Theoretical Treatment on the Hydrostatic Stability of Ships (Part 2:) Stable Attitude in an Inclined State

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Summary

In this paper, a theoretical treatment on the hydrostatic stability of ships is presented. As the simplest hull form, a columnar ship with rectangular cross-section, which is made of homogeneous squared timber with arbitrary breadth and material, is chosen.

In the Part 2 of this theme, the stable attitude in an inclined state of the ship, which is not stable in the upright state with horizontal deck, is analyzed by means of ship's hydrostatics. By doing so, the dependence of the inclined attitude on the breadth and material of the ship will be clarified.

Keywords : ***Hydrostatic Stability, Columnar Ship, Rectangular Cross-Section, Arbitrary Breadth and Material, Stable Attitude, Inclined State***

1. Introduction

In the previous paper(1) as a typical example problem(2),(3) related to the hydrostatic stability of ships, we solved the condition under which the ship floats stably in the upright state with horizontal deck, in terms of the positional relationship among the center of buoyancy, center of gravity and metacenter. At that time, the target hull form is a columnar ship with a rectangular cross-section, which is made of homogeneous squared timber with arbitrary breadth and material.

On the other hand, if the above conditions are not satisfied, under what inclined attitude does the ship float ? is also of interest from a mechanical point of view. Igarashi et al. of the National Defense Academy of Japan have elucidated this problem in detail based on geometrical considerations concerning the center of buoyancy and the center of gravity for the squared timber with square(4) and rectangular(5) cross-sections.

In this 2nd paper, as an extension of the previous 1st paper(1), we describe a theoretical treatment for solving the stable attitude of a columnar ship with a rectangular cross-section in an inclined state. The one of the authors gave an solution for the inclined attitude and published it in the journal(6) "*NAVIGATION*" of Japan Institute of Navigation at 2021.

We summarize the above solutions consistently and introduce them in this paper.

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2. Material α and Breadth β as Setting Variables

In this paper, α and β are defined as the setting variable, as in the previous paper (1). α (hereinafter called the ***material***) is the ratio of the specific weight γ_t of the columnar ship (t in the subscript is the initial letter of *timber*) to that γ_w of water (w in the subscript is the initial letter of *water*), and β (hereinafter called the ***breadth***) is the aspect ratio of the breadth βh to the depth h of the cross-section as follows :

$$\left.\begin{aligned} &\alpha \equiv \frac{\gamma_t}{\gamma_w} \quad (where,\ 0<\alpha \le 1) \\ &\beta \equiv \frac{breadth}{depth} = \frac{\beta h}{h} \quad (where,\ \beta > 0) \end{aligned}\right\} \quad \cdots\cdots (1)$$

Here, when γ_w is fresh water, α represents the specific gravity of the columnar ship.

3. Stable Conditions in the Upright State for a Columnar Ship with Rectangular Cross-Section

In the previous paper(1), the condition for stable floating in the upright state with deck horizontal can be written as the relation between α and β in Eq. (1) as follows :

$$\beta^2 - 6\alpha\,(1-\alpha) > 0 \quad \cdots\cdots (2)$$

Hence, it was explained that the above condition can be divided into the following cases(1) :

● Stable conditions for breadth β with fixed material α

$$\left.\begin{aligned} &\beta > \sqrt{6\alpha\,(1-\alpha)} \\ &\quad e.g.\quad \alpha = \frac{1}{2} \rightarrow \beta >= \frac{\sqrt{6}}{2} \fallingdotseq 1.225 \end{aligned}\right\} \quad \cdots\cdots (3)$$

● Stable conditions for material α with fixed breadth β

i) In the case of $\beta > \dfrac{\sqrt{6}}{2}$ for wide breadth,

the floating body is always stable regardless of material α .

ii) In the case of $\beta < \dfrac{\sqrt{6}}{2}$ for narrow breadth,

it is then stable in both lighter and heavier materials than wood with $\alpha = \dfrac{1}{2}$ as shown below :

$$\left.\begin{array}{l} 0 < \alpha < \dfrac{1}{2} - \kappa \quad (\textit{Light Material}) \\ \dfrac{1}{2} + \kappa < \alpha \le 1 \quad (\textit{Heavy Material}) \\ \quad \textit{where,} \left\{ \begin{array}{l} \kappa \equiv \dfrac{\sqrt{3(3-2\beta^2)}}{6} \\ \textit{e.g.} \quad \beta = 1 \rightarrow \kappa = \dfrac{\sqrt{3}}{6} \fallingdotseq 0.289 \end{array} \right. \end{array}\right\} \cdots\cdots (4)$$

4. Stable Attitude for an Inclined Columnar Ship with Rectangular Cross-Section

In this chapter, we will try to find out what kind of inclined state is stable when the stable condition in the upright state described in the previous chapter is not satisfied. For this purpose, let's analyze the inclined attitude, *i.e.* the heel angle, of the columnar ship.

