# An Exact Theoretical Mass of the X(3872)

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The mass of the X(3872) has been determined experimentally to an accuracy of almost 0.01 MeV. Using n-sphere surface volume factoring, an algebraic expression for the mass of the X(3872), which involves ‘h’, can be found, and, because Planck’s constant (6.62607015 E-34 J/s) was recently declared exact, that means that the mass of X(3872) can be expressed with an accuracy of any number of digits. In this paper, the n-sphere surface volume factoring technique is explained and the implications of its success in finding exact hadron masses is laid out, the biggest of which is that hadrons could be made of higher dimensional matter.

## 1. Introduction

For the past 58 years, the assumption that quarks are point particles attracted to one another by a strong force and orbiting one another in 3d space has not proven to be a useful model for making progress in our understanding of the nature of subatomic particles. A much more usefull model is to assume that quarks, rather than being point particles, are volumes of energy that occupy simple multiples of n-sphere surface volumes. Surface volumes rather than interior volumes of n-spheres were chosen for the assumed shape of quarks because of particle spin. Surface n-sphere volumes are *unbounded* spaces whereas interior n-sphere volumes are *bounded* spaces. It is assumed that circulation causing spin would more easily occur in the unbounded surface of an n-sphere than in its bounded interior, therefore, the decision to use surface volume formulae rather than interior volume formulae for quark volumes/masses.

For the problem of which quark should be assigned to which n-sphere surface volume formula, the experimentalists have already roughly determined the masses of the quarks, so it is logical to follow their lead and assign the six quarks, in mass order, to the 2-sphere through 7-sphere surface volume formulae, as shown below. (Sn is an abbreviation for the surface volume formula of an n-sphere.)

| | | |
|---|---|---|
| u | S2 = | $2\ \pi^1 r^1$ |
| d | S3 = | $4\ \pi^1 r^2$ |
| s | S4 = | $2\ \pi^2 r^3$ |
| c | S5 = | $8/3\ \pi^2 r^4$ |
| b | S6 = | $\pi^3 r^5$ |
| t | S7 = | $16/15\ \pi^3 r^6$ |

Notice that this quark model can be extended to allow for an unlimited number of different types of quarks - one for each dimension of n-sphere.

## 2. Hypersphere surface volume factoring

It has been discovered by trial and error, that the 3d mass of a hadron is a simple multiple of the n-sphere surface volume formulae associated with its quarks multiplied together along with Planck’s constant’s coefficient ‘h’, which is 6.62607015. For instance, the experimentalists say the Ds+ meson has quark content ‘cs’ and has a mass of 1967.0 +/-1.0. Multiplying the associated formulae for ‘c’ and ‘s’ together (while setting r=1), along with ‘h’, then dividing that into 1967.0 should result in an integer or small denominator fraction.

$$\text{csh} = S_5 S_4 h = (8/3\ \pi^2 r^4)\ (2\ \pi^2 r^3)\ h = 3442.343842 \text{ MeV}$$

$$\mathbf{1967.0} / 3442.343842 = 0.571412994 = \mathbf{4/7}$$

And it does. 1967.0 = **4/7** S5S4h. Maybe this is a fluke. Let's try Ds(2460). Its experimental mass (one of them) is 2458.9 +/- 1.5. Dividing 2458.9 by S5S4h should again result in an integer or small denominator fraction.

$$csh = S_5S_4h = (8/3\ \pi^2 r^4)\ (2\ \pi^2 r^3)\ h = 3442.343842\ \text{MeV}$$

$$\mathbf{2458.9} / 3442.343842 = 0.714309817 = \mathbf{5/7}$$

And again the result is a small denominator fraction, 2458.9 = **5/7** S5S4h. These two examples are not flukes or coincidences. All hadrons can be factored this way. More examples of n-sphere surface volume factoring of hadrons are given in the appendix in Table 1. *Examples of Hypersphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each*.

