \omega_x}{\partial x^2} - \frac{\partial^2 \omega_y}{\partial x \partial y} - \frac{\partial^2 \omega_z}{\partial x \partial z} = 0$	(1.14)
2	$\frac{\partial^3 u_z}{\partial y^2 \partial z} - \frac{\partial^3 u_y}{\partial y \partial z^2} = 0$	(1.12)	$\frac{\partial^2 \omega_y}{\partial y^2} - \frac{\partial^2 \omega_z}{\partial y \partial z} - \frac{\partial^2 \omega_x}{\partial x \partial y} = 0$	(1.15)
3	$\frac{\partial^3 u_x}{\partial z^2 \partial x} - \frac{\partial^3 u_z}{\partial z \partial x^2} = 0$	(1.13)	$\frac{\partial^2 \omega_z}{\partial z^2} - \frac{\partial^2 \omega_x}{\partial x \partial z} - \frac{\partial^2 \omega_y}{\partial y \partial z} - = 0$	(1.16)
4	$\frac{\partial^3 u_z}{\partial x^2 \partial y} - \frac{\partial^3 u_y}{\partial x^2 \partial z} = 0$		$\frac{\partial^2 \omega_x}{\partial y \partial z} = 0$	
5	$\frac{\partial^3 u_x}{\partial y^2 \partial z} - \frac{\partial^3 u_z}{\partial y^2 \partial x} = 0$		$\frac{\partial^2 \omega_y}{\partial x \partial z} = 0$	
6	$\frac{\partial^3 u_y}{\partial z^2 \partial x} - \frac{\partial^3 u_x}{\partial z^2 \partial y} = 0$		$\frac{\partial^2 \omega_z}{\partial x \partial y} = 0$	

§2. One-dimensional problem

Let us consider the application of the compatibility equations for the problem of a laminar flow around a horizontal plate with a velocity u_f (Fig. 2) [2, 5 - 7].

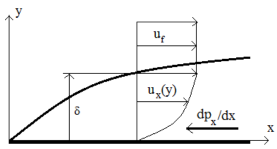

Fig. 2. Scheme of laminar flow on a horizontal plate

Near the surface of the plate, the influence of the viscous friction force is manifested and a boundary layer is formed within which the flow velocity is variable. The vertical velocity distribution is found in various ways: by integrating the equation of motion or by solving the equations of the laminar boundary layer theory [2, 5–7].

We use the compatibility equation and find the dependence ux(y) using equation (1.11). Since the flow is flat, the differential equation will take the form:

$$\frac{\partial^3 u_x}{\partial x \partial y^2} = 0.$$

After integration, we get:

$$u_x(y) = C_{1x} y^2 + C_{2y} y + C_{3y}.$$

To find the constants C_{2y} and C_{3y}, we take the following boundary conditions: for $y=0$, $u_x(y)=0$; at $y=\delta$, $u_x(y)=u_f$. As a result, we obtain the following particular solution:

$$u_x(y) = C_{1x}(y^2 - y \cdot \delta) + u_f \frac{y}{\delta}. \quad (2.1)$$

The same problem was solved using the equation of motion for a Newtonian fluid. The corresponding particular solution has the form: [5]

$$u_x(y) = \frac{1}{4\mu}\frac{dp_x}{dx}\left[y^2 - y\cdot\delta(x)\right] + u_f\,\frac{y}{\delta(x)}, \tag{2.2}$$

where μ is the dynamic viscosity, dp_x/dx is the change in pressure along the flow.

It follows from equation (2.1) that in order to fulfill the condition of flow continuity, the velocity distribution along the vertical must be parabolic and it does not depend on the properties of a particular medium.

Comparison of (2.1) and (2.2) shows, that using the compatibility equations, it is impossible to find a constant, containing the properties of a continuous medium. At the same time, the velocity distribution law $u_x(y)$ does not depend on the method of solving the problem. Characteristically, the use of equation (1.11) makes it possible to find the velocity distribution law much more simply than using the equations of motion for a laminar flow or using the theory of a laminar boundary layer [2, 5].

With an arbitrary choice of coordinate axes, other compatibility equations are applied and the velocity distribution law does not change. Thus, the equations found have the property of invariance.

§ 3. Closure of the equations of motion

Consider the closure of the equation of motion on the example of the Stokes equation, which for incompressible liquid has the form [2, 7]:

$$G - \frac{1}{\rho} grad\ p + \nu\cdot\nabla^2 u = \frac{du}{dt}. \tag{3.1}$$

Equation (3.1) does not contain the influence of the angular velocity (ω) and requires the transformation of terms that take into account the influence of inertia and viscous friction.

