Title: Non-Stellar Black Holes

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Abstract:

This article derives an equation similar to the Schwarzschild Equation for a Stellar Black Hole. The equation relaxes and shifts the condition of a Black Hole forming from a Star collapsing at the end of its life, to a condition in which only the equation and values need to be satisfied, even for masses much lower than stellar masses. In fact, microscopic Black Holes formed right after the Inflationary Period ended up following the Big Bang. The equation indicates that Black Holes could form right up to their radii of the order of Planck's Length. The equation also acts as the first step towards the unification of General Relativity with Quantum Mechanics.

The Principle of Equivalence and General Relativity

In order to account for special relativity, we must change the original laws of motion. Relativistically speaking, the Lorentz force law and other conventional electromagnetism laws are still applicable.

Newton's theory of gravitation, despite being extremely effective at harmonising experimental observations, is conceptually dubious simply because it is an action-at-a-distance theory. The gravitational force of interaction between bodies is thought to be transmitted instantly and at an infinite rate, in contrast to the relativistic precondition that a signal's maximum speed be limited to c, the speed of light.

There are also issues with how the law of gravitation interprets the masses. One example is the inertial and gravitational masses being equal, which according to classical theory, is an accident. Indeed, this equality has some physical significance.

According to the relativistic concept of mass energy, even particles with zero rest mass display mass-like characteristics (e.g., inertia and weight). But the classical theory does not include such particles. If gravity affects them, we must figure out how to include this information in a gravitational theory.

Einstein developed his principle of equivalence in 1911, which served as the foundation for a brand-new theory of gravitation.

He published his general relativity theory in 1916, which states that gravitational effects move at the speed of light and that accelerated (non-inertial) observers are subject to the same laws of physics. The experiment strongly supports the equivalence principle. Let's first examine this.

The Principle of Equivalence

Consider two reference frames: one that is nonaccelerating (inertial) and has a consistent gravitational field, and the other that is uniformly accelerating relative to an inertial frame but without a gravitational field.

Since two of these frames are physically equivalent, experiments performed in them under otherwise identical circumstances ought to produce the same outcomes. This is known as Einstein's equivalence principle.

Consider a spacecraft at rest in an inertial reference frame S with a constant gravitational field at the earth's surface.

Released objects within the spacecraft will fall in the gravitational field with an acceleration, say g; an object at rest, such as an astronaut seated on the ground, will experience a force opposing its weight.

Let the spacecraft now travel to a location far from the earth's gravitational field in outer space. With respect to the inertial frame S, its rockets are propelling and accelerating the spaceship, our new frame S'.

In other words, the ship moves away from the earth faster than the point at which the earth's gravitational field (or any other gravitational field) becomes detectable.

The conditions inside the spacecraft will now be the same as when it was at rest on the surface of the earth, in accordance with the Principle of Equivalence. An object that the astronaut releases inside the craft will experience a g-force acceleration toward the spaceship.

For example, the astronaut sitting on the floor would experience a force identical to that which previously balanced its weight, in accordance with the Principle of Equivalence, if the object were at rest with respect to the spaceship.

The astronaut could not distinguish between (1) a scenario in which his spaceship was accelerating relative to a particular frame of inertia in a region with zero gravitational fields and (2) a scenario in which his spaceship was unaccelerated in a frame of inertia where a uniform gravitational field existed, in accordance with the Principle of Equivalence, from observations made in his frame.

Indeed, it follows that if a body is in a uniform gravitational field— such as an elevator in a building on earth—and is at the same time accelerating in the direction of the field with an acceleration whose magnitude equals that due to the field—such as the same elevator in free fall —then particles in such a body will behave as though they are in an inertial reference frame with no gravitational field.

Unless a force is applied to them, they will not accelerate.

This is the situation that occurs inside an artificial satellite, where, in relation to the satellite, the items the astronaut releases don't fall; instead, they seem to float in space. The astronaut is also free of the force that counterbalanced gravity before launch (he feels weightless).

Similar to how the special theory of relativity implied that we could only speak of the relative velocity of a reference frame, Einstein noted that the principle of equivalence suggests that we could only speak of the relative acceleration of a reference frame and not an absolute one.

This special relativity analogy is purely formal because if we grant that there is no absolute gravitational field, then there is also no absolute acceleration. Inertial mass and gravitational mass are equivalent, which is another conclusion that can be drawn from the principle of equivalence (it is not an accident).

The Gravitational Red Shift

Let's now use the principle of equivalence to examine any potential gravitational effects that the classical theory may have missed.

Consider a vacuum chamber that is firmly fixed to the spacecraft, has an emitter atom at rest inside, and has a detector attached to the interior.

Consider a photon—a pulse of radiation—emitted by an atom A that is at rest in frame S, such as an earthly spacecraft. The photon falls through a distance d through this field before it is absorbed by detector D, falling through a uniform gravitational field g that is directed downward in S.

Let's examine the analogous circumstance to explore the impact of gravity on the photon. In a frame S' with no gravitational field, the spaceship far from earth, the emitter atom, the vacuum chamber, and a detector are separated by a distance d, with the frame S' (the spaceship, for example) accelerating uniformly upward in relation to S with a = g.

The atom is at rest in this frame when the photon is released, but it is moving with a velocity of v along with the detector and the rocket body. A photon is in free space once it is emitted. When the photon reaches the detector, its speed is at, where t is the photon's time of flight. However, since a = g and t is (roughly) d/c, the detector's speed during absorption is gd/c. In actuality, the detector approaches the emitter with an independent velocity of v = gd/c. The Doppler formula gives us $1 + gd/c^2$ because the frequency received, ν', is higher than the frequency emitted, ν.

$\nu' = \nu (1 + gd/c^2)$ Equation (1)

According to the equivalence principle, the atom, the photon, the detector and the spaceship at rest on earth under the influence of gravity should all produce the same result in frame S.

However, A and D are both at rest in this rest frame, so the increase in frequency cannot be attributed to the Doppler effect.

There is a gravitational field, though; the outcome in S' suggests that the photon may be affected by it. Let's investigate this possibility by giving the photon a gravitational mass equal to its inertial mass, E/c^2. The photon then gains energy $(E/c^2)gd$ by falling a distance d in a gravitational field of strength g.

How can we link the frequency ν to the energy E? The relationship in quantum theory is $E = h\nu$, where h is the Planck constant. Let's use this relationship for the time being so that the photon's energy upon absorption at D is equal to both its initial emission energy and the energy gained during its descent from A to D, or $h\nu + (h\nu/c^2)g\ d$. The same result as in frame S', i.e. equation (2), is obtained if we designate this absorption energy E', which is equal to hν' (l).

Our findings (Eqs. (l) and (2)) are easily generalised to photons that are emitted from stars' surfaces and observed on earth. Here, we assume that the gravitational field does not necessarily need to be uniform and that the disparity solely influences the outcome in gravitational potential between the source and the observer.

We know that the force of attraction F for a photon leaving a star with mass $M_\odot$ and $R_\odot$ as its radius is $F = m_p g = G m_p M_\odot / R_\odot$ for a photon with inertial mass m_p.

As a result, it is clear that $g = GM_\odot/R_\odot$, where $M_\odot$ is the star's mass and $R_\odot$ is its radius. We obtain the emergent photon's energy loss during ascent through the star's gravitational field. We must substitute $R_\odot$ for d in order to account for a photon that is emitted from the star's surface and emerges from its core. Equation (1) then changes to $\nu' = (1 - G_\odot M_\odot / R_\odot c^2)$, minus, because the photon now loses energy instead of gaining it.

Since the light which is in the visible region of the spectrum will be shifted in frequency toward the red end, this effect is also known as the gravitational red shift.

Let us now assume a massive body such that ν' of the emergent photon is zero. This photon has zero energy. Hence, it can't escape the body's gravitational pull.

In that case, since ν is non-zero, we have $(1 – GM_\odot/R_\odot c^2) = 0$

That is, $GM_\odot/R_\odot c^2 = 1$ (3)

Or $R_\odot = GM_\odot/c^2$ is the condition for such a star for which photons can't escape its pull. Or a Black Hole. Interestingly, the equation is just half Schwarzschild's Radius for a Black Hole.

This body is not observed for a world outside this body, but its gravitational pull is experienced.

Hence, Einstein's Principle of Equivalence directly leads to the radius of a Black Hole with a particular mass. There is no restriction on $M_\odot$, just that the mass $M_\odot$ satisfies the particular relationship with its radius $R_\odot$. Which is simply Mass Density $\rho_\odot$.

For any particular mass distribution, the corresponding $R_\odot$ could be calculated by applying the principle of least action for all forces in equilibrium.

Schwarzschild's use of limiting cases and simplifications for a very complex Einstein's Relativistic Equation is optional to achieve the equation for Event Horizon for a massive body.

The removal of the limitation of a stellar mass, and shifting the criterion to a specific condition leads to a situation where there cold be microscopic Black Holes right down to the limit approaching Planck's Length. The equation and condition also leads to the first step towards the unification of General Relativity and Quantum Mechanics.

Reference:

Section C, Supplementary Topic C, The Principle of Equivalence and General Relativity, **Introduction To Special Relativity** Wiley (1968), pages 211 to 218, by Robert Resnick.