Classical Kaluza-Klein Vacuum Fluctuations

John French

email: johnfrenchng2@yahoo.com

Published in *Physics Essays*, Vol 31, No. 2 (2018)

https://physicsessays.org/

Abstract

We look at the classical Kaluza-Klein theory allowing the metric to vary in the fifth dimension. Considering only linear terms we then use a Lorentz invariant distribution of the metric terms to find out how the metric terms vary with respect to frequency. We find that they blow up at high frequency, so that non-linear terms would have to be taken into account.

I. Introduction

Kaluza and Klein[1-3] extended the Einstein field equations to 5 dimensions and were able to obtain the Einstein-Maxwell equations if the metric in the fifth dimension was taken to represent the electromagnetic potential. For a review see Applequist, Chodos and Freund[4], Bailin and Love[5], and Overduin and Wesson[6]. Most papers, including those of Kaluza and Klein, take the metric to be independent of the fifth dimension. When the metric is allowed to vary in the fifth dimension then it can be expanded in a Fourier expansion with respect to the fifth dimension. The higher order modes are then generally interpreted as massive spin two particles, and neglected in the low energy limit. For example see Bailin and Love[5]. Wesson[7] allows the metric to vary in the fifth dimension, and interprets the extra terms as a stress-energy tensor in Einstein's field equations.

In this paper we also make a Fourier expansion of the metric with respect to the fifth dimension, but take the terms as purely classical and don't associate any spin two particles with them. Instead we require their distribution to be invariant under a Lorentz transformations and find the resulting expectation values.

II. Basic equations

Following Klein[2] we will write the metric in the form

$$ds^2 = \varphi^2\left(dx^1 + \alpha_\mu dx^\mu\right)(dx^1 + \alpha_\nu dx^\nu) + g_{\mu\nu}dx^\mu dx^\nu \qquad (1)$$

where φ, α_μ, and $g_{\mu\nu}$ are functions of x^1 and x^μ. The coordinate x^1 represents the fifth dimension, and x^μ, where the μ can be any Greek index, represents the four dimensional space-time coordinates. x^0 will be used to represent time, and x^i, where the i can be any Latin index, is used to represent the three-space coordinates. We will also use the Einstein summation conversion where repeated indices indicate a summation.

Again following Klein[3] we take the fifth dimension to be closed with respect to x^1 so that $x^1 + 2\pi a$ comes back to the same point for some constant a. φ, α_μ, and $g_{\mu\nu}$ are taken as functions of x^1, and x^μ.

Following Einstein and Bergmann[8] if we now make the transformation $x^{1\prime} = D^{-1}\int_0^{x^1}\varphi dx^1$, where $D = \int_0^{2\pi a}\varphi dx^1$, then the new φ will be independent of x^1. In this case the curvature tensor components take the form

$$R_{11} = {R_{11}}' \qquad (2)$$

$$R_{1\mu} = R_{\mu 1} = R_{\mu 1}{}' + \alpha_\mu R_{11}{}' \tag{3}$$

$$R_{\mu\nu} = \alpha_\mu \alpha_\nu R_{11}{}' + \alpha_\mu R_{\nu 1}{}' + \alpha_\nu R_{\mu 1}{}' + R_{\mu\nu}{}' \tag{4}$$

where

$$R_{11}{}' = g^{\mu\nu}\left\{(\varphi^2 \alpha_{\mu,1})_{;\bar{\nu}} - \varphi\varphi_{;\mu\nu} - \varphi^2 \alpha_{\mu,1}\,\alpha_{\nu,1} - \frac{1}{2}\,g_{\mu\nu,11}\right\}$$
$$+ \frac{1}{4}\,\{-g^{\mu\nu}{}_{,1}\,g_{\mu\nu,1} + \varphi^4 F^{\mu\nu} F_{\mu\nu}\} \tag{5}$$

$$R'_{\mu 1} = -g^{\nu\gamma}[\frac{1}{2}\varphi\varphi_{,\nu}\,F_{\mu\gamma} + \frac{1}{2}(\varphi^2 F_{\mu\nu})_{;\bar{\gamma}} + \varphi^2 \alpha_{\gamma,1}\,F_{\nu\mu}$$
$$+ \frac{1}{2}\varphi\{(\varphi^{-1} g_{\nu\gamma,1})_{;\bar{\mu}} - (\varphi^{-1} g_{\mu\gamma,1})_{;\bar{\nu}}\}] \tag{6}$$