We use the equation for calculating the total acceleration of a particle in the form of Gromeka-Lamb []:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + grad\left(\frac{u^2}{2}\right) + 2\cdot\left[\vec{\omega}\times\vec{u}\right]. \tag{3.2}$$

In accordance with the definition, this equation characterizes the turbulent flow regime within the framework of the averaged model.

We transform $\nabla^2 u$ in order to determine the influence of linear and angular velocity.

For the x-axis, we have:

$$\nabla^2 u_x = \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}.$$

We express the second and third terms in terms of the first derivative, add zero in parentheses and represent it as two identical terms with different signs.

$$\frac{\partial^2 u_x}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u_x}{\partial y} - \frac{\partial u_y}{\partial x} + \frac{\partial u_y}{\partial x}\right) = \frac{\partial^2 u_y}{\partial x\partial y} - \frac{\partial (curl\ u)_z}{\partial y} = \frac{\partial^2 u_y}{\partial x\partial y} - 2\frac{\partial \omega_z}{\partial y}$$

$$\frac{\partial^2 u_x}{\partial z^2}=\frac{\partial}{\partial z}\left(\frac{\partial u_x}{\partial z}-\frac{\partial u_z}{\partial x}+\frac{\partial u_z}{\partial x}\right)=\frac{\partial^2 u_z}{\partial x\partial z}+\frac{\partial (curl\ u)_y}{\partial z}=\frac{\partial^2 u_z}{\partial x\partial z}+2\frac{\partial \omega_y}{\partial z}.$$

Thus, $\nabla^2 u_x$ is a function of two variables - linear and angular velocity.

$$\nabla^2 u_x=\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}+2\left(\frac{\partial \omega_y}{\partial z}-\frac{\partial \omega_z}{\partial y}\right). \quad (3.3)$$

The viscosity of the liquid is also present in the second term of equation (3.1).
For the x-axis we have [].

$$p_{xx}=-p+2\mu\frac{\partial u_x}{\partial x}=-p_x \quad \text{or} \quad p=p_x+2\mu\frac{\partial u_x}{\partial x},$$

Then, the term from the pressure will take the form:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=-\frac{1}{\rho}\frac{\partial}{\partial x}\left(p_x+2\mu\frac{\partial u_x}{\partial x}\right)=-\frac{1}{\rho}\frac{\partial p_x}{\partial x}-2\nu\frac{\partial^2 u_x}{\partial x^2}. \quad (3.4)$$

where the normal stress is $p_{xx}= - p_x$ according to the sign rule for stress and pressure.

Taking into account (3.3) and (3.4), we obtain:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\cdot\nabla^2 u_x=-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+\nu\left[-\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}+2\left(\frac{\partial \omega_y}{\partial z}-\frac{\partial \omega_z}{\partial y}\right)\right],$$

for x-axis:

$$X-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+\nu\cdot\left[-\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_y}{\partial x\partial y}+\frac{\partial^2 u_z}{\partial x\partial z}-2(rot\ \omega)_x\right]-\frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)=\frac{\partial u_x}{\partial t}+2(u_z\omega_y-u_y\omega_z). \quad (3.5)$$

Or in vector form [6]:

$$G-\frac{1}{\rho}div\ p+\nu\cdot\varphi(u,\omega)-grad\left(\frac{u^2}{2}\right)=\frac{\partial u}{\partial t}+2[\vec{\omega}\times\vec{u}], \quad (3.6)$$

where $\varphi(u,\omega)$ is a function that takes into account the effect of two velocities on viscous friction.

Thus, the Stokes equation has the second form (3.6), which takes into account the influence of linear and angular velocity. This means that it is intended to describe the turbulent flow regime within the framework of the averaged model, as well as the compatibility equation of Table 2.

Equation (3.6) contains nine unknowns (p_x, p_y, p_z, u_x, u_y, u_z, ω_x, ω_y, ω_z) and forms a closed system with six equations (1.5) - (1.10).

The Stokes equation (3.6) has two special cases. For the laminar flow regime, according to the condition $\omega=0$, we obtain:

$$G-\frac{1}{\rho}divp+\nu\cdot\varphi(u)-grad\left(\frac{u^2}{2}\right)=\frac{\partial u}{\partial t}. \quad (3.7)$$

This equation has six unknowns (p_x, p_y, p_z, u_x, u_y, u_z) and can be closed by three equations (1.11) - (1.13) (Table 3).

Under the condition $u = 0$, we obtain the second particular case of equation (3.6), which characterizes a vortex flow with a fixed axis [6].

$$G - \frac{1}{\rho} divp + \nu \cdot \varphi(\omega) = \frac{\partial(\omega \cdot r)}{\partial t}. \qquad (3.8)$$

This equation has six unknowns (p_x, p_y, p_z, ω_x, ω_y, ω_z) and can be closed by three compatibility equations (1.14) - (1.16) (Table 3).

