

## CARDINALITY BETWEEN NATURAL AND REAL NUMBERS

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Let  $S = [0,1,1)$ , whose cardinality is the same as that of  $\mathbb{R}$ , and  $f(n) = n \times 10^{-l}$ , where  $l$  is the number of digits of  $n \in \mathbb{N}$ ; for example, for  $n = 1$ ,  $l = 1$  and  $f(1) = 0.1$ . Then,

Sea  $S = [0,1,1)$ , cuya cardinal es la misma que la de  $\mathbb{R}$ , y  $f(n) = n \times 10^{-l}$ , donde  $l$  es el número de dígitos de  $n \in \mathbb{N}$ ; por ejemplo, para  $n = 1$ ,  $l = 1$  y  $f(1) = 0,1$ . Entonces,

| $n$ | $f(n)$ | $n$      | $f(n)$     |
|-----|--------|----------|------------|
| 1   | 0.1    | 200      | 0.2        |
| 2   | 0.2    | ⋮        | ⋮          |
| 3   | 0.3    | 2000     | 0.2        |
| ⋮   | ⋮      | ⋮        | ⋮          |
| 10  | 0.1    | 6500     | 0.65       |
| ⋮   | ⋮      | ⋮        | ⋮          |
| 15  | 0.15   | 65000    | 0.65       |
| ⋮   | ⋮      | ⋮        | ⋮          |
| 100 | 0.1    | 650000   | 0.65       |
| ⋮   | ⋮      | ⋮        | ⋮          |
| 150 | 0.15   | 98765... | 0.98765... |
| ⋮   | ⋮      | ⋮        | ⋮          |

Note that  $f(n)$  is surjective, thus  $|\mathbb{N}| > |S| = c$ .

The above result either contradicts Cantor ( $\mathbb{R}$  is denumerable) or infinite sets do not exist.

Note que  $f(n)$  es sobreyectiva, por lo tanto  $|\mathbb{N}| > |S| = c$ .

El resultado anterior o bien contradice a Cantor ( $\mathbb{R}$  es numerable) o bien no existen conjuntos infinitos ( $|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}| = |\mathbb{R}|$ ).