This method of factoring, using the quark content of a hadron to calculate a unit of factorization, works well if the quark content of a hadron is known, but in many cases the quark content is not known. From factoring experience, the quark content of only about 25% of hadrons have been determined correctly. It might be even less. It's definitely not more. When the quark content of a hadron is not known or is in doubt, there is an alternative method of factoring that can be employed. It is based on the fact that when two or more n-sphere surface volume formulae are multiplied together, the result is another n-sphere surface volume formula (usually), except for a difference in the constant of multiplication. This n-sphere surface formula can then be used, after multiplying by 'h', as a unit of factorization. If a hadron factors with that formula, say it is S6 (S6h is the factoring unit), then you know the quark content of that hadron has to be one of the combinations of quarks that when their associated surface volume formulae are multiplied together, results in an S6 similar formula.

This reduces the search for the correct quark content, because only three combinations of quarks when multiplied together form an **S6** similar formula. They are **ddu**, **sd**, and **cu**. There are about 150 different quark combinations that are compatible with n-sphere surface formulae between S4 and S21, so first determining which n-sphere surface formula will factor the hadron in question narrows down the search of its quark content to just the possibilities associated with the factoring unit employed, which could be from 1 to about 12 possibilities, instead of 150. A table showing the quark combinations associated with each factoring unit from dimension 4 to 21 is found in the appendix. It's called: Table1. *Quark Content Possibilities by Factoring Unit Used*. Only quark combinations that result in an n-sphere surface volume formula are listed. Not all quark combinations, when multiplied together, result in an n-sphere surface volume formula. Any quark combination containing two or more even dimension n-sphere quarks (**uu**, **ss**, **bb**, **su**, **bu**, **bs**, **ssu**, **bbs**, etc.), does not result in an n-sphere surface volume formula when the quarks' n-sphere surface volume formulae are multiplied together, so quark combinations of that description are not included in the table. It is assumed that the only quark combinations that exist are the ones that yield an n-sphere surface volume formula when the n-sphere surface volume formulae of the quarks in the combination are multiplied together.

You may agree that **ss** mesons do not exist, because there is no PDG category of **ss** mesons, but you may not agree that **bb** and **bs** mesons do not exist because PDG has categories of those types of mesons filled with particles. However, n-sphere surface volume factoring of the masses of the particles listed in those PDG categories shows that the quark contents assigned to those particles are incorrect. For instance, the first bb meson listed by PDG, the ηb(1S), factors very convincingly with S14h, as shown below, and S14 has (π, r) powers of (7,13), whereas 'bb' has (π, r) powers of (6,10). So, the ηb(1S) cannot be a 'bb' meson (contingent on whether the S14h factoring is the correct factoring, of course, which it seems to be). Several other so called 'bb' mesons factor convincingly as 'cccc' tetraquarks.

| Particle | ExpMass | Error | Factoring | ThrMass | dm | dm/Error |
|---|---|---|---|---|---|---|
| ηb(1S) | 9394.8 | 2.7/3.1 | **169.000 S14h** = | 9394.8390 | .0390 | 1.4% |

Likewise for the first particle in PDG's 'bs' category, the $Bs^0$. It factors convincingly as a 'cccc' tetraquark, as shown below.

| Particle | ExpMass | Error | Factoring | ThrMass | dm | dm/Error |
|---|---|---|---|---|---|---|
| $Bs^0$ | 5366.90 | .28/.23 | $\mathbf{67\ c^4 h/(2^1 3^4 5^1 7^2)}$ = | 5366.9017 | .0017 | 0.6% |

Although *all* the bb and bs mesons have not be factored, the ones that have been are *not* bb or bs mesons. So there is evidence supporting the claim that the only quark combinations that exist are the ones that yield an n-sphere surface volume formula when the n-sphere surface volume formulae of the quarks in the combination are multiplied together.