$$R_{\mu\nu}{}' = \frac{1}{2}\varphi^{-2}\{(\varphi^2 \alpha_{\mu,1})_{;\bar{\nu}} + (\varphi^2 \alpha_{\nu,1})_{;\bar{\mu}}\} - \varphi^{-1}\varphi_{;\mu\nu} - \alpha_{\mu,1}\,\alpha_{\nu,1}$$
$$+ \frac{1}{4} g^{\gamma\delta}\{2 g_{\gamma\mu,1}\,F_{\delta\nu} + g_{\gamma\delta,1}\,F_{\nu\mu}\} - \frac{1}{2}\,\varphi^2 g^{\gamma\delta} F_{\gamma\mu} F_{\delta\nu}$$
$$+ \frac{1}{2}\,\varphi^{-2}\left\{g^{\gamma\delta}\left(g_{\gamma\mu,1}\,g_{\delta\nu,1} - \frac{1}{2} g_{\gamma\delta,1}\,g_{\mu\nu,1}\right) - g_{\mu\nu,11}\right\} + g^{\gamma\delta} R_{\mu\gamma\nu\delta} \tag{7}$$

A comma represents a partial derivative, a semicolon represents a covariant derivative, and

$$()_{,\bar{\mu}} = ()_{,\mu} - \alpha_\mu ()_{,1} \tag{8}$$

$$()_{;\bar{\mu}} = ()_{;\mu} - \alpha_\mu ()_{,1} \tag{9}$$

where () represents a tensor of any rank and for tensors A, A_μ, $A_{\mu\nu}$ we have

$$A_{;\mu} = A_{,\mu} \tag{10}$$

$$A_{\mu;\nu} = A_{\mu,\nu} - \Gamma^{\gamma}_{\mu\nu} A_{\gamma} \tag{11}$$

$$A_{\mu\nu;\gamma} = A_{\mu\nu,\gamma} - \Gamma^{\delta}_{\mu\gamma} A_{\delta\nu} - \Gamma^{\delta}_{\gamma\nu} A_{\mu\delta} \tag{12}$$

where $\Gamma^{\gamma}_{\mu\nu} = g^{\gamma\delta}\Gamma_{\delta\mu\nu}$ and

$$\Gamma_{\mu\nu\gamma} = \frac{1}{2}\left(g_{\mu\nu,\overline{\gamma}} + g_{\mu\gamma,\overline{\nu}} - g_{\nu\gamma,\overline{\mu}}\right) \tag{13}$$

$g^{\mu\nu}$represents the inverse metric of $g_{\mu\nu}$, that is $g^{\mu\gamma} g_{\gamma\nu} = \delta^{\mu}_{\nu}$. We also have

$$F_{\mu\nu} = \alpha_{\mu,\overline{\nu}} - \alpha_{\nu,\overline{\mu}} \tag{14}$$

$$F^{\mu\nu} = g^{\mu\gamma} g^{\nu\delta} F_{\gamma\delta} \tag{15}$$

$$R_{\mu\gamma\nu\delta} = \frac{1}{2}\{g_{\mu\delta,\overline{\gamma}\overline{\nu}} + g_{\gamma\nu,\overline{\mu}\overline{\delta}} - g_{\gamma\delta,\overline{\mu}\overline{\nu}} - g_{\mu\nu,\overline{\gamma}\overline{\delta}}\} + \Gamma^{\lambda}_{\mu\delta}\Gamma_{\lambda\gamma\nu} - \Gamma^{\lambda}_{\mu\nu}\Gamma_{\lambda\gamma\delta} \tag{16}$$

The eqs. (2-7) for R_{11}, $R_{\mu 1}$ and $R_{\mu\nu}$ agree with Bejancu[9] when φ = 1, with Wesson[7] when α_{μ} = 0, and with Wehus and Revndal[10] when the metric is independent of the fifth dimension.

