

# MEASURING WEALTH AND REMUNERATION GAPS

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## ABSTRACT

The paradigm that our societies should provide equal access to opportunities and a sufficient level of well-being to citizens regardless of their personal characteristics (gender, race, religion...) is one of the pillars, perhaps an axiom, of our political systems. And the link detected between the possession of income and wealth and access to opportunities means that the measurement of economic inequality (or gap) between different social groups is an indirect indicator to evaluate the degree of compliance with the previous axiom; that is, the degree to which said groups have equal access to opportunities and well-being.

The mathematical characterization of the gap between groups is therefore key in this assessment. However, most proposals to assess economic gaps between groups use mathematical characterizations of the income or wealth of each group to be compared, whose inadequacy has already been demonstrated. Thus, to make progress on this issue, this text presents an alternative characterization of the income or wealth of the groups, based on the economic concept of *equivalent average income*, which states a relationship between average income, economic inequality, and generated welfare. This allows us to contrast the optimality of the different possible characterizations against the current axiomatic for inequality indexes. Likewise, the conceptualization itself implies the formulation of an inequality index which, compared to the most commonly used indexes (Gini, Pietra, Atkinson, Theil), provides the advantage of being able to compute zero or negative income or wealth values.

KEYWORDS: Gender Pay Gap; Ethnic Pay Gap; Remuneration Gap; Wealth Gap; Economic inequality.

## 1 INTRODUCTION: ASSESSING GENDER AND RACE WEALTH OR INCOME INEQUALITY

The paradigm that excessive economic inequality among citizens implies an unacceptable inequality of rights and opportunities, which in turn generates instability and reduces the resilience of societies, dates back to antiquity. We find rules aimed at limiting economic inequality in the Code of Urukagina (ca. 2400 BC). Later examples have been the *Seisachtheia* (or release of burdens) made by Solon (ca. 594 BC) which is considered a first step towards the Athenian *Demokratia*; the agrarian laws enacted during the Roman Republic (e.g., *Lex Licinia*, ca. 350 BC; Tiberius Gracchus, 134 BC), or the limitation by Lucullus (ca. 80 BC) of the percentage of income allocated to pay debts to a maximum of 25% (Author, 2017).

However, few rules of this era proposed what the optimal inequality should be, which in turn implied defining a criterion to measure it, perhaps the most interesting being Plato's (*Laws*, ca. 349 BC) who proposed a ratio of 4:1 as a measure of the maximum admissible wealth gap<sup>1</sup>.

In contrast to these *normative* approaches to the issue, the search for a quantitative characterization of societies' economic inequality existing began at the end of the 19<sup>th</sup> century. Subsequently, throughout the 20<sup>th</sup>-century studies reviewed the differences in income and wealth between different groups in society. After the mid-twentieth century, studies were published reviewing the different remuneration that men and women perceived for their work, which was designated as the Gender Pay or Wage Gap. These studies have had different goals for measuring this Remuneration Gap (RG)<sup>2</sup> and consequently have adopted different perspectives on how the phenomenon should be quantitatively assessed/measured<sup>3</sup>, which has led to different mathematical characterizations of the Gender Remuneration Gap (GRG):

On the one hand, the majority of *official institutions and agencies* aim to measure the inequality of opportunities implied in the different remuneration for each gender workers. In order to assess it, these organizations have adopted an easy mathematical approach. Firstly, they characterize Men and Women Remuneration using a central tendency measure -it can be the Arithmetic Mean/Average ( $R_a$ ) or Median ( $R_{med}$ ) Remuneration-, then they calculate the GRG as the percentage that the difference between Men and Women Remuneration implies in relation to men remuneration. That is, the GRG is calculated in a two-steps procedure (NJNCHEs, 2011; ILO, 2016):

1. Each group of workers -men  $R_M$  and women  $R_W$ - is characterized by its Average or Median Remuneration.

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<sup>1</sup> Noteworthy, at this time a great difference in welfare is admitted between certain groups of inhabitants (for example, between citizens and *metics* -or even slaves-), and often between men and women.

<sup>2</sup> The goal of limiting the Gap is to provide equality of opportunities between people (as well as maximise society's welfare). In order to achieve this equality, the fundamental aspect is not the wage (salary) but the total compensation (salary plus contributions to pension plans, bonus, severance pay...) a person may expect to perceive for his/her work depending on his/her gender, ethnic origin or disability status,... Therefore, herein analyses review the total annual worker's remuneration, and the term we use is Remuneration Gap

<sup>3</sup> For a review of two main perspectives (Supply side and Demand side) see Russell *et al* (2017).

