The Liquid Metallic Hydrogen Model of the Sun: Modelling a Density Profile of the Chromosphere

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Abstract

In the last years, the liquid metallic hydrogen model has proven to be a viable alternative to the standard solar model, almost exclusively due to the work of Pierre-Marie Robitaille (2002, 2009, 2011, 2013). By modeling the density of both the liquid metallic and the molecular state of hydrogen from first principles, the pressure at the phase transition can be estimated, resulting in about 550 GPa. In the liquid metallic hydrogen model, this phase transition defines the photosphere which can therefore be considered real surface, as it is consistent with many observations. However, considerable pressure must be exerted by the above chromosphere, which is assumed to consist of molecular hydrogen, albeit in a compressed, liquid form. Deriving a relation between pressure and density from the above considerations, it can be shown that the chromosphere has an approximate thickness of not more than 8000 km, in agreement with observations.

1 Introduction

There is a vast amount of evidence in favor of the liquid metallic hydrogen model, involving opacity arguments, the visible surface across the entire electromagnetic spectrum, the phenomenology of surface waves and coronal mass ejections, and many others. All this has been outlined in detail by Pierre-Marie Robitaille, including a thorough discussion of the historical development that led to the currently accepted standard model based on a gaseous sun. Long after this model had been established, Wigner in Huntington, in (1935), proposed a metallic state of hydrogen. Such a state could explain the apparent surface of the sun that caused the standard solar model so much trouble to understand. This curious property of hydrogen arises because it can be seen as the '0th' alkali metal,[1] providing the unique opportunity to organize itself in both the molecular and metallic state. Of course, this vastly complicates the phase diagram, which is a matter of ongoing theoretical research. In 2016 however, the metallic state has been realized for the first time in the laboratory (Silveira 2016) under huge pressure though this is still debated. There is good reason however to assume that once the metallic state is reached, it can remain metastable at considerably lower pressure. However, this will be extremely difficult to quantify, and one might also doubt that the effect can lower the necessary pressure over orders of magnitude. Then, metastability in phase transitions is usually observed in very controlled conditions, something that cannot easily be assumed for the highly dynamical surface of the sun. Thus the following, rather crude calculation should demonstrate the possibility of a metallic phase in the photosphere without resorting to the metastability argument.

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[1] It is interesting to note that the boiling temperature of alkali elements increases from the (heavy) rubidium with 937 K, and 1050 K (potassium) to lithium which boils at 1650 K. It is therefore reasonable to assume that such a state of the even lighter hydrogen exists at several thousand Kelvin.

Another caveat will be that besides the molecular and the metallic state, other lattices may exist, such as two-dimensional, hexagonal structures similar to graphite. These should behave like semi-metals, and there are even hints that such a semi-metallic state, emitting as a black body, forms the photosphere. Although the following results are in agreement with the pressure observed by Silveira (2016), I restrict to an approximate calculation. The details needed for a concrete modeling of the photosphere that may involve several semi-metallic states would go beyond the scope of this first approach. The most striking difference the standard solar model and the liquid metallic hydrogen model lies in the vastly different densities of the photosphere. The standard solar model assumes those to be in the order of $10^{-7}\frac{g}{cm^3}$, with an even lower value in the chromosphere. On the other hand, a metallic state requires densities in the order of $0.7\frac{g}{cm^3}$, while the sun has an average density of $1.4\frac{g}{cm^3}$. iIt is clear that the metallic state could only be achieved under huge pressure in the order of several 10^{11} Pa. In the following, the density and pressure of the two states is modelled from first principles, in order to understand how the necessary pressure may build up in the photosphere.

2 Model

2.1 Metallic lattice

The approach by Gross (2012), sections 3.5.1 - 3.5.3 is followed. In order to estimate the binding energy of the metallic state, a lattice of positive point charges is adopted and a cloud of negative charges, whereby the charge of a single electron is assumed to be distributed continuously over a sphere with radius r_A (half the distance of the atomic trunks, in this case the protons). r_A is usually called the Wigner-Seitz radius. Since hydrogen has just one electron, we may write $n = (\frac{4}{3}\pi r_A^3)^{-1}$ and obtain for the average kinetic energy of an electron (m electron mass, e, electron charge):

$$E_{kin} = \frac{3}{5}\Big(\frac{9\pi}{4}\Big)^{\frac{2}{3}}\frac{\hbar^2}{2mr_A^2} \approx 2.21\frac{\hbar^2}{2ma_B^2}\Big(\frac{a_B}{r_A}\Big)^2 \tag{1}$$