III. Linear Field Equations

We will take the field equations to be a 5D generalization of the Einstein vacuum field equations

$$R_{11} = 0 \tag{17}$$

$$R_{\mu 1} = 0 \tag{18}$$

$$R_{\mu\nu} = 0 \tag{19}$$

Now write $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ and $\varphi = 1 + \varepsilon$, where $\eta_{\mu\nu}$ is the diagonal Minkowski tensor with $\eta_{00} = -1$, $\eta_{0i} = 0$ and $\eta_{ij} = \delta_{ij}$ along $\eta^{\mu\nu} = \eta_{\mu\nu}$. If we only keep linear terms in $h_{\mu\nu}$, ε and α_μ then eqs. (17-19) reduce to

$$\eta^{\mu\nu}\left\{\alpha_{\mu,\nu 1} - \varepsilon_{,\mu\nu} - \frac{1}{2} h_{\mu\nu,11}\right\} = 0 \tag{20}$$

$$\eta^{\nu\gamma}\{ F_{\mu\nu,\gamma} + h_{\nu\gamma,1\mu} - h_{\mu\gamma,1\nu} \} = 0 \tag{21}$$

$$\alpha_{\mu,\nu 1} + \alpha_{\nu,\mu 1} - 2\varepsilon_{,\mu\nu} - h_{\mu\nu,11} + \eta^{\gamma\delta}\{h_{\mu\delta,\gamma\nu} + h_{\gamma\nu,\mu\delta} - h_{\gamma\delta,\mu\nu} - h_{\mu\nu,\gamma\delta}\} = 0 \tag{22}$$

Now consider the allowable coordinate transformations such that $\varphi_{,1} = 0$. Define new coordinates $x^{1'}$ and $x^{\mu'}$ such that

$$x^1 = x^{1'} + f(x^{\mu'}, x^{1'}) \tag{23}$$

$$x^\mu = x^{\mu'} + f^\mu(x^{\nu'}, x^{1'}) \tag{24}$$

If we only keep linear terms in f , f^{μ}, and the metric components $h_{\mu\nu}$, ε and α_{μ}, then the new metric components take the form

$$\varepsilon' = \varepsilon + f_{,1'} \quad (25)$$

$$\alpha'_{\mu} = \alpha_{\mu} + f_{,\mu'} + \eta_{\mu\nu} f^{\nu}{}_{,1'} \quad (26)$$

$$h'_{\mu\nu} = h_{\mu\nu} + \eta_{\mu\gamma} f^{\gamma}{}_{,\nu'} + \eta_{\gamma\nu} f^{\gamma}{}_{,\mu'} \quad (27)$$

Then in order for ε' to be independent of $x^{1'}$ we need

$$\varepsilon'_{,1'} = \varepsilon_{,\mu} f^{\mu}{}_{,1'} + f_{,1'1'} = 0 \quad (28)$$

Taking $\varepsilon_{,\mu} f^{\mu}{}_{,1'}$ as a second order effect we then need $f_{,1'1'} = 0$ and thus take f to be independent of $x^{1'}$. To see what further restriction we can make, take the $x^{1'}$ derivative of eq. (26) to obtain

$$\alpha'_{\mu,1'} = \alpha_{\mu,1'} + f_{,\mu'1'} + \eta_{\mu\nu} f^{\nu}{}_{,1'1'} \quad (29)$$

Since $f_{,\mu'1'} = 0$ eq. (29) will be zero if we take $f^{\mu}{}_{,1'1'} = -\eta^{\mu\nu}\alpha_{\nu,1'}$. Thus we can take α'_{μ} to be independent of $x^{1'}$ in this linear approximation.

Now dropping the prime, expand the metric in a Fourier expansion in x^{1} so that

$$h_{\mu\nu} = \sum_{n=-\infty}^{\infty} a_{\mu\nu}^{n} \, e^{inx^1/a} \qquad (30)$$

$h_{\mu\nu}$ is real so we need $a_{\mu\nu}^{-n} = a_{\mu\nu}^{n*}$ where $a_{\mu\nu}^{n*}$ is the complex conjugate of $a_{\mu\nu}^{n}$. We then take $a_{\mu\nu}^{n}$ for $n \geq 0$ to be the independent degrees of freedom. Eqs. (20-22) then become

$$\eta^{\mu\nu} a_{\mu\nu}^{n} = 0 \qquad (31)$$

$$\eta^{\nu\gamma}\{a_{\nu\gamma,\mu}^{n} - a_{\mu\gamma,\nu}^{n}\} = 0 \qquad (32)$$

$$\frac{n^2}{a^2} a_{\mu\nu}^{n} + \eta^{\gamma\delta}\{a_{\mu\delta,\gamma\nu}^{n} + a_{\gamma\nu,\mu\delta}^{n} - a_{\gamma\delta,\mu\nu}^{n} - a_{\mu\nu,\gamma\delta}^{n}\} = 0 \qquad (33)$$

where we have set $\alpha_{\mu,1} = 0$ and have only considered the $n > 0$ terms. Eqs. (31-33) reduce to the conditions

$$\eta^{\mu\nu} a_{\mu\nu}^{n} = 0 \qquad (34)$$

$$\eta^{\nu\gamma} a_{\mu\nu,\gamma}^{n} = 0 \qquad (35)$$

$$\eta^{\gamma\delta} a_{\mu\nu,\gamma\delta}^{n} = \frac{n^2}{a^2} a_{\mu\nu}^{n} \qquad (36)$$