As shown in Fig. 1, we shall assume that a columnar ship of length L with a rectangular cross-section of depth h and breadth βh, which is made of homogeneous material and of squared timber of specific weight γ_t, floats stably in a lateral inclined state of heel angle θ to the starboard side from an upright state.

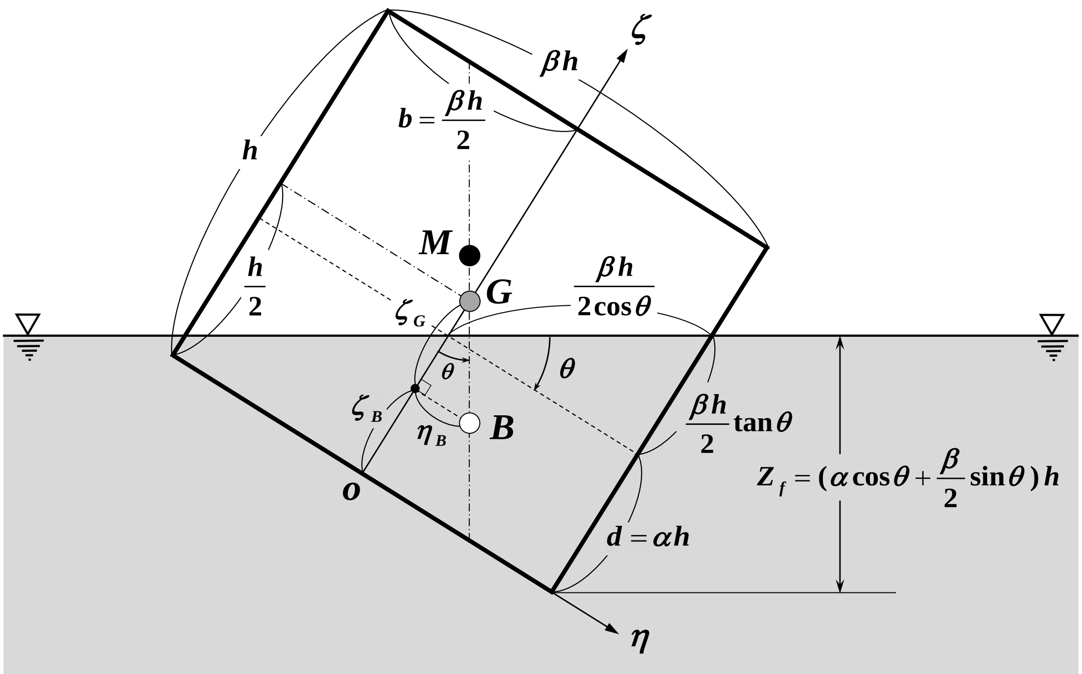

Fig. 1 The columnar ship, with rectangular cross-section of length L, floating stably in a lateral inclined state.

First, in order to determine the draft, we need to find the cross-sectional area A_w under the water surface at lateral inclination.

Since its underwater shape is a trapezoid with height βh, the lengths of its upper and lower bases can be calculated by taking into account the increase or decrease $\frac{\beta h}{2}\tan\theta$ of the port and starboard submerged lengths with respect to the draft d in the upright state. So, the underwater area A_w is obtained as follows :

$$A_w = \frac{1}{2}\left\{ (d - \frac{\beta h}{2}\tan\theta) + (d + \frac{\beta h}{2}\tan\theta) \right\} \cdot \beta h$$
$$= \beta h d \quad \cdots\cdots(5)$$

Here, the above result is equal to the area of the rectangle, which is the underwater shape in the upright state.