2. The GRG is calculated as the difference between the two previous characterizations, divided by men's Remuneration  $R_M$ <sup>4</sup>.

$$GRG = \frac{R_M - R_W}{R_M} \quad (1)$$

This approach has been incorporated in recent regulations (e.g.: in the UK, the Equality Act 2010 (2017); in Spain, RD 902/2020 on equal pay between women and men...).

On the other hand, *social and economic scientists* have sought to review whether remuneration difference between genders is due to actual gender discrimination or to differences in other workers' characteristics which imply different value for companies. This paradigm, started with the *Human Capital Theory*, which assumes the premise that different workers' qualities such as education and experience should be differently compensated<sup>5</sup>. These scientists have undertaken studies which also imply a several-step procedure mathematical modelling, which builds on Oaxaca (1973) and Blinder (1973) regression and decomposition analyses:

1. A theoretical relation between objective variables and economic compensation is modelled.
2. Deviation of the actual remuneration structure for each gender regarding the expected compensation according to the theoretical model is calculated
3. The gap is computed as the difference between both genders' deviations.

These studies initially resorted to each group characterization by the Average and Median Remuneration - $R_a$  and  $R_{med}$ - (e.g., Sanborn, 1966) but later incorporated also the Geometric Mean ( $R_{gm}$ ) Remuneration (Oaxaca, 1973: 696)<sup>6</sup>.

Therefore, both types of Gender Remuneration Gap analyses use some type of central tendency measure (Arithmetic Average, Median, or Geometric Average) to characterize the income received by each group of workers. These two approaches have also been adopted for

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<sup>4</sup> While this approach has been adopted because most times the GRG favours men (therefore, men's remuneration is usually higher), we show several examples where the opposite is the case. Hence, in this article, we use the formula  $GRG = \frac{R_M - R_W}{\max(R_M; R_W)}$ .

<sup>5</sup> These studies measure the part of the Remuneration Gap that can be explained by 'human capital' differences (being *education* and *work experience* the most fundamental variables), then consider the unexplained gap is caused by gender discrimination (e.g., Sanborn, 1966; Oaxaca, 1973; Becker, 1975...). However, the last decades' educational change has led women to outperform men in education in many countries, and work experience differences –still favoring men- have been minimized. Thus, the importance of classical human capital theory variables for the GRG in most developed countries is challenged by latest studies (e.g., Heinze, 2010; Blau & Khan, 2017). Yet, these studies show other variables with appreciable influence on the GRG, such as occupation type, job level ('glass ceiling' effect); or difference between types of firms men and women work.

<sup>6</sup> Following this path, disaggregated analyses are undertaken decomposing gender pay gaps for different percentiles throughout the wage distribution (e.g., Heinze, 2010; Blau & Khan, 2017).

studies reviewing the Ethnic and Disability Remuneration Gaps (and will most likely be adopted in forthcoming regulations of these gaps<sup>7</sup>), as well as in studies reviewing the Wealth Gap<sup>8</sup>.

Thus, usual approaches have assumed the suitability of mean, median, and geometric mean to characterize groups' income. Yet, these three characterizations fail to meet some basic requirements:

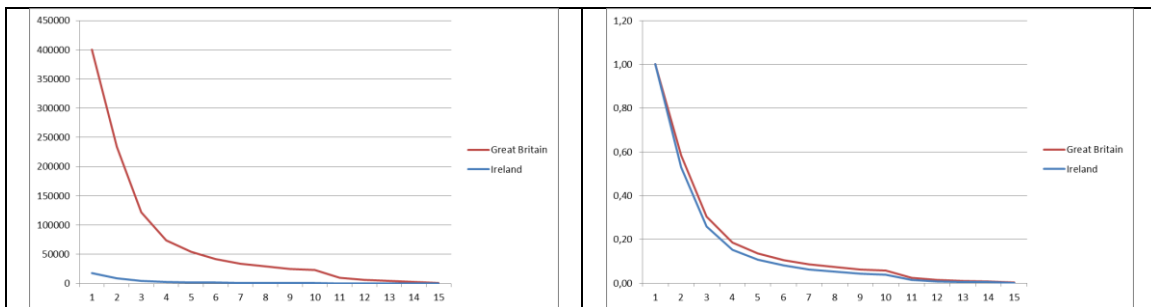
- The mean fails to capture the often high inequality in the distribution of Income and wealth.
- The median often fails to meet the monotonicity requirement.
- In the presence of zero values, the geometric mean also breaches the monotonicity requirement.