Eqs. (34-36) are the same equations as found in Bailin and Love[5]. Following Mandel and Wolf[11], expand $a_{\mu\nu}^{n}$ in a Fourier series in a box of size L^3 so that

$$a^n_{\mu\nu} = \frac{1}{\sqrt{L^3}} \sum_{\mathbf{k}} a^n_{\mu\nu\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \tag{37}$$

where $\mathrm{k}_{\boldsymbol{i}} = \frac{\boldsymbol{2\pi}}{L} \mathrm{n}_{\boldsymbol{i}}$ with $\mathrm{n}_{\boldsymbol{i}}$ an integer, and a bold letter indicates a vector. Using eq. (37) in eqs. (34-36) they then reduce to

$$\eta^{\mu\nu} a^n_{\mu\nu\mathbf{k}} = 0 \tag{38}$$

$$a^n_{\mu 0\mathbf{k},0} = ik^i a^n_{\mu i\mathbf{k}} \tag{39}$$

$$k^2 a^n_{\mu\nu\mathbf{k}} + a^n_{\mu\nu\mathbf{k},00} = -\frac{n^2}{a^2} a^n_{\mu\nu\mathbf{k}} \tag{40}$$

For a solution write

$$a^n_{\mu\nu\mathbf{k}} = a^{n+}_{\mu\nu\mathbf{k}} e^{i\omega t} + a^{n-}_{\mu\nu\mathbf{k}} e^{-i\omega t} \tag{41}$$

When eq. (41) is applied to eqs. (38-40) , we obtain the condition that $\omega^2 = \frac{n^2}{a^2} + k^2$ and

$$a^{n\pm}_{00\mathbf{k}} = \delta^{ij} a^{n\pm}_{ij\mathbf{k}} \tag{42}$$

$$a^{n\pm}_{0i\mathbf{k}} = \pm \frac{k^j}{\omega} a^{n\pm}_{ij\mathbf{k}} \tag{43}$$

with the $a^{n+}_{ij\mathbf{k}}$ and $a^{n-}_{ij\mathbf{k}}$ arbitrary up to the constraint equation

$$(\omega^2\delta^{ij} - k^i k^j)a^{n\pm}_{ij\mathbf{k}} = 0 \tag{44}$$

IV. Invariants

We would like to find a distribution of $a^{n+}_{ij\mathbf{k}}$ and $a^{n-}_{ij\mathbf{k}}$ which is invariant under Lorentz transformations, so we want to see how $a^{n+}_{ij\mathbf{k}}$ and $a^{n-}_{ij\mathbf{k}}$ transform under a Lorentz transformation. To accomplish this, we can write

$$a^n_{\mu\nu} = \frac{1}{\sqrt{8\pi^3}}\int d^3k\, a^n_{\mu\nu}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}} \tag{45}$$

where

$$a^n_{\mu\nu}(\mathbf{k}) = (\frac{L}{2\pi})^{\frac{3}{2}}a^n_{\mu\nu\mathbf{k}} = a^{n+}_{\mu\nu}(\mathbf{k})e^{i\omega t} + a^{n-}_{\mu\nu}(\mathbf{k})e^{-i\omega t} \tag{46}$$