The weight W and the buoyant force F_B of this columnar ship can be obtained as follows, respectively :

$$\left. \begin{aligned} W &= \gamma_t V_t = \gamma_t \cdot \beta h \cdot h \cdot L \\ F_B &= \gamma_w V_w = \gamma_w A_w L = \gamma_w \cdot \beta h d \cdot L \end{aligned} \right\} \quad \cdots\cdots(6)$$

Here, the weight W of the former is obtained as the product of the specific weight γ_t and the total volume V_t of the columnar ship. And the buoyant force F_B of the latter is obtained as the product of the specific weight γ_w of water and the displacement volume V_w of underwater portion, according to Archimedes' principle. Then V_w is obtained by the product of the cross-sectional area A_w in Eq. (5) and the ship's length L.

The floating body is stable under the following conditions where the weight W and buoyancy F_B are in equilibrium.

$$W = F_B \quad \cdots\cdots(7)$$

Substituting W and F_B in Eq. (6) into both sides of the above, we obtain as :

$$\gamma_t \cdot \beta h \cdot h \cdot L = \gamma_w \cdot \beta h \cdot d \cdot L \quad \cdots\cdots(8)$$

By solving the above equation, the undetermined draft d in the upright state can be determined as α times the depth h of the ship, as follows :

$$d = \frac{\gamma_t}{\gamma_w} h = \alpha h \quad \cdots\cdots(9)$$

In this paper, to simplify the problem, it is assumed that the deck, *i.e.* upper side of a rectangular cross-section, is in the air and the bottom, *i.e.* lower side of a rectangle, is in the water over the entire breadth even when the ship is laterally inclined, as shown in Fig. 1. That is, we will discuss the case in which the cross-sectional shape under the water surface is trapezoidal, as calculated in Eq. (5).

The above assumptions would impose the following conditions, where the increase or decrease $\frac{\beta h}{2}\tan\theta$ of submerged length due to the lateral inclination does not exceed the freeboard $h-d$ or the draft d in the upright state, while divided into two cases around $\alpha=\frac{1}{2}$.

$$\frac{\beta h}{2}\tan\theta \le \begin{cases} h-d=(1-\alpha)\,h & (\ \textit{for Heavy Material of}\ \ \alpha \ge \frac{1}{2}\ ) \\ d=\alpha h & (\ \textit{for Light Material of}\ \ \alpha < \frac{1}{2}\ ) \end{cases} \quad \cdots\cdots (10)$$

Therefore, the heel angle θ is limited to small inclination within the following range.

$$\theta \le \begin{cases} \tan^{-1}\left(\dfrac{2\,(1-\alpha)}{\beta}\right) & (\ \textit{for Heavy Material of}\ \ \alpha \ge \frac{1}{2}\ ) \\ \tan^{-1}\left(\dfrac{2\alpha}{\beta}\right) & (\ \textit{for Light Material of}\ \ \alpha < \frac{1}{2}\ ) \end{cases} \quad \cdots\cdots (11)$$

For example, it means the following setting range.

$$\left.\begin{array}{l} \beta=1\,,\ \alpha=\dfrac{1}{2} \\ \rightarrow\ \tan\theta \le 1\ \ \therefore \theta \le \dfrac{\pi}{4} \end{array}\right\} \quad \cdots\cdots (12)$$

The position of the center of buoyancy $B\,(\eta_B\,,\zeta_B)$ in the inclined state by heel angle θ is obtained by Hori[(7)~(11)] as the center of hydrostatic pressure. As shown in Fig. 1, in the inclined coordinate system, which is fixed to the ship and has its origin at the center of the ship's bottom, the position $(\eta_B\,,\zeta_B)$ is calculated as follows[(7),(10),(11)] when the draft and half-breadth of the ship in upright state are f and b, respectively.

$$\left.\begin{array}{l} \eta_B=\dfrac{b^2}{3f}\tan\theta \\ \zeta_B=\dfrac{f}{2}+\dfrac{b^2}{6f}\tan^2\theta \end{array}\right\} \quad \cdots\cdots (13)$$

This result is consistent with the result of Eq. (A-5) described in the Appendix A-1 of the above Hori's papers[(10),(11)]. In the Appendix, the centroid of the trapezoid, which is the cross-sectional shape under the water surface at lateral inclination, is calculated geometrically from the areal moment. Therefore, it can be seen that Eq. (13) coincides with the well-known center of buoyancy certainly.