## 3. Factoring and mass of the X(3872)

The chart on the next page graphs all the experimental mass data for X(3872) reported by PDG, plus one data point from another source. As can be seen from the chart, 11 of the 18 data points are arranged nearly symmetrically around the theoretical mass **3871.6806**, which factors as (**18 – 16/3600) S8h**. That factoring is very likely an expression of the exact mass of the **X(3872)**. Its factoring expression can be reduced to (*4049*/225) S8h. (All masses are in units of MeV/$c^2$)

(18 – 16 / 3600) S8h = 3871.6806
(16196 / 900) S8h = 3871.6806
(4**4049* / 900) S8h = 3871.6806
**(*4049* / 225) S8h** = 3871.6806

In terms of decimal numbers, it is 17.99555 S8h, or (18 - .00444) S8h. It is interesting to note that there is another theoretical mass the same distance above 18 S8h as X(38712) is below it that factors similarly to X(3872). As you can see from the factorings below, the X(3872)'s mass is less than 18 S8h by 0.00444 S8h whereas the other particle, the one that factors similarly, is greater than 18 S8h by the same amount. It factors with a prime number that is only two bigger than 4049. That prime is the second prime in the twin prime pair (*4049, 4051*).

| | | | | | |
|---|---|---|---|---|---|
| (18 - .00444) | S8h | **=** | *4049* /225 S8h | = | **3871.6806** |
| 18 | S8h | = | | = | 3872.6368 |
| (18 + .00444) | S8h | = | *4051* /225 S8h | = | 3873.5930 |

The question raised by this factoring is why is it significant? Why is a strong peak in hadron production observed at (18 - .00444) S8h and only weak production observed at 18.0000 S8h?

## 4. Quark content of the X(3872)

Since X(3872) factors with S8h or S9h (both have the same power of π in their equations, so, if one factors X(3872), the other one will too), it can have any of the quark content possibilities listed on the S8h and S9h lines in Table 1. *Quark Content Possibilities by Factoring Unit Used*, in the appendix. They are:

| | | | | |
|---|---|---|---|---|
| S8h = | (4, 7) | dddu | dds | cs , bd, tu |
| S9h = | (4, 8) | dddd | ddc | cc , td |

So, X(3872) could be a meson, baryon, or tetraquark, of the types shown above. If you do a native factoring of X(3872) , that is, construct and use a factoring unit consisting of any of the allowed quark combinations consistent with S8h or S9h factoring, you will always get the prime number *4049* as part of the factoring. Here are some examples:

| Quark Content | Native Factoring | ThrMass (MeV) |
|---|---|---|
| **cs** | *4049*/3600 **csh** | = 3871.6806 |
| **cc** | *4049*/4800 **cch** | = 3871.6806 |
| dddu | *4049*/86400 **ddduh** | = 3871.6806 |
| dddd | *4049*/172800 **ddddh** | = 3871.6806 |

Since the X(3872) factors with S8h (and also with S9h) it has to have one of these quark contents:

| | | |
|---|---|---|
| dddu | dds | cs , bd, tu |
| dddd | ddc | cc , td |

If it's a tetraquark, as many suspect, the only possibilities for its quark content are dddu and dddd.

# Mass Spectrum of X(3872)'s Experimental Mass Data

S8h Factoring / 'dddu' Tetraquark?

