THE DARK MATTER PHENOMENON IS A RESULT OF THE EXISTENCE OF YUKAWA POTENTIAL

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Abstract: *The analysis of the rotation curves of the galaxies, the analysis of the gravitational lenses as well as the fluctuations of the relict radiation point to the existence of dark matter, which is not subject to strong or electromagnetic interaction. The second possibility is a modification of Newtonian dynamics - MOND theory. So far, no dark matter has been found, and the MOND theory still stands only on observation, without a deeper theoretical explanation. A theoretical starting point could be the introduction of the gravitational Yukawa potential, which explains the observations within the framework of the MOND theory sufficiently well.*

1: GRAVITATIONAL YUKAWA POTENTIAL

In the period about 1990 - 2015, the model of Expansive Nondecelerative Universe (ENU) was developed and was successfully applied to clarify several problems of cosmology.
The model of the Expansive Nondecelerative Universe (ENU) makes it possible to locate the energy density of the gravitational field [1], [2].
The Schwarzschilds metric can be expressed as follows:

$$ds^2 = e^{\gamma} c^2 dt^2 - e^{\lambda} dr^2 - r^2 \left(d\vartheta^2 + \sin^2\vartheta \, d\varphi^2 \right) \tag{1}$$

Then for the density of localized gravitational energy [3] it holds:

$$T_0^1 = \varepsilon_g = -\frac{c^4}{8\pi G} \mathrm{e}^{-\lambda} \frac{d\lambda}{r\,dt} = -\frac{c^4 r_g}{8\pi G a r^2} \tag{2}$$

r_g is the gravitational radius of a body with mass m and a is the gauge factor of the Universe. Its currently value is
$a = 1.31 \times 10^{26}\,\mathrm{m}$

The relation (2) expresses the energy density of the gravitational field of a body with the mass m in the distance r. It is obvious that in the ENU model, the matter-based energy and gravitational field energy are exactly compensated in the scale of the whole Universe and the total energy of the Universe is thus zero. The gravitational force is a far-reaching force and acts in principle up to infinity; it is measurable, however, only to a certain distance called the effective range r_{ef}. Its meaning lies in a postulate that in the ENU, the effect of gravitation can be displayed only at such a distance, in which the absolute value of the gravitational energy density is higher than the critical energy density of the Universe. The effective range can be expressed as:

$$r_{\mathrm{ef}} = \left(r_g a \right)^{\frac{1}{2}} \tag{3}$$

Non - relativistic Yukawa gravitational potential can be expressed as:

$$\Phi = \Phi_N \, \mathrm{e}^{-\frac{r}{r_{\mathrm{ef}}}} \tag{4}$$

where Φ_N is the Newton potential

$$\Phi_N = -G \int \frac{\rho}{r} dV = -\frac{Gm}{r} \tag{5}$$

Within the distances shorter that the effective range, this potential is almost identical to the Newton potential. At distances $r > r_{\mathrm{ef}}$, the potential approaches zero value. Objects of the Universe, passing from the smallest ones to the highest agglomerations, such as galaxies, galaxy clusters up to superclusters contain still more and more matter, however, with still lesser and lesser mass density. In the mentioned order, the mean energy density decreases. In the ENU model it is supposed that the largest objects such as superclusters or giant elliptical galaxies have a limited size since their gravitational energy density approaches nearly critical density. Their dimensions reach almost the size of the effective radius.

2: MOND THEORY

MOND theories claim that, in extremely weak gravitational fields, the dynamics of Newtonian mechanics change, which is reflected in the rotation curves of the galaxies. For that the rotation speeds are constant, regardless of the distance. That is why the rotational curves are flat (Fig. 1).