Here, in order to conform to the notation of this paper, f and b in Eq. (13) are replaced as follows respectively.

$$\left.\begin{array}{l} f\ =\ d\ =\alpha h \\ b\ =\ \dfrac{1}{2}\beta h \end{array}\right\} \quad \cdots\cdots (14)$$

Thereby, η_B and ζ_B can be written as follows :

$$\left.\begin{aligned}\eta_B &= \frac{\beta^2 \tan\theta}{12\alpha} h \\ \zeta_B &= \frac{\alpha h}{2} + \frac{\beta^2 \tan^2\theta}{24\alpha} h\end{aligned}\right\} \quad \cdots\cdots (15)$$

Next, the center of gravity of the ship is located at the centroid of the rectangular cross-section (*i.e.*, at the center of the figure), even after inclining, since homogeneous materials are assumed. Therefore, using the fact that the sum of ζ_B and ζ_G is equal to $\frac{h}{2}$, ζ_G in Fig. 1 can be obtained as follows :

$$\begin{aligned}\zeta_G &= \frac{h}{2} - \zeta_B \\ &= \frac{1-\alpha}{2} h - \frac{\beta^2 \tan^2\theta}{24\alpha} h \\ &= \frac{12\alpha(1-\alpha) - \beta^2 \tan^2\theta}{24\alpha} h \quad \cdots\cdots (16)\end{aligned}$$

In order for the ship to float while maintaining the inclined state shown in Fig. 1, the center of buoyancy B and the center of gravity G must first be located on the same vertical line. Therefore, the following relationship is required between η_B and ζ_G.

$$\left.\begin{aligned}&\frac{\eta_B}{\zeta_G} = \tan\theta \\ &\therefore\ \eta_B = \zeta_G \tan\theta\end{aligned}\right\} \quad \cdots\cdots (17)$$

Here, by using Eqs. (15) and (16) for η_B and ζ_G, the following relationship is obtained.

$$\beta^2 \tan^2\theta = 2\left\{6\alpha(1-\alpha) - \beta^2\right\} \quad \cdots\cdots (18)$$

The tangent of the inclined attitude θ for a given material α and breadth β is then obtained by the following equation.

$$\tan\theta = \frac{\sqrt{2\left\{6\alpha(1-\alpha) - \beta^2\right\}}}{\beta} \quad \cdots\cdots (19)$$

When the interior of the radical symbol of the right-hand side of the above equation is positive, there exists a solution for the heel angle θ. This result coincides with Eqs. (1-h) and (4-f) of Igarashi and Nakamura[(5)]. This requires that the interior of the braces in the numerator of the above equation take positive values, as follows :

$$6\alpha(1-\alpha) - \beta^2 > 0 \quad \cdots\cdots (20)$$

The inequality above is the inverse condition in which the inequality sign is opposite to the stable condition in the upright state in Eq. (2) of Chapter 3, and the validity of the analysis in this chapter can be confirmed.

Finally, it is necessary to examine whether the above-mentioned inclined attitude is stable or not. For this purpose, let's consider determining the location of *the metacenter M,* meaning *the center of inclination*.

The metacentric radius $\overline{BM}$ can be calculated by using the basic formula [13],[14],[15] of naval architecture as follows :

$$\overline{BM} = \frac{I_{CL}}{V_w} \quad \cdots\cdots (21)$$

Here, I_{CL} is the quadratic moment about the center line of water plane, and V_w is the underwater volume of a ship.

I_{CL} in the numerator of the above formula can be calculated as follows, since the water plane at inclination is a rectangle of length L and breadth $\dfrac{\beta h}{\cos\theta}$.

$$I_{CL} = \frac{1}{12}\left(\frac{\beta h}{\cos\theta}\right)^3 L \quad \cdots\cdots (22)$$

And the denominator V_w can be obtained by using d in Eq. (9) for A_w in Eq. (5) and as follows :

$$V_w = A_w L = \beta h d \cdot L = \alpha\beta h^2 L \quad \cdots\cdots (23)$$

By using the obtained results I_{CL} and V_w into Eq. (21), $\overline{BM}$ can be determined independently of the length L of the columnar ship as follows :

$$\overline{BM} = \frac{\frac{1}{12}\left(\frac{\beta h}{\cos\theta}\right)^3 L}{\alpha\beta h^2 L} = \frac{\beta^2}{12\alpha\cos^3\theta}h \quad \cdots\cdots (24)$$