Hereby, we have used the Bohr radius

$$a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2} = 52.9\ \text{pm}. \tag{2}$$

Additionally, we seek an expression for the potential energy of the electrostatic interaction. Therefore, we consider the electric potential at a distance are from a point source, taking into consideration that this source is partially shielded by the homogeneous electron charge. For the potential, we obtain

$$= \frac{e - e(\frac{r}{r_A})^3}{4\pi\epsilon_0 r}, \tag{3}$$

whereby $r < r_A$. The electric volume charge of the density within a spherical shell of thickness dr at a distance r of the point source $+e$ contributes the following amount to the electrostatic energy:

$$dE_{pot} = \frac{e\big(1-(\frac{r}{r_A})^3\big)}{4\pi\epsilon_0 r}\rho 4\pi r^2\ dr = -\frac{3e^2}{4\pi\epsilon_0}\Big(\frac{r}{r_A^3} - \frac{r^4}{r_A^6}\Big)dr \tag{4}$$

Therefore, for the potential energy, we obtain

$$E_{pot} = \int_0^{r_A} = -\frac{9e^2}{40\pi\epsilon_0}\frac{1}{r_A} = -\frac{9}{5}\frac{\hbar^2}{2ma_B^2}\Big(\frac{a_B}{r_A}\Big). \tag{5}$$

Hence, the total energy writes

$$E = \frac{\hbar^2}{2ma_B^2}\Big(\frac{3}{5}\Big(\frac{9\pi}{4}\Big)^{\frac{2}{3}} - \frac{9}{5}\Big(\frac{a_B}{r_A}\Big)\Big). \tag{6}$$

After rewriting as a function of particle density n and substituting r_A by $(\frac{4}{3}\pi n)^{\frac{1}{3}}$, we obtain the numerical value for E (in eV) as a function of (molar) volume

$$E = 0.155226\ v^{-\frac{2}{3}} - 1.76024\ v^{-\frac{1}{3}}. \tag{7}$$

2.2 Compressed molecules

Intuitively, liquids are believed to be essentially incompressible. It is clear however, that under conditions of the huge pressures in consideration, density must become a function of pressure. The following approach should be seen as a 'proof of concept' rather than a fully realistic computation of this extreme state of hydrogen. The shortcomings of this approach will be discussed later. The key question, however, is: How do you compress an atom or a molecule, respectively? The size of an atom is determined by Bohr's radius, which is again determined by the condition that one entire de Broglie wavelength λ_B must fit into its circumference. However, λ_B can vary as a matter of principle, while a smaller wavelengths corresponds to a higher velocity, corresponding to the formula

$$\lambda_B = \frac{h}{mv} \tag{8}$$

Although m is velocity-dependent, we can still use a non-relativistic a calculation for a wide range of v. Naturally, a smaller λ_B influences the energy (which is $E_H = -13,6eV$ in the ground state) in two ways: a higher kinetic energy due to the increased velocity raises the energy, while the closer distance to the nucleus still lowers the potential energy. Since the former raise is quadratic and the latter just linear, it is clear that there is an overall loss of binding energy, as expected. Assuming the volume of an atom to be proportional to λ_B^3, one obtains:

$$E_{mol} = -2E_H(\frac{\rho}{\rho_0})^{\frac{1}{3}} + E_H(\frac{\rho}{\rho_0})^{\frac{2}{3}}, \tag{9}$$

as a function of density ρ and

$$E_{mol} = -2E_H(\frac{v}{v_0})^{-\frac{1}{3}} + E_H(\frac{v}{v_0})^{-\frac{2}{3}}. \tag{10}$$

as a function of molar volume v. Numerically, this compares to (7) as

$$0.797671\ v^{-\frac{2}{3}} - 6.58736\ v^{-\frac{1}{3}}. \tag{11}$$

It is thus clear that for small densities the molecular state is energetically preferable while for large densities, the positive term in the metallic state does not grow as much. The transition occurs around a density of 424 kg/m^3. As fig. (1) shows, this corresponds to about 0.022 $\frac{l}{mol}$. At this stage, we do not distinguish between the atomic and the molecular state, since at $t = 5800\ K$, the temperature of the photosphere, the Gibbs factor $\exp(-E_H/kT)$ still yields a negligible amount of atoms compared to molecules. However, one might add half the binding energy of the H_2 molecule, 2.37 eV, to E_H.