From Landau and Lifshitz[12] we have that the integral over k space transforms as

$$\int d^3k = \int \frac{\omega}{\omega'}d^3k\,' \tag{47}$$

under a Lorentz transformation. k' is the transformed k vector, and ω' is the frequency associated with it. Using eqs. (45-47) and the fact that $\mathbf{k}\cdot\mathbf{x} - \omega t$ is invariant under Lorentz transformations, we find that under a Lorentz transformation $a^{n\pm}_{\mu\nu}(\mathbf{k})$ transforms as

$$a^{n\pm}_{\mu'\nu'}{}'(\mathbf{k}') = \frac{\omega}{\omega'}\Lambda^{\mu}_{\mu'}\Lambda^{\nu}_{\nu'}a^{n\pm}_{\mu\nu}(\mathbf{k}) \tag{48}$$

where the $\Lambda^{\mu}_{\mu'}$ are Lorentz transformation matrices. Using eqs. (47) and (48), we have the invariants $\omega^2 a^{\mu\nu n\pm *}(\mathbf{k}) a^{n\pm}_{\mu\nu}(\mathbf{k})$ and $\int \frac{1}{\omega} d^3k$. We have set $a^{\gamma\delta n\pm}(\mathbf{k}) = \eta^{\gamma\mu}\eta^{\delta\nu} a^{n\pm}_{\mu\nu}(\mathbf{k})$ and a * represents the complex conjugate. Thus

$$\int \frac{1}{\omega} d^3k\, \omega^2 a^{\mu\nu n\pm *}(\mathbf{k}) a^{n\pm}_{\mu\nu}(\mathbf{k}) = \int d^3k\, \omega a^{\mu\nu n\pm *}(\mathbf{k}) a^{n\pm}_{\mu\nu}(\mathbf{k}) \tag{49}$$

transforms as a scalar under a Lorentz transformation.

V. Probability Distribution and Expectation Values

We will take the distribution of the $a^{n\pm}_{\mu\nu}(\mathbf{k})$ to be invariant under a Lorentz transformation and assume the following Gaussian form

$$P = Cexp(-\sum_{n>0} \frac{1}{\sigma_n^2} \int d^3k\, \omega (a^{\mu\nu n+*}(\mathbf{k}) a^{n+}_{\mu\nu}(\mathbf{k}) + a^{\mu\nu n-*}(\mathbf{k}) a^{n-}_{\mu\nu}(\mathbf{k})))$$

$$= Cexp(-\sum_{n>0} \frac{1}{\sigma_n^2} \sum_{\mathbf{k}} \omega (a^{\mu\nu n+*}_{\mathbf{k}} a^{n+}_{\mu\nu\mathbf{k}} + a^{\mu\nu n-*}_{\mathbf{k}} a^{n-}_{\mu\nu\mathbf{k}}))$$

$$= Cexp(-\sum_{n>0} \frac{1}{\sigma_n^2} \sum_{\mathbf{k}} \omega a^{\mu\nu n\pm *}_{\mathbf{k}} a^{n\pm}_{\mu\nu\mathbf{k}}) \tag{50}$$

where C is a normalization constant and σ_n is an input to determine the width of the distribution.

Using eqs. (42) and (43) we can express eq. (50) in terms of the degrees of freedom $a^{n\pm}_{ij\mathbf{k}}$ as

$$P = Cexp(-\sum_{n>0} \frac{1}{\sigma_n^2} \sum_{\mathbf{k}} \omega (\delta^{ij} a^{n\pm *}_{ij\mathbf{k}}\, \delta^{kl} a^{n\pm}_{kl\mathbf{k}} - 2\delta^{ij} \frac{k^l k^k}{\omega^2} a^{n\pm *}_{il\mathbf{k}} a^{n\pm}_{jk\mathbf{k}} + a^{ijn\pm *}_{\mathbf{k}} a^{n\pm}_{ij\mathbf{k}})) \tag{51}$$

subject to the constraint equation (44).

To find the constant C, normalize the probability P so that

$$\prod_{n>0} \prod_{\mathbf{k}} \int da_{ij\mathbf{k}}^{n+R} \int da_{ij\mathbf{k}}^{n+I} \int da_{ij\mathbf{k}}^{n-R} \int da_{ij\mathbf{k}}^{n-I} P = 1 \qquad (52)$$

subject to the constraint eq. (44). We have set

$$\int da_{ij\mathbf{k}}^{n\pm R} = \int_{-\infty}^{\infty} da_{11\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{12\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{13\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{22\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{23\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{33\mathbf{k}}^{n\pm R} \qquad (53)$$

with the same idea for $\int da_{ij\mathbf{k}}^{n\pm I}$ where $a_{ij\mathbf{k}}^{n\pm} = a_{ij\mathbf{k}}^{n\pm R} + i a_{ij\mathbf{k}}^{n\pm I}$.