$\overline{BG}$ in the inclined state is then obtained below by using the trigonometric ratio with η_B in Eq. (15), as shown in Fig. 1.

$$\overline{BG} = \frac{\eta_B}{\sin\theta} = \frac{\beta^2}{12\alpha\cos\theta}h = \overline{BM}\cos^2\theta \quad \cdots\cdots (25)$$

Thereby, the metacentric height $\overline{GM}$ can be determined by subtracting Eq. (25) from Eq. (24), as follows :

$$\begin{aligned}\overline{GM} &= \overline{BM} - \overline{BG} \\ &= \frac{\beta^2(1-\cos^2\theta)}{12\alpha\cos^3\theta}h = \frac{\beta^2\sin^2\theta}{12\alpha\cos^3\theta}h \\ &= \overline{BM}\sin^2\theta \geq 0 \quad \cdots\cdots (26)\end{aligned}$$

From this result, the metacenter M is always located above the center of gravity G, since $\overline{GM}$ takes a positive value regardless of the heel angle θ, material α and breadth β. Therefore, it can be seen that the inclined attitude θ determined by Eq. (19) is constantly a stable state. However, it is necessary to check that the calculated θ is within the assumed small heel angle in Eq. (11).

Here, let us take few considerations on $\overline{GM}$. Eq. (19) shows that when $\beta^2 = 6\alpha(1-\alpha)$, which corresponds to Eq. (30) in next chapter, the inside of the radical symbol is zero and $\tan\theta = 0$, so the floating body is an upright state with heel angle $\theta = 0$. At this time, since $\overline{GM} = 0$ from Eq. (26), M and G coincide and the floating hydrostatic state is neutral. On the other hand, when α and β satisfy the above condition, $\overline{GM}$ for the upright state shown in Eq. (13) of the previous paper(1) is also zero. Hence, it can be seen that the equation for the upright state and the Eq. (26) for the inclined state derived in this paper are connected consistently at $\theta = 0$ in the neutral state between both formulas for the metacentric height $\overline{GM}$.

For example, in the states β and α below, the heel angle θ, $\overline{BG}$ and $\overline{GM}$ are calculated as follows by Eqs. (19), (25), and (26).

$$\left.\begin{aligned} &\beta = 1\,,\ \alpha = \frac{1}{2} \ \to\ \theta = \frac{\pi}{4} \\ &\therefore\ \overline{GM} = \overline{BG} = \frac{\sqrt{2}}{6}\,h \end{aligned}\right\} \quad \cdots\cdots (27)$$

This state corresponds to the case where the diagonal line of the square cross-section is aligned with the water line, and the heel angle θ is also within the setting range of Eq. (12). And $\overline{BG}$ and $\overline{GM}$ also coincide with the results described in examples of many textbooks(16), (17), (18).

In addition, Fig. 1 shows the following states, and the inclined attitude θ and the positions of B, G and M are also drawn accurately.

$$\left.\begin{aligned} &\alpha = 0.4\,,\ \beta = 1.1 \ \to\ \theta = 31.7^\circ \\ &\therefore\ \overline{GM} = 0.113\,h\,,\ \overline{BG} = 0.296\,h \\ &\quad Z_f = 0.629\,h \end{aligned}\right\} \quad \cdots\cdots (28)$$

Here, Z_f in the above Eq. (28) is the water depth at the starboard side of the ship's bottom, and is calculated by the following equation.

$$\begin{aligned} Z_f &= (\,d + \frac{\beta h}{2}\tan\theta\,)\cos\theta \\ &= (\,\alpha\cos\theta + \frac{\beta}{2}\sin\theta\,)\,h \end{aligned} \quad \cdots\cdots (29)$$

5. Calculation Results for the Stable Inclined Attitude θ

In this chapter, the dependence of the stable attitude θ at lateral inclined state on the breadth β and material α of the columnar ship is grasped.

Fig. 2 shows the dependence of the above on breadth β when α is a fixed, and Fig. 3 shows that on material α when β is a fixed. The results in both figures are obtained by calculating the heel angle θ in Eq. (19) using an Excel spreadsheet.