2.3 Pressure

If $E(v)$ describes the energy content of a state as a function of volume, the pressure can simply be obtained by

$$p(v) = \frac{d}{dv}E(v). \tag{12}$$

This is easily retransformed into a function of density, see fig.(2). The numerical results are

$$p_{LMH}(v) = -0.0561674\rho^{\frac{4}{3}} + 0.0099062\rho^{\frac{5}{3}} \tag{13}$$

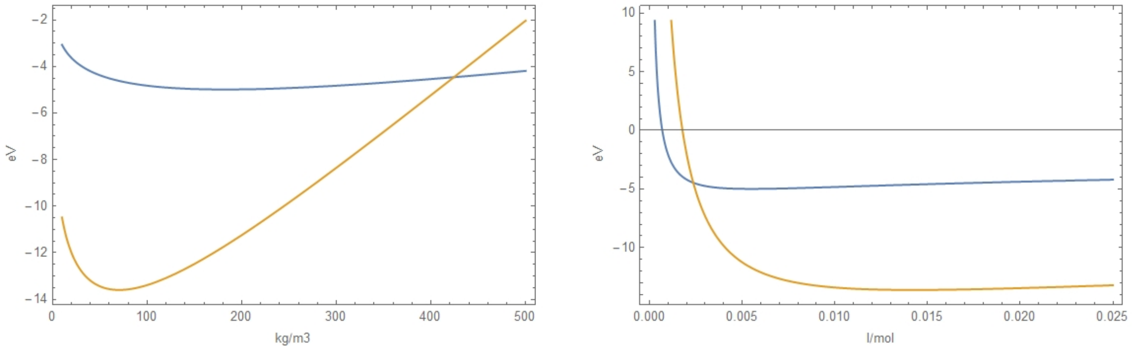

Figure 1: Engery of the liquid metallic state (blue) and the molecular state (yellow). Function of density (left) and volume (right).

for the liquid metallic state and

$$p_{H2}(v) = -0.210196\rho^{\frac{4}{3}} + 0.0509057\rho^{\frac{5}{3}} \tag{14}$$

for the molecular state.

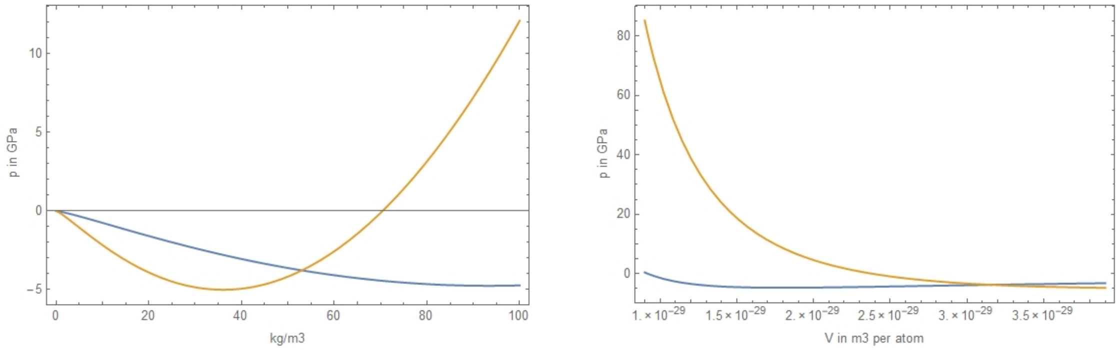

Figure 2: Pressure in the liquid metallic state and in the molecular state. Function of density (left) and volume per atom (right).

2.4 Integrating chromospheric density

By numerically inverting eq.(13) and eq.(14),we can now plot density as a function of pressure. This is done by a *Mathematica* routine which interpolates between previously computed points (ρ, p). The respective curves are shown in fig.(3).

Having expressed density as a function of pressure, it is now easy to model the pressure that the entire chromosphere exerts on the photospheric layer that represents the phase transition from metallic to molecular hydrogen. We can determine from eq.(14) that a pressure of $p_{tr} = 548\ GPa$ is needed to transform molecular to metallic hydrogen, since this amount is necessary compress molecular hydrogen to a density of $424\ kg/\mu^3$. It should be emphasized that the molecular state in the chromosphere, despite the high temperatures in the range of $3000 - 4000\ K$, can still be seen as a liquid, since due to the high pressure, the density exceeds $70kg/m^3$ by far. This is the reason why temperature can be left out for the purposes of the present discussion.