Table 4 lists the numbers of the three equations of motion and the corresponding compatibility equations for each flow regime.

Table 4. An example of the closure of the equations of motion

N	Equation of fluid motion	Closing equations	Flow regime
1	$G - \frac{1}{\rho} divp + \nu \cdot \varphi(u,\omega) - grad\left(\frac{u^2}{2}\right) = \frac{\partial u}{\partial t} + 2[\vec{\omega} \times \vec{u}]$	(1.5) – (1.10)	Turbulent
2	$G - \frac{1}{\rho} divp + \nu \cdot \varphi(u) - grad\left(\frac{u^2}{2}\right) = \frac{\partial u}{\partial t}$	(1.11) –(1.13)	Laminar
3	$G - \frac{1}{\rho} divp + \nu \cdot \varphi(\omega) = \frac{\partial(\omega \cdot r)}{\partial t}$	(1.14) –(1.16)	Vortex with fixed axis

All equations of motion take into account the influence of body forces, pressure forces, viscous friction and inertia. A comparison of the influencing factors in the equations of motion and equations of compatibility shows, that linear and angular velocities are present in both equations, while pressure, time, body force, and thermophysical properties of the medium are present only in the equations of motion.

Thus, the pressure distribution $p(x, y, z, t)$ can be found in only one way - from the equation of motion, the distribution of linear and angular velocity /$u(x, y, z, t)$, $\omega(x, y, z, t)$/ can be found from the equations of motion and compatibility.

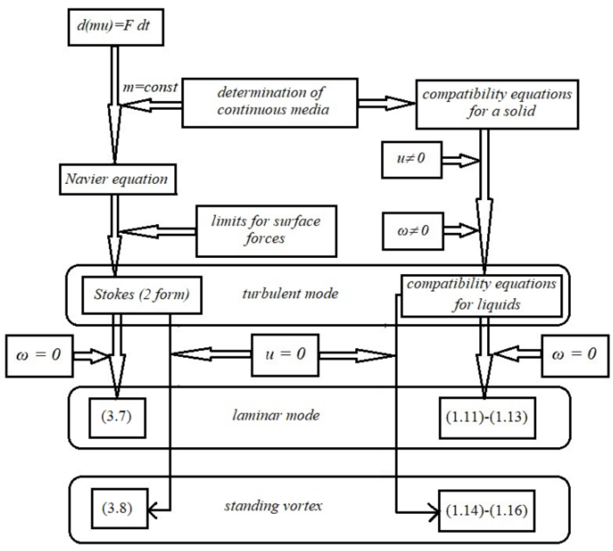

fig. 3. Example of closure of the equation of motion

On fig. 3 shows an example of a scheme for deriving closed systems of equations of motion for three flow regimes - turbulent, laminar, and a vortex with a fixed axis. From the scheme it follows that the derivation of the general equation of motion (Stokes), the compatibility equation for the liquid and their decay are performed in parallel according to similar schemes.

The derivation of the equation of motion is based on the law of conservation of momentum, and the definition of a continuous medium is used to derive the closing equations. The decay of the equations of motion and the equations of compatibility is performed according to the same conditions: $\omega=0$ and $u=0$ (Fig. 3).

As a result, three closed systems of equations for three modes are obtained, which are shown by rectangles.

A similar conclusion (left side of the diagram) can be made for another general equation of fluid motion obtained from the Navier equation under other constraints [5]. The scheme for deriving the compatibility equations for a liquid (right side) remains unchanged.

SUMMARY

A transformation of the compatibility equations for a solid body (Saint-Venant) has been performed with the aim of using it in fluid mechanics to close the equations of motion. The resulting compatibility equations for a fluid in a turbulent flow regime have two particular cases: in the absence of the angular velocity of rotation of particles, a laminar flow regime ($\omega=0$) and in the absence of a linear velocity, a vortex flow with a fixed axis ($u = 0$). The general equation of motion (3.1) [6] has similar special cases under the same conditions. This made it possible to perform the closure of the fluid motion equations for the corresponding regimes.

One of the resulting compatibility equations is used to solve a well-known particular problem - a viscous laminar flow around a horizontal plate. Comparison with the solution of the equation of motion made it possible to establish the general and distinctive properties of the two solution methods.

The joint use of the equations of motion and exact closing equations is relevant for the numerical solution of three-dimensional problems with various boundary and initial conditions. In the future, equations (1.5) - (1.10) can be used to close the equations of motion of non-Newtonian fluids, and also as part of a more complex system that describes the flow of electrolytes and plasma.

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