To do this calculation, we will choose special coordinate system for each value of **k** such that in this coordinate system x^1 points in the **k** direction. In this case the constraint eq. (44) becomes

$$a_{11\mathbf{k}}^{n\pm} = -(1 + \frac{a^2}{n^2} k^2)(a_{22\mathbf{k}}^{n\pm} + a_{33\mathbf{k}}^{n\pm}) \qquad (54)$$

so that integrating over $a_{11\mathbf{k}}^{n\pm}$ we find we just need to replace $a_{11\mathbf{k}}^{n\pm}$ in the probability distribution, eq. (51), by $-(1 + \frac{a^2}{n^2} k^2)(a_{22\mathbf{k}}^{n\pm} + a_{33\mathbf{k}}^{n\pm})$ so that now eq. (53) is replaced by

$$\int da_{ij\mathbf{k}}^{n\pm R} = \int_{-\infty}^{\infty} da_{12\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{13\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{22\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{23\mathbf{k}}^{n\pm R} \int_{-\infty}^{\infty} da_{33\mathbf{k}}^{n\pm R} \qquad (55)$$

The normalization condition eq. (52) then reduces to the condition

$$C = \prod_{n>0}\prod_{\mathbf{k}}[\sqrt{\frac{24\omega}{\pi^5}}\frac{n^2}{a^2\sigma_n^5}]^4 \quad (56)$$

Next look at the expectation values given by

$$< a_{ij\mathbf{k}'}^{n'\pm} > = \prod_{n>0}\prod_{\mathbf{k}}\int da_{ij\mathbf{k}}^{n+R}\int da_{ij\mathbf{k}}^{n+I}\int da_{ij\mathbf{k}}^{n-R}\int da_{ij\mathbf{k}}^{n-I}\,(a_{ij\mathbf{k}'}^{n'\pm}P) \quad (57)$$

$$<a_{ij\mathbf{k}'}^{n'\pm}a_{kl\mathbf{k}''}^{n''\pm}> = \prod_{n>0}\prod_{\mathbf{k}}\int da_{ij\mathbf{k}}^{n+R}\int da_{ij\mathbf{k}}^{n+I}\int da_{ij\mathbf{k}}^{n-R}\int da_{ij\mathbf{k}}^{n-I}\,(a_{ij\mathbf{k}'}^{n'\pm}a_{kl\mathbf{k}''}^{n''\pm}P) \quad (58)$$

where in eq. (58) the $a_{ij\mathbf{k}'}^{n'\pm}$ and $a_{kl\mathbf{k}''}^{n''\pm}$ terms represent the real or imaginary parts, along with either the + or – components, so that eq. (58) actually represents ten different equations. Using the same methods as used for finding C, we find that

$$< a_{ij\mathbf{k}}^{n\pm} >= 0 \quad (59)$$

$$< a_{ij\mathbf{k}}^{n\pm R}a_{kl\mathbf{k}'}^{n'\pm I} > = < a_{ij\mathbf{k}}^{n\pm R}a_{kl\mathbf{k}'}^{n'\mp I} > = < a_{ij\mathbf{k}}^{n+R}a_{kl\mathbf{k}'}^{n'-R} >=< a_{ij\mathbf{k}}^{n+I}a_{kl\mathbf{k}'}^{n'-I} > = 0 \quad (60)$$

along with

$$< a_{ij\mathbf{k}}^{n\pm R}a_{kl\mathbf{k}'}^{n'\pm R} > = < a_{ij\mathbf{k}}^{n\pm I}a_{kl\mathbf{k}'}^{n'\pm I} > = \delta^{nn'}\delta_{\mathbf{k}\mathbf{k}'} < a_{ij\mathbf{k}}^{n}a_{kl\mathbf{k}}^{n} > \quad (61)$$

where in the frame of reference where **k** is aligned with x^1 we have

$$< a_{11\mathbf{k}}^{n}a_{11\mathbf{k}}^{n} > = \frac{1}{3n^4}a^4\sigma_n^2\omega^3 \quad (62)$$

$$< a^n_{11\mathbf{k}} a^n_{22\mathbf{k}} > = < a^n_{11\mathbf{k}} a^n_{33\mathbf{k}} > = -\frac{1}{6n^2} a^2 \sigma_n^2 \omega \tag{63}$$

$$< a^n_{12\mathbf{k}} a^n_{12\mathbf{k}} > = < a^n_{13\mathbf{k}} a^n_{13\mathbf{k}} > = \frac{1}{4n^2} a^2 \sigma_n^2 \omega \tag{64}$$