S8h = 215.1464901 MeV/$c^2$

**Res** = (1/3600)**S8h**

| | n | (18+n/3600) S8h | ExpMass | Error | dm | dm/Error | Ref |
|---|---|---|---|---|---|---|---|
| | -26 | 3871.0830 | | | | | |
| | -25 | 3871.1427 | | | | | |
| | -24 | 3871.2025 | | | | | |
| | -23 | 3871.2623 | | | | | |
| | -22 | 3871.3220 | **3871.3** | 0.6/0.1 | 0.0220 | 7.3% | |
| | -21 | 3871.3818 | **3871.4** | 0.6/0.1 | 0.0182 | 3.0% | |
| | -20 | 3871.4416 | | | | | |
| | -19 | 3871.5013 | | | | | |
| | -18 | 3871.5611 | | | | | |
| | -17.500 | 3871.5909 | **3871.59** | 0.06/0.03 | 0.0009 | 1.5% | |
| | -17 | 3871.6209 | **3871.61** | 0.16/0.19 | 0.0109 | 6.8% | |
| | -16.666 | 3871.6407 | **3871.64** | 0.06/0.01 | 0.0007 | 1.2% | |
| **(4*4049)/900 S8h** | **-16** | **3871.6806** | **Key Mass** | | | | |
| | -15.75**0** | 3871.6955 | **3871.695** | .067/.068 | 0.0005 | 0.7% | |
| | -15 | 3871.7404 | | | | | |
| | -14 | 3871.8001 | **3871.8** | 3.1/3.0 | 0.0001 | 0.0% | |
| | -13 | 3871.8599 | **3871.85** | 0.27/0.19 | 0.0099 | 3.4% | |
| | -12 | 3871.9197 | **3871.9** | 0.7/0.2 | 0.0197 | 2.8% | |
| | -11.500 | 3871.9495 | **3871.95** | 0.48/0.12 | 0.0005 | 0.1% | |
| | -11 | 3871.9794 | | | | | |
| | -10.500 | 3872.0093 | **3872.0** | 0.6/0.5 | 0.0093 | 1.5% | |
| | -10 | 3872.0392 | | | | | |
| | -9 | 3872.0990 | | | | | |
| | -8 | 3872.1587 | | | | | |
| | -7 | 3872.2185 | | | | | |
| | -6 | 3872.2782 | | | | | |
| | -5 | 3872.3380 | | | | | |
| | -4 | 3872.3978 | | | | | |
| | -3 | 3872.4575 | | | | | |
| | -2 | 3872.5173 | | | | | |
| | -1 | 3872.5771 | | | | | |
| **Key Mass** **18 S8h** | **0** | 3872.6368 | **3872.6** | 0.5/0.4 | 0.0368 | 7.4% | [2] |
| | 1 | 3872.6966 | | | | | |
| | 2 | 3872.7563 | | | | | |
| | 3 | 3872.8161 | | | | | |
| | 4 | 3872.8759 | **3872.9** | 0.6/0.4 | 0.0241 | 4.0% | |
| | 5 | 3872.9356 | | | | | |
| | 6 | 3872.9954 | **3873** | 1.8/1.6 | 0.0046 | 0.2% | |
| | 7 | 3873.0552 | | | | | |
| | 8 | 3873.1149 | | | | | |
| | 9 | 3873.1747 | | | | | |
| | 10 | 3873.2345 | | | | | |
| | 11 | 3873.2942 | **3873.3** | 1.1/1.0 | 0.0058 | 0.5% | |
| | 12 | 3873.3540 | | | | | |
| | 13 | 3873.4137 | **3873.4** | 1.4 | 0.0137 | 1.0% | |
| | 14 | 3873.4735 | | | | | |
| | 15 | 3873.5333 | | | | | |
| **(4*4051)/900 S8h** | **+16** | **3873.5930** | **Key Mass** | | | | |
| | 17 | 3873.6528 | | | | | |
| | 18 | 3873.7126 | | | | | |
| | 19 | 3873.7723 | | | | | |
| | 20 | 3873.8321 | | | | | |
| | 21 | 3873.8918 | | | | | |
| | 22 | 3873.9516 | | | | | |
| | 23 | 3874.0114 | | | | | |
| | 24 | 3874.0711 | | | | | |
| | 39 | 3874.9676 | | | | | |
| | 40 | 3875.0273 | | | | | |
| | 41 | 3875.0871 | **3875.1** | 0.7/0.5 | 0.0129 | 1.8% | |
| | 42 | 3875.1469 | | | | | |
| | 43 | 3875.2066 | **3875.2** | 0.7 | 0.0066 | 0.9% | |
| | 44 | 3875.2664 | | | | | |

Source of ExpMass and Error data except [n]: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

## 5. Conclusion

If one assumes, as factoring results suggest, that the mass of the X(3872) equals *exactly* **(4049/225) S8h,** then the mass of the X(3872) can be expressed with an accuracy of any number of digits. This follows because the coefficient of 'h' is now (as of 2019) assumed to have the *exact value* **6.62607015**, and **S8**, the surface volume formula of an 8-sphere ($S8=(1/3)\pi^4 r^7$), has an *exact value* also, therefore, the value of the mass of the X(3872) can be expressed with an accuracy of any number of digits.