Since $\theta = 0$ means that the ship floats with its deck horizontal and is the limit point at which the inequality sign in Eq. (2) becomes an equality sign, α and β satisfy the following relationship at that point.

$$\beta^2 - 6\alpha\,(1-\alpha) = 0 \qquad \cdots\cdots (30)$$

Thereby the intersection with β - axis in Fig. 2 is obtained by Eq. (3), and that with α - axis in Fig. 3 is obtained by Eq. (4), replacing the inequality sign in both equations by an equality sign.

In both Figs. 2 and 3 above, the heel angles θ of materials α and $1-\alpha$ are obtained equally, as can be seen from the factors in the radical symbol of Eq. (19). The angle θ becomes smaller as breadth β becomes wider. And θ is largest for materials with $\alpha = 0.5$ such as wood, and is smaller as α becomes heavier or lighter than that.

The reason why the point is not plotted in the case of $\beta < 1$ for $\alpha = 0.5$, $\beta < 1.06$ for $\alpha = 0.4, 0.6$ and $\beta < 1.04$ for $\alpha = 0.3, 0.7$ in Fig. 2 is because the heel angle θ exceeds the range of the small inclination in Eq. (11).

Similarly, in Fig. 3, the part of the curve at $\beta = 1.05$, the narrowest of the 4 states with breadth β, is broken off and no point can be placed because it exceeds the range of small inclination angles in Eq. (11) and the inclined attitude θ cannot be calculated using Eq. (19) in Chapter 4. In detail, in the lighter case of $0.32 < \alpha < 0.43$, the bottom of the ship partially rises into the air and the underwater shape becomes triangular, while in the heavier case of $0.57 < \alpha < 0.68$, the deck partially sinks into the water and the underwater shape becomes pentagonal, as both cases are different from the trapezoidal shape assumed in the present theory.

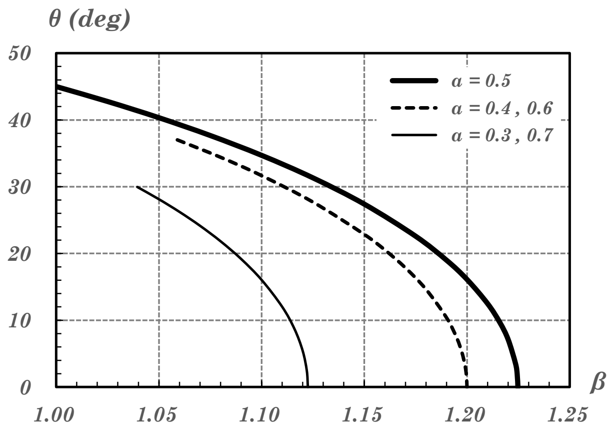

Fig. 2 The dependence of the stable inclined attitude θ on breadth β .

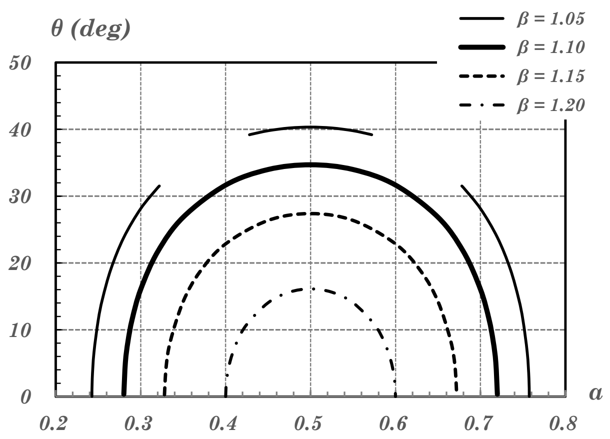

Fig. 3 The dependence of the stable inclined attitude θ on material α .

Igarashi *et al.*[(4),(5)] provide a detailed analysis of all inclined state, including cases of large heel angles (where part of the deck sinks into the water or part of the ship's bottom rises into the air), which cannot be calculated in this paper. And they have perfectly elucidated the dependence on α and β by organizing all cases in maps and tables and verifying them experimentally, so we encourage to read their paper for anyone interested.

Fig. 4 illustrates the attitudes of the four states when the material is fixed at $\alpha = 0.5$ and the breadth $\beta = 1.0$, 1.1, 1.2 and 1.3, including the positions of B, G and M. It can be seen how the heel angle θ decreases as the breadth β increases.