The above issues imply that the three characterizations may provide different assessments, showing the need for deepen our understanding on the characterization of the income or wealth of a group of people.

## 2 THE CHARACTERIZATION OF ECONOMIC INEQUALITY BETWEEN GROUPS AND ITS IMPACT ON WELL-BEING. THE CONCEPT OF AVERAGE EQUIVALENT INCOME

### 2.1 CHARACTERIZATION OF THE INCOME OF A SET OF INDIVIDUALS

Most studies that measure the difference in income or wealth between two groups (e.g., men and women, whites and non-whites...) use a central tendency measure such as the arithmetic mean, the median, or the geometric mean. However, given the non-linearity of income and wealth distributions, the suitability of such measures is very limited. This non-linearity was first highlighted in 1897 by Pareto, who divided the population into income/wealth steps, obtaining highly skewed distributions.



**Image 01. Income distribution in Great Britain and Ireland.** By grouping the individuals by income level Pareto detected a skewed distribution. Furthermore, although the distribution of income expressed in monetary units (left image) shows an appreciable difference between the two countries, the normalized values (right image) draw almost overlapping curves. Source: Own elaboration based on Pareto, 1897: 305.

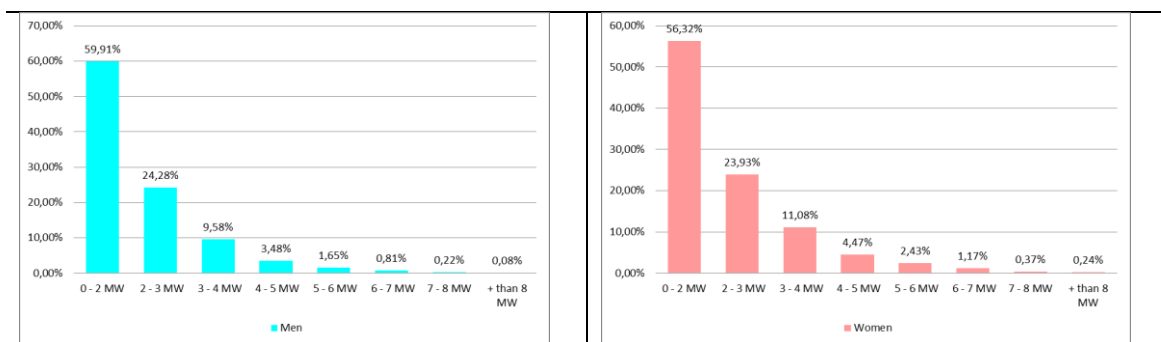
<sup>7</sup> E.g., Adams *et al*, 2018 use Average pay for calculations. An early regulation limiting the Ethnic or Disability Pay Gap was South Africa *Equal Employment Act 55 (1998)*, which however, did not state the calculation criterion for the gap (Reg. 21). Most authors expect forthcoming UK regulation limiting the ethnic and disability pay gaps to follow the guidelines set in *Equality Act 2010 Regulations 2017*, which sets mandatory report of both mean and median pay differences for the whole set and for each quartile of workers (Reg. 2.1).

<sup>8</sup> E.g., Sullivan *et al*, 2015; Darity *et al*, 2018 use Median Wealth in their analysis

In 1953, Champernowne modelled the curve seeking to explain why real-life income distributions approached Pareto curves, taking into consideration the effects of age and occupational stratification of the population. He obtained some approximations, by setting a lower probability of increasing income in high-income levels, and higher in low-income levels.

Subsequently, Simon (1955) detected these highly skewed distribution curves in several phenomena<sup>9</sup>. The author proposed there was a *non-linear underlying probability structure*. Furthermore, regarding income, he stated this underlying probability distribution enabled the assignment of probabilities to the way future income would be distributed among the current (or new) individuals of the set.

Later, studies confirmed these skewed income/wealth distribution curves, also when the distribution is reviewed independently for each sex.



**Image 02. Distribution of workers in Spain in 2020 according to income levels (referring to the Minimum Wage), differentiating by sex (men on the left and women on the right). Pareto obtained distributions skewed toward the lower income brackets similar to those we obtain today. Source: Own elaboration based on INE (2020).**

This last issue raises the difficulty of characterizing both distributions prior to their comparison. How is it possible to quantitatively characterize skewed distributions consistently, so that their comparison is also consistent?