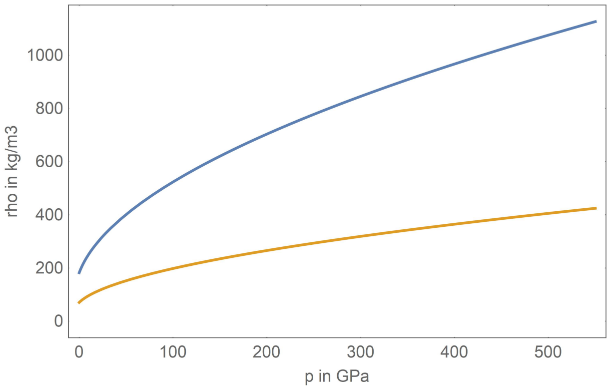

Figure 3: Density as a function of pressure for the liquid metallic (blue) and the molecular (yellow) state.

One may thus just numerically stratify layers of molecular hydrogen on top of the photosphere, calculate the density ρ and via

$$dp = -\rho \; g \; dz, \tag{15}$$

with $g = 274 \; m/s^2$, determine what height h_c is needed to obtain 548 GPa. The result, as shown in fig. (4), is $h_c = 7900 \; km$, in good agreement with the observations. Of course, on top of this liquid molecular layer there is still a barometric atmosphere with half thickness of about 70 km, which however does contribute very few to the mass column.

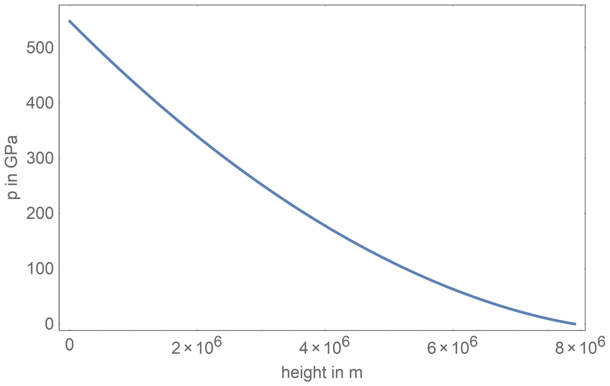

Figure 4: Pressure as a function of height in the chromosphere

3 Outlook

Such an approximate approach thus strengthens the overall picture of the liquid metallic hydrogen model of the sun. While the metallic (actually semi-metallic) state is able to emit blackbody radiation at $T = 5800 \; K$, the chromosphere is widely transparent, except for the Fraunhofer absorption lines. However, there are still many details to be refined in the coarse picture outlined here. The Fraunhofer absorption lines do not correspond exactly to the chromospheric emission spectrum. This is just one detail of a wealth of spectroscopic evidence that points to chemical reactions in the chromosphere, which are only possible if the density is orders of magnitude higher than assumed in the standard solar model.[2] Then, as stressed in a series of articles by Robitaille, the transition from the molecular to the metallic state is probably not occurring in such a direct way. Likely instead, there are intermediate states of semi-metals, presumably with a graphite-like structure. Such crystalline structures are completely common also in liquids at high temperature, if one restricts to properly small time and length scales. There is one shortcoming in the above assumptions that however supports the hypothesis of an intermediate semi-metal state occurring at a pressure inferior to the metallic transition. Assuming a shortening of de Broglie wavelengths in a hydrogen atom would of course alter the spectrum of the hydrogen molecule (or atom), something which is not observed, or at least not clearly identified yet. Hence, though the present model becomes increasingly unrealistic for higher pressures, the transition to the metallic state sooner or later must occur. According to Nellis 2013 the transition pressure at temperatures around 3000 K is even lower, around 140 GPa. In any case, prior to the metallic state, a semimetal state, possibly graphite-like, will form that is already capable of producing a blackbody spectrum. This would require an even less thick chromosphere.

The above considerations were justified by the fact that the temperature at the photosphere is relatively small and corresponds only to about 0.5 eV, thus enabling us to treat the chromosphere as a (molecular) liquid. Below the photosphere, two scenarios are possible: a metallic state with relatively uniform density, as Robitaille (2011) has argued, backed by the coincidence that the average density of the sun is in teh same range ($1.4 \frac{g}{cm^3}$). On the other hand, if both temperature and pressure rise right beneath the photosphere, there is also the possibility that the liquid metallic hydrogen model merges into the standard solar model that treats almost the entire interior of the sun as a plasma. In this case, no big discrepancy is expected when comparing the sun's total mass to the integrated densities of the present model. Finally, I have not taken into consideration the amount of helium in the sun, which is almost one fourth in mass. However, the above methods can easily be generalized, while the rough estimate computed will surely not change the order of magnitude.

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[2] See a series of videos in Robitaille's YouTube channel "SkyScholar".