Fig. 5 shows the five attitudes for material $\alpha = 0.25$, 0.3, 0.5, 0.7 and 0.75, with the breadth fixed at $\beta = 1.06$. It can be found that the heel angle θ decreases symmetrically around $\alpha = 0.5$ even if the draft increases or decreases as the material α becomes heavier or lighter than that.

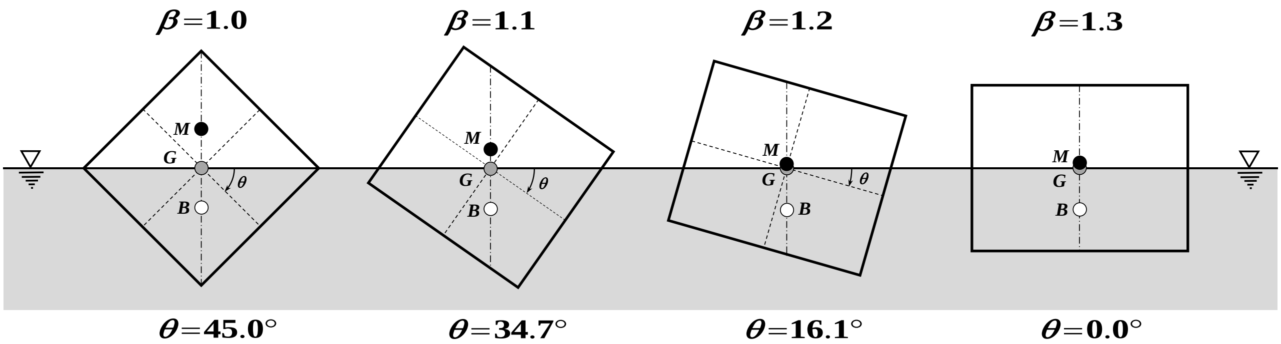

Fig. 4 The four attitudes for breadth β = 1.0, 1.1, 1.2, 1.3 with the material fixed at α = 0.5.

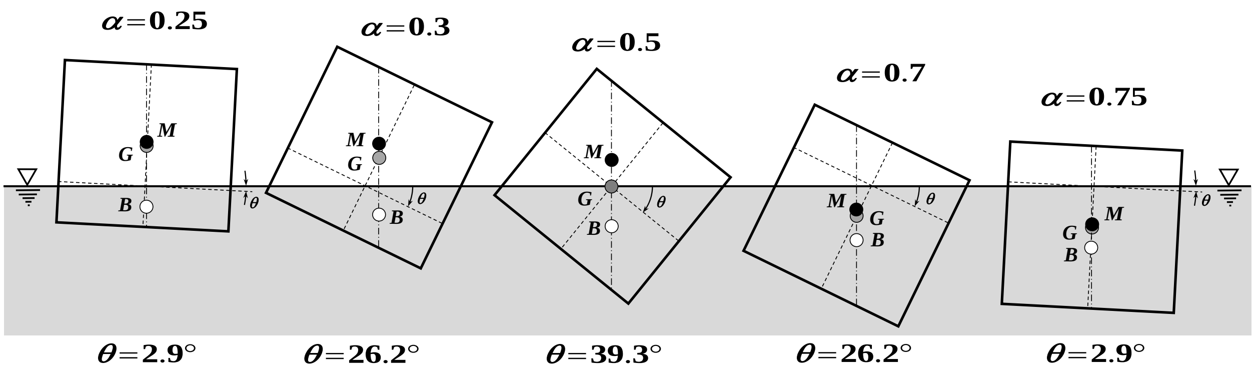

Fig. 5 The five attitudes for material α = 0.25, 0.3, 0.5, 0.7, 0.75 with the breadth fixed at β = 1.06.

6. Verificational Experiment

Fig. 6 compares the model experiment (left) and the calculation results (right) for the case of material $\alpha = 0.458$ and breadth $\beta = 1.15$.

The model of the columnar ship is length $L = 30cm$, depth $h = 10.0cm$, breadth $\beta h = 11.5cm$, and weight $W = 18.09N$. Two pieces of chemical wood were pasted together in the center at the top and bottom, and the model was manufactured by *Space Model Co., Ltd.* in Nagasaki, Japan. The verificational experiment was conducted by floating its model in a small water tank.

The inclined attitude was $\theta = 27.5°$ in the experiment and the calculated results are as follows by Eqs. (19), (25), (26) and (29).

$$\left.\begin{array}{l} \theta = 26.7° \ (\alpha = 0.458,\ \beta = 1.15) \\ \overline{GM} = 0.068\,h\ ,\ \overline{BG} = 0.269\,h \\ Z_f = 0.668\,h \end{array}\right\} \qquad (31)$$

We consider that the reason why there is a difference of about 1° between the two is that the heel angle θ in the experiment was obtained by measuring $\tan\theta$ from photographs and that the center of gravity position G may be slightly off-center due to the manufacturing process of the model. Therefore we are able to verify that the theory in this paper can correctly calculate the actual inclined state.

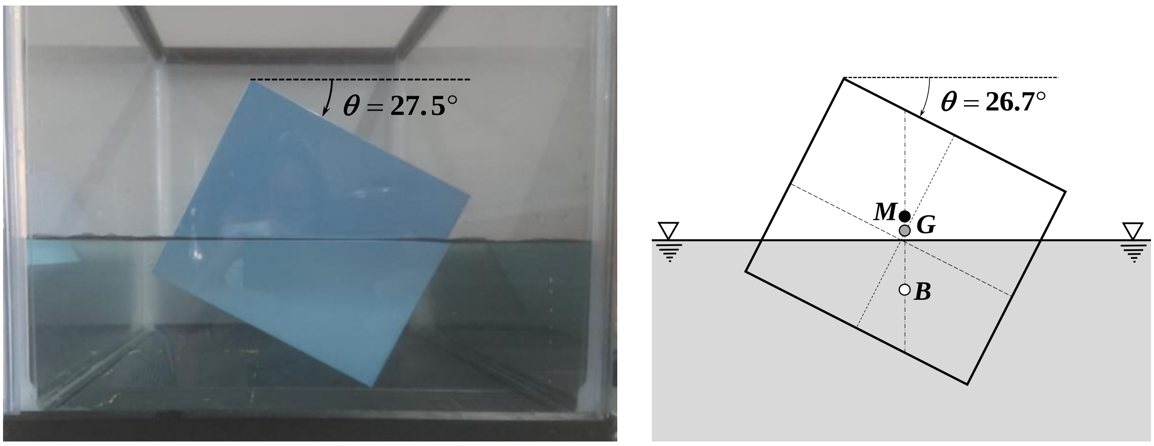

Fig. 6 The Comparison of experimental (left) and calculated (right) results
for material $\alpha = 0.458$, breadth $\beta = 1.15$.

7. Afterword

In this paper, as an applied example which is an extension of the previous paper [1], a theoretical treatment for solving the stable attitude of a columnar ship with a rectangular cross-section in a lateral inclined state is explained in an easy-to-understand manner, in which the inclined states are limited to a small heel angle (where the deck is not submerged and the ship's bottom is not floated), in order to understand the stability theory of ships.

The authors would be very happy if this paper could be of assistance to teachers and students who will teach and learn this field in the future, going one step forward from the basic examples in the previous paper [1].

Acknowledgments

In closing this paper, we would like to pay tribute to two valuable papers [4],[5] written by *Tamotsu IGARASHI,* Professor Emeritus of *the National Defense Academy of Japan*. The reason is that the authors were deeply impressed by both of their papers.

We then would like to express my heartfelt gratitude to *Dr. Yoshihiro KOBAYASHI*, former professor of *Sojo University* and current president of *Como-Techno Co., Ltd.* in Nagasaki, Japan. He always gave warm encouragement to the author's research and recommended that this study should be published in English. We are greatly inspired by the vigorous academic spirit with which he writes about the results of his research in books [18],[19].

Finally from the 1st author [20], let me express the following thanks. I would like to communicate my deepest gratitude to my late teacher, *Pr. Masato KURIHARA* [21]~[23], who cordially taught me the theory of *"Hydrostatics of Ships"* with detailed figures and formulas on the blackboard when I was a 1st year undergraduate student and learned my 1st specialized subject of naval architecture in *the College of Naval Architecture of Nagasaki* in Japan. Therefore, I am following the appearance of my teacher at that time from more than 40 years ago as an exemplary example, when I currently lecture *Hydrostatics of Floating Bodies* [24]~[27] and *Theory of Ship Stability* [28]~[30] to 2nd year students at my university [31],[32].

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‡ Bold text in the list means that there is a HyperLink

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