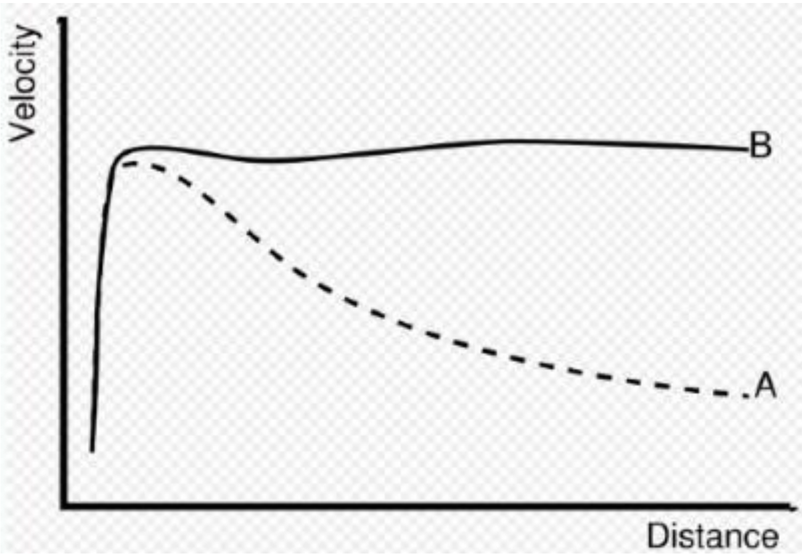

Fig 1: *Rotation curves for Newtonian dynamics – A and real measured rotation curves – B*

According to a large number of observations, the mentioned change occurs when the gravitational acceleration is smaller or equal to the limit value
$g_0 = 1.2 \times 10^{-10} \mathrm{m/s}^2$
It follows from relation (4) that the change in dynamics occurs in the case $r > r_{\mathrm{ef}}$
then it holds for gravitational acceleration:

$$g = g_N \, \mathrm{e}^{-\frac{r}{r_{\mathrm{ef}}}} \tag{6}$$

In the case of $r = r_{ef}$ we use relation (6) to get the values

$$g_N = 3.415\times10^{-10}\,\text{m/s}^2 \tag{7}$$

$$g = 1.25\times10^{-10}\,\text{m/s}^2 \tag{8}$$

The last value is very close to the value of the limit acceleration g_0 in the MOND theory [4],[5].
If the distance r exceeds the value of the effective range, the surrounding energy density is equal to the critical value – there is an external field with a constant energy density. Therefore in the region $r > r_{\text{ef}}$ it holds:

$$g = n g_N e^{-\frac{r}{n r_{\text{ef}}}} \tag{9}$$

where: $n = r/r_{\text{ef}}$ in $r > r_{\text{ef}}$
$n = 1$ in $r < r_{\text{ef}}$

Then in regions where $r > r_{\text{ef}}$ and $g < g_0$ it will hold:

$$g \sim \frac{\sqrt{m}}{r} \tag{10}$$

The relation (10) is in agreement with measurements and predictions of MOND theory.
The rotation speeds are thus constant, regardless of the distance.

CONCLUSIONS

To date, no trace of the existence of dark matter has been found. On the contrary, theoretical observations are starting to prefer MOND theories [6], [7].
There are three essential so far known facts about MOND
1: A change in the rotation dynamics occurs when the acceleration is smaller or equal $g_0 = 1.2\times10^{-10}\,\text{m/s}^2$
2: The rotation speed there does not depend on the distance, but is constant for every distance $r > r_{\text{ef}}$.
3: The relation (10) applies atypically for acceleration.
All three facts are known, but no theory can explain them. So here is an opportunity to use Yukawa potential. All three measurements can be explained in the way that a critical energy density is reached at the effective distance and it will continue to be constant for all $r > r_{\text{ef}}$. Acceleration g_0 corresponds to the critical density, and further, the rotational speeds will be the same and independent of r. Finally, the equation (10) holds for $r > r_{\text{ef}}$.
All three facts can be explained by formula (9). In that case, the existence of dark matter would be useless.
The stated Yukawa potential represents a possible theoretical starting point for MOND theories. However, we do not want to exclude the possibility of the existence of axions.

References:

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