## 2.2 CHARACTERIZING A SKEWED DISTRIBUTION

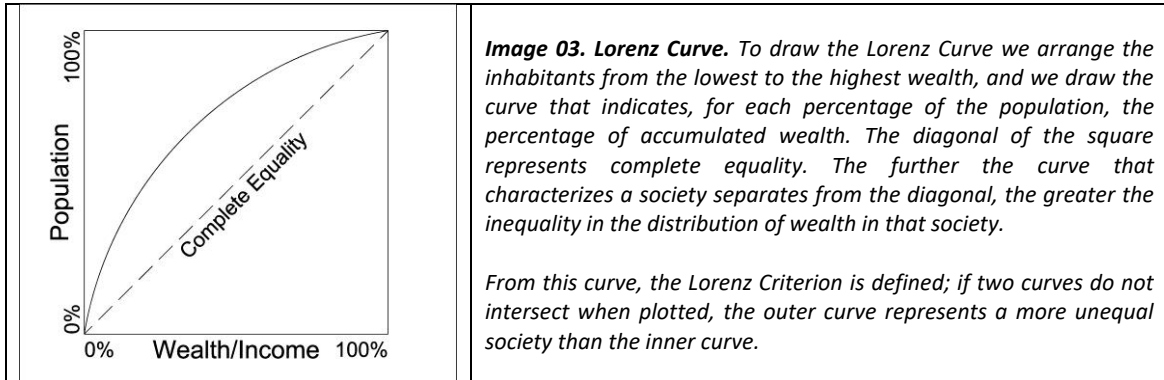
Since Pareto's early contribution, we find two main approaches to the characterization of skewed distributions. Some experts have focused on measuring the inequality or concentration implicit in the distribution. Others have focused on proposing a value that represents the allocation of economic resources implicit in the distribution. We begin by reviewing the first approach.

### 2.2.1 CHARACTERIZING THE CONCENTRATION OR INEQUALITY OF DISTRIBUTION

A first contribution was made by Pareto himself, who suggested 1) defining income or wealth steps, and 2) counting the variation in the number of people in each step over time, to evaluate whether inequality was increasing or decreasing.

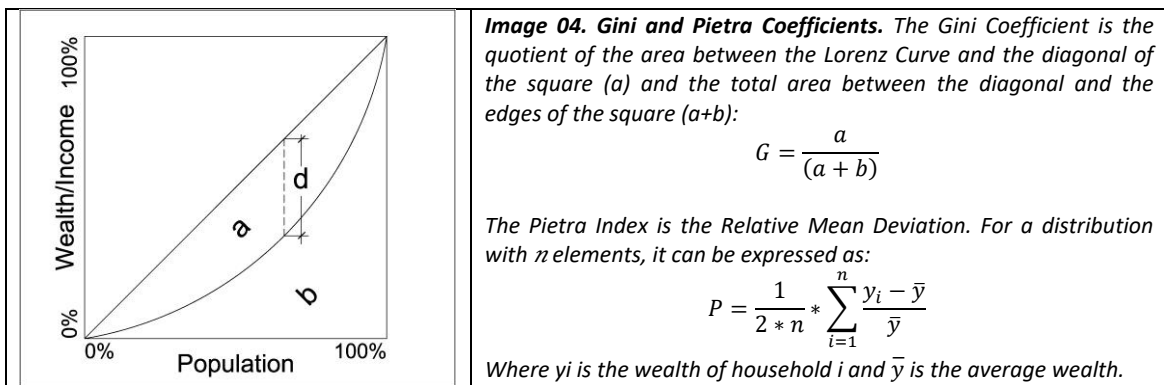
<sup>9</sup> The author found the same probability function underlies diverse phenomena as "distributions of words in prose samples by their frequency of occurrence; distributions of scientists by number of papers published; distributions of cities by populations; distributions of incomes by size; and distributions of biological genera by number of species" (Simon, 1955; 1)

A few years later, Max O. Lorenz (1905) stated that Pareto's approach was incomplete, thus it was necessary to account at the same time for both changes in the percentage of people and accumulated wealth. To this end, he proposed to graphically represent the inequality of societies by means of a curve.



In 1914, Corrado Gini proposed a characterization relating the area between the Lorenz Curve and the diagonal with half of the square, to obtain a coefficient between 0 and 1 that expressed the existing inequality in each society.

In 1915, Pietra (cited in Neves & Perez-Duarte 2019) proposed using the Mean Relative Deviation. This index is the proportion of total wealth that would have to be transferred from some citizens to other citizens to achieve complete equality. Graphically, it is the maximum vertical distance between the Lorenz curve and the diagonal of the square.



By 1920, in addition to the Gini Coefficient, the deviation between quartiles, the mean relative deviation (Pietra Index) and the standard deviation were being used to characterize the inequality of distributions (Dalton, 1920). However, the above two coefficients can provide equal values for very different income distributions. Therefore, alternative coefficients were proposed. Some decades later, Herfindahl and Hirschman (1950) proposed an index to measure market economic concentration, and in 1967 Theil used Shannon's entropy to measure the concentration of income.

However, the question of the optimal quantitative characterization of inequality remains currently unanswered, as subsequent studies (e.g., Winship & Schwartz, 1978; Neves & Pérez-Duarte 2019) have shown that different inequality measures can provide different

measurements and rankings and that existing indices cannot compute negative wealth values (Neves & Pérez-Duarte 2019)<sup>10</sup>.

### **2.2.2 CHARACTERIZING BY MEANS OF A VALUE THE ALLOCATION THAT THE DISTRIBUTION IMPLIES TO THE INDIVIDUALS: THE AVERAGE EQUIVALENT INCOME OR WEALTH**

A first contribution for characterizing skewed distribution by a unique value was proposed by McAlister (1879). The author reviewed several types of data sets and concluded that the arithmetic mean does not correctly characterize some distributions. When a data set is characterized by a value, we expect that when choosing a subset (or an element) at random, the *error by excess or default is similar*<sup>11</sup>. However, for some distributions the arithmetic mean does not meet this condition.

Later contributions were made by Pigou (1912) and Dalton (1920) who proposed that it was possible to approximately characterize these distributions by logarithmic laws. Besides, Dalton (1920:348) clarified *the goal was not to measure the inequality of the distribution of income, but the “effects of the distribution of income upon the distribution and total amount of economic welfare”*. The author highlighted that what was needed was the relation between inequality and welfare, since *“inequality, though it may be defined in terms of economic welfare, must be measured in terms of income” (ibid, 349)*.

Dalton stated that given a certain amount of wealth in a society the maximum welfare created by such wealth would be obtained in the case it was equally distributed among the individuals, while any departure from complete equality would imply a reduction in the obtained welfare. *Dalton suggested the harmonic or geometric mean of the incomes could be the relevant variable to characterize the welfare generated by an unequal wealth/income distribution.*

Dalton assumed the diminishing marginality of income and wealth in the creation of welfare (or collective utility) and suggested any function characterizing welfare should comply with *the principle of transfers*. If a transfer of income took place from a richer to a poorer individual, (which did not reverse the position of said individuals in the ordering, being the maximum admissible transfer that which generated equality of both incomes), inequality was diminished.

The author stated that this principle was neither satisfied by the arithmetic mean nor by the dispersion of the quartiles, although it was satisfied by the Gini coefficient, which he stated was the best available indicator of inequality. Besides, the author suggested that the income that generates economic welfare was that which is *above the cost of subsistence, and therefore, that the inequality that would be relevant is that which is obtained once the cost of subsistence was subtracted from the income of individuals*<sup>12</sup>.

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<sup>10</sup> Neves & Perez (2019:36ff) assessed the variation in households' wealth of different European countries between 2010 and 2014 using Gini, Pietra, Theil and Generalized Entropy, and show that the indices provide different values and may even indicate different sign variation.

<sup>11</sup> The author stated that the characterization of a data set must incorporate a probability assignment, and referred to the concepts of *expected value and expected error*. He stated that both the probability of making a mistake by default or by excess and the amount of said defect/excess must be assessed.

<sup>12</sup> See ANNEX 2. A COMPLEMENTARY CONCEPT: THE APPROPRIABLE SURPLUS GAP

In 1939, Wedgwood continued Dalton's ideas and calculated that "in the case of the logarithmic function the level of welfare associated with the actual distribution of income in Great Britain in 1919-1920 was only 77% of what it would have been had income been equally distributed" (Wedgwood cited in Atkinson, 1970:250)

In 1967, Aigner & Heins suggested that different concentration formulas implied different conceptualizations of inequality and advocated making them explicit. The authors showed that different inequality coefficients could provide different valuations and rankings of income distributions, making it impossible to establish a collective preference among these states.

In 1970 Atkinson continued the above paradigm and suggested it was necessary to relate income to social welfare. He proposed the concept of *equally distributed equivalent level of income* as "the level of income per head which if equally distributed would give the same level of social welfare" as the reviewed distribution (*