$$< a^n_{23\mathbf{k}} a^n_{23\mathbf{k}} > = \frac{1}{4\omega} \sigma_n^2 \tag{65}$$

$$< a^n_{22\mathbf{k}} a^n_{22\mathbf{k}} > = < a^n_{33\mathbf{k}} a^n_{33\mathbf{k}} > = \frac{1}{3\omega} \sigma_n^2 \tag{66}$$

$$< a^n_{22\mathbf{k}} a^n_{33\mathbf{k}} > = -\frac{1}{6\omega} \sigma_n^2 \tag{67}$$

with the other $< a^n_{ij\mathbf{k}} a^n_{kl\mathbf{k}} >$ equal to zero.

In a general frame of reference these results reduce to

$$< a^n_{ij\mathbf{k}} a^n_{kl\mathbf{k}} > = \frac{\sigma_n^2}{4\omega} \{d_{ik} d_{jl} + d_{il} d_{jk} - \frac{2}{3} d_{ij} d_{kl}\} \tag{68}$$

where

$$d_{ij} = \delta_{ij} + \frac{a^2}{n^2} k_i k_j \tag{69}$$

Now $\omega^2 = \frac{n^2}{a^2} + k^2$, so for very large **k** values, that is for $k \gg \frac{n}{a}$, we have $\omega \sim k$. Then from eqs. (68) and (69) it is apparent that the expectation values of $a^n_{ij\mathbf{k}}$ blow up as $\sigma_n^2 \frac{a^4}{n^4} k^3$ and that nonlinear terms would have to be taken into account to see what would happen at very high frequencies.

A way out of this would be to only have transverse fluctuations so that $k^j a_{ij\mathbf{k}}^{n\pm} = 0$ with the constraint equation (44) requiring that $\delta^{ij} a_{ij\mathbf{k}}^{n\pm} = 0$. From eqs. (42) and (43) we would then need $a_{00\mathbf{k}}^{n\pm} = a_{0i\mathbf{k}}^{n\pm} = 0$ which would not be the case when we transform to a new frame, so this idea is not frame independent.

VI. Conclusions

If the fluctuations were finite and died off at very high frequency they could have been used in the rest of the Kaluza-Klein equations as an extra fluctuating force in Maxwell's equations, and perhaps Einstein's field equations. In that way they might be interpreted as a possible source for a hidden variable theory similar to the ideas of Nelson's stochastic mechanics[13]. Since the fluctuations blow up at high frequency nonlinear terms would have be taken into account, and that is beyond the scope of this investigation. Bergia, Cannata, and Pasini[14] have also looked at the idea of metric fluctuations being the source of quantum mechanics, but consider conformal metric fluctuations.

Since the five dimensional fluctuations blow up at high frequency it is interesting to compare them to the vacuum fluctuations of the electromagnetic field. The five dimensional vacuum fluctuations blow up as ω^3 while those of the vacuum electromagnetic field go as ω, for example see Mandel and Wolf[11].

References

[1]Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Phys. Math. Klasse **10**, 966 (1921).

[2]O. Klein, Z. Phys. **37**, 895 (1926).

[3]O. Klein, Nature **118** 516 (1926).

[4]T. Applequist, A. Chodos and P. Freund *Modern Kaluza-Klein theories* (Addison-Wesley Press, Reading Mass.,1987).

[5]D. Bailin and A. Love, Rep. Prog. Phys. **50**, 1087 (1987).

[6]J. M. Overduin and P. S. Wesson, Phy. Reports **283**, 303 (1997).

[7]P. S. Wesson, *Space-Time-Matter, Modern Kaluza-Klein Theory* (World Scientific, 1999).

[8]A. Einstein and P. Bergmann, Ann. Math. **39**, 683 (1938).

[9]A. Bejancu, Prog. Theor. Phys. **128**, 541 (2012).

[10]I. K. Wehus, and F. Ravndal, Int. J. of Mod. Phys. A **19**, 4671 (2004).

[11]L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge U.P., 1995).

[12]L. D. Landau and E. M. Lifshitz , *The Classical Theory of Fields* (Pergamon Press, New York, 1975).

[13]E. Nelson, *Quantum Fluctuations* (Princeton University Press, Princeton NJ, 1985).

[14]S. Bergia, F. Cannata, and A. Pasini, Phy. Letts A **137**, 21 (1989).