Theoretical Mass of the X(3872)

(4049/225) S8h = **3871.6806** MeV/$c^2$ ( 8 digits of accuracy)

(4049/225) S8h = **3,871,680,616** eV/$c^2$ (10 digits of accuracy)

## 6. Commentary on the Quark Model

If it's true that hadron masses can be expressed as simple multiples of n-sphere surface volumes then what does it mean? Does it mean that hadrons are made of higher dimensional matter? If so, then much of the current quark model is incorrect. If matter exists in multiple higher dimensions - as n-sphere surface volume factoring suggests - then the *strong force*, which is currently assumed to be a central 3d force, would have to operate according to a different force law for every higher dimensional space. Could there be a different strong force for every higher dimensional space? Perhaps, or perhaps the strong force is a concept that has outlived its usefulness. If quarks are not point particles, but rather waves, then maybe the concept of a strong central force is not necessary for explaining hadron structure.

One may object to the idea hadrons are made of higher dimensional matter by arguing that our space is 3d and will not accommodate higher dimensional matter within it. True, 3d space cannot contain higher dimensional matter *entirely* within it, but it can *intersect it*, because 3d space has zero thickness in the fourth and higher dimensional directions. That, it seems, is a little known fact in the physics community, but it's true. 3d space has zero thicknes in the fourth and higher dimensional directions, which means 4d space is immediately adjacent to '*every point'* in our 3d space. So, wherever a hadron is in our 3d space, parts of it can extend out into the higher dimensional space that is immediately adjacent to our 3d space. If you still don't believe it's true, think of a 2d plane in 3d space. You can easily see that '*every point*' in the 2d plane is immediately adjacent to 3d space (in two opposite 3d directions). Likewise, 4d space is immediately adjacent to every point in 3d space (in two opposite 4d directions). It's a mathematical truth, so, it is mathematically possible, at least, that hadrons could be made of higher dimensional matter, and still exist (partially) in our 3d space.

## 7. References

[1] P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

[2] Study on X(3872) from effective field theory with pion exchange interaction,
arXiv: 1304.0846v1 [hep-ex] 3 Apr 2013

## 8. Appendix

Table 1. Quark Content Possibilities by Factoring Unit Used

Table 2. Examples of Hypersphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each

Table 3. Hypersphere surface volume formulae

Table 4. Values of Hypersphere Surface Volume Units of Factorization

# Table 1. Quark Content Possibilities by Factoring Unit Used

Note: s=du c=dd b=sd=cu t=cd

| Factoring Unit<br>If…….<br>Mass factors with | | | Quark Content Possibilities<br>Then…….<br>Hadron has one of these Quark Contents | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| u | S2h = | (1, 1) | | | | | | | | | |
| d | S3h = | (1 ,2) | | | | | | | | | |
| s | S4h = | (2, 3) | du | | | | | | | | |
| c | S5h = | (2, 4) | dd | | | | | | | | |
| b | S6h = | (3, 5) | ddu | sd, cu | | | | | | | |
| t | S7h = | (3, 6) | ddd | cd | | | | | | | |
| v | S8h = | (4, 7) | dddu | dds | cs , bd, tu | | | | | | |
| w | S9h = | (4, 8) | dddd | ddc | cc , td | | | | | | |
| x | S10h = | (5, 9) | ddddu | ddds | dcs | bc, ts | | | | | |
| y | S11h = | (5, 10) | ddddd | dddc | dcc | t c | | | | | |
| z | S12h = | (6, 11) | dddddu | sdddd | csdd | ccs | tb, vc | | | | |
| | S13h = | (6, 12) | dddddd | cdddd | ccdd | ccc | t t, | | | | |
| | S14h = | (7, 13) | ddddddu | ddddds | dddcs | dccs | bcc | tv | | | |
| | S15h = | (7, 14) | ddddddd | dddddc | dddcc | dccc | t cc | tw | | | |
| | S16h = | (8, 15) | dddddddu | dddddds | ddddcs | ddccs | cccs | btc | wv | | |
| | S17h = | (8, 16) | dddddddd | ddddddc | ddddcc | ddccc | cccc | t t s | ww | | |
| | S18h = | (9, 17) | ddddddddu | ddddddds | dddddcs | dddccs | dcccs | cccb | t t b | wx | |
| | S19h = | (9, 18) | ddddddddd | dddddddc | dddddcc | dddccc | dcccc | ccct | t t t | wy | |
| | S20h = | (10, 19) | dddddddddu | dddddddds | ddddddcs | ddddccs | ddcccs | ccccs | cccv | t t w | xy |
| | S21h = | (10, 20) | dddddddddd | ddddddddc | ddddddcc | ddddccc | ddcccc | ccccc | cccw | t t x | yy |

All quark combinations for the factoring units from S4h to S9h are shown. For the factoring units from S10h to S21h not all possible quark combinations are shown, especially for the triquarks (qqq, baryons) and the diquarks (qq, mesons). This was done so the table wouldn't look too complex and potentially confusing.

The parentheses enclosing two integers separated by a comma that is just to the right of the factoring units, such as the (1,2) in the line S3h = (1,2), means the surface volume formula of that factoring unit has the powers 1 and 2 for 'π' and 'r'. In the case of S3h, $S3 = 4\pi^1r^2$. 'π' is raised to the power 1, and 'r' is raised to the power 2, that's why it's written S3h = (1,2). Using this parentheses notation for surface volume formula representation makes it easy to determine which factoring unit will factor which quark combinations, or vice versa, which quark combinations can be factored by which factorung unit.

For instance, if you want to know which factoring unit will factor 'ddd', since 'd' = S3 = (1,2), just add the corresponding integers together of the product (1,2)(1,2)(1,2). You are multiplying numbers together ( 'π' and 'r') that are raised to integer powers, and, powers add, so you get (3,6). Now find the line with (3,6) in it. It is S7h = (3,6). So the factoring unit needed to factor 'ddd' is S7h.

Table 2. Examples of Hypersphere Surface Volume Factoring of Some Hadron Masses Showing a Compatible Quark Content for Each

| Subatomic Particle | ExpMass | Error | HSSV Factoring | | ThrMass | Compatible QuarkContent |
|---|---|---|---|---|---|---|
| ρ(770) | 775.02 | 0.35 | **4.44444 S5h** | = | 775.071 | dd |
| η | 547.865 | 0.031 | **2.66666 S6h** | = | 547.8660 | ds |
| Δ(1232) | 1232.9 | 1.2 | **6.00000 S6h** | = | 1232.698 | ddu |
| K(1430) | 1438 | 8/4 | **7.00000 S6h** | = | 1438.148 | uc |
| Δ(1700) | 1643 | 6/3 | **8.00000 S6h** | = | 1643.598 | ddu |
| $Xi^0$ | 1314.86 | 0.20 | **6.00000 S7h** | = | 1314.878 | ddd |
| $Xi^-$ | 1321.71 | 0.07 | **6.03125 S7h** | = | 1321.727 | ddd |
| a2(1700) | 1721 | 11/44 | **8.00000 S8h** | = | 1721.172 | cs |
| Ds | 1967.0 | 1.0/1.0 | **64/7 S8h** | = | 1967.053 | cs |
| Ds(2460) | 2458.9 | 1.5 | **80/7 S8h** | = | 2458.817 | cs |
| B2(5747) | 5737.2 | 0.7 | **26.66666 S8h** | **=** | 5737.239 | bd |
| Ds | 1967.0 | 1.0/1.0 | **10.00000 S9h** | = | 1967.053 | cc |
| Ds(2460) | 2458.9 | 1.5 | **12.50000 S9h** | = | 2458.817 | cc |
| Ds(2700) | 2688 | 4 | **13.66666 S9h** | = | 2688.307 | cc |
| Ds(2700) | 2710 | 2 | **13.77777 S9h** | = | 2710.163 | cc |
| Bj(5732) | 5704 | 4/10 | **29.00000 S9h** | = | 5704.455 | cc |
| Ds(2212) | 2112.2 | 0.4 | **12.5000 S10h** | = | 2112.195 | bc |
| Ω(2250) | 2253 | 13 | **13.3333 S10h** | = | 2253.008 | dcs |
| Ds1(2536) | 2534.6 | 0.3/0.7 | **15.0000 S10h** | = | 2534.634 | bc |
| Ds2(2572) | 2572.2 | 0.3/1.0 | **15.2222 S10h** | = | 2572.185 | bc |
| Ds0(2590) | 2591 | 13 | **15.3333 S10h** | = | 2590.960 | bc |
| Pc(4337) | 4337 | 7/4 | **25.6666 S10h** | = | 4337.041 | ddddu |
| Pc(4457) | 4449.8 | 1.7/2.5 | **26.3333 S10h** | = | 4449.692 | ddddu |
| Y(4500) | 4506 | 11 | **26.6666 S10h** | = | 4506.017 | ddddu |
| b1(1235) | 1236 | 16 | **9.0000 S11h** | = | 1235.936 | ddddd |
| X(2175) | 2197.4 | 4.4 | **16.0000 S11h** | = | 2197.219 | ddddd |
| Z(3985) | 3982.5 | 1.8 | **29.0000 S11h** | = | 3982.461 | ddddd |
| X(4660) | 4669 | 21/3 | **34.0000 S11h** | = | 4669.092 | ddddd |
| Ds(2860) | 2866.6(avg) | | **27.0000 S12h** | = | 2866.605 | bt |
| $D(3000)^0$ | 2971.8 | 8.7 | **28.0000 S12h** | = | 2972.775 | bt |
| $D(3000)^0$ | 3008.1 | 4.0 | **28.3333 S12h** | = | 3008.165 | bt |
| Dsj(3040) | 3044 | 8 | **28.6666 S12h** | = | 3043.555 | bt |
| Λ | 1115.59 | 0.08 | **14.2222 S13h** | = | 1115.599 | ccc |
| Ω | 1673.4 | 1.7 | **21.3333 S13h** | = | 1673.398 | ccc |
| Xi(1950) | 1952 | 11 | **24.8888 S13h** | = | 1952.298 | ccc |
| Σ(2230) | 2234 | 25 | **28.4444 S13h** | = | 2231.198 | ccc |
| Xi(2500) | 2505 | 10 | **31.9375 S13h** | = | 2505.195 | ccc |
| fj(2220) | 2223.9 | 2.5 | **40.0000 S14h** | = | 2223.630 | vt |
| Xc0(1P) | 3415.5 | 0.4/0.4 | **61.4400 S14h** | = | 3415.496 | ccsd |
| Xc2(1P) | 3557.8 | 0.2/4 | **64.0000 S14h** | = | 3557.808 | ccsd |
| ηb(1S) | 9394.8 | 2.7/3.1 | **169.0000 S14h** | = | 9394.839 | vt |
| f0(980) | 977.3 | 0.9/3.7 | **99.7500 S18h** | = | 977.298 | cccb |
| f0(980) | 982.2 | 1.0/8.1 | **100.2500 S18h** | = | 982.197 | cccb |
| f0(980) | 984.7 | 0.4/2.4 | **100.5000 S18h** | = | 984.646 | cccb |

Source of ExpMass and Error data: P.A. Zylaet al.(Particle Data Group), Prog. Theor. Exp. Phys.2020, 083C01 (2020) and 2021 update

# Table 3. Hypersphere Surface Volume Formulae

(Dimension 2 - Dimension 21)

| Sphere Dimension | Sn | Surface Volume Formula | $(\pi^x, r^y)$ (x, y) |
|---|---|---|---|
| 2 | S2 = | $2 \ \pi^1 r^1$ | (1, 1) |
| 3 | S3 = | $4 \ \pi^1 r^2$ | (1, 2) |
| 4 | S4 = | $2 \ \pi^2 r^3$ | (2, 3) |
| 5 | S5 = | $8/3 \ \pi^2 r^4$ | (2, 4) |
| 6 | S6 = | $\pi^3 r^5$ | (3, 5) |
| 7 | S7 = | $16/15 \ \pi^3 r^6$ | (3, 6) |
| 8 | S8 = | $1/3 \ \pi^4 r^7$ | (4, 7) |
| 9 | S9 = | $32/105 \ \pi^4 r^8$ | (4, 8) |
| 10 | S10 = | $1/12 \ \pi^5 r^9$ | (5, 9) |
| 11 | S11 = | $64 / 945 \ \pi^5 r^{10}$ | (5, 10) |
| 12 | S12 = | $1 / 60 \ \pi^6 r^{11}$ | (6, 11) |
| 13 | S13 = | $128 / 10395 \ \pi^6 r^{12}$ | (6, 12) |
| 14 | S14 = | $1 / 360 \ \pi^7 r^{13}$ | (7, 13) |
| 15 | S15 = | $256 / 135135 \ \pi^7 r^{14}$ | (7, 14) |
| 16 | S16 = | $1 / 2520 \ \pi^8 r^{15}$ | (8, 15) |
| 17 | S17 = | $512 / 2027025 \ \pi^8 r^{16}$ | (8, 16) |
| 18 | S18 = | $1 / 20160 \ \pi^9 r^{17}$ | (9, 17) |
| 19 | S19 = | $1024 / 34459425 \ \pi^9 r^{18}$ | (9, 18) |
| 20 | S20 = | $1 / 181440 \ \pi^{10} r^{19}$ | (10, 19) |
| 21 | S21 = | $2048 / 654729075 \ \pi^{10} r^{20}$ | (10, 20) |

# Table 4. Values of Hypersphere Surface Volume Units of Factorization

(Dimension 2 - Dimension 21)

| Sphere Dimension | Unit of Factorization | | Formula | | Value (MeV/$c^2$) |
|---|---|---|---|---|---|
| 2 | S2h | = | $2\ \pi^1 r^1 h$ | = | 41.63282661 |
| 3 | S3h | = | $4\ \pi^1 r^2 h$ | = | 83.26565322 |
| 4 | S4h | = | $2\ \pi^2 r^3 h$ | = | 130.7933822 |
| 5 | S5h | = | $8/3\ \pi^2 r^4 h$ | = | 174.3911763 |
| 6 | S6h | = | $\pi^3 r^5 h$ | = | 205.4497644 |
| 7 | S7h | = | $16/15\ \pi^3 r^6 h$ | = | 219.1464153 |
| 8 | S8h | = | $1/3\ \pi^4 r^7 h$ | = | 215.1464901 |
| 9 | S9h | = | $32/105\ \pi^4 r^8 h$ | = | 196.7053624 |
| 10 | S10h | = | $1/12\ \pi^5 r^9 h$ | = | 168.9756582 |
| 11 | S11h | = | $64 / 945\ \pi^5 r^{10} h$ | = | 137.3262492 |
| 12 | S12h | = | $1 / 60\ \pi^6 r^{11} h$ | = | 106.1705373 |
| 13 | S13h | = | $128 / 10395\ \pi^6 r^{12} h$ | = | 78.44057013 |
| 14 | S14h | = | $1 / 360\ \pi^7 r^{13} h$ | = | 55.59076334 |
| 15 | S15h | = | $256 / 135135\ \pi^7 r^{14} h$ | = | 37.91204905 |
| 16 | S16h | = | $1 / 2520\ \pi^8 r^{15} h$ | = | 24.94907624 |
| 17 | S17h | = | $512 / 2027025\ \pi^8 r^{16} h$ | = | 15.88056197 |
| 18 | S18h | = | $1 / 20160\ \pi^9 r^{17} h$ | = | 9.797479330 |
| 19 | S19h | = | $1024 / 34459425\ \pi^9 r^{18} h$ | = | 5.869441980 |
| 20 | S20h | = | $1 / 181440\ \pi^{10} r^{19} h$ | = | 3.419965454 |
| 21 | S21h | = | $2048 / 654729075\ \pi^{10} r^{20} h$ | = | 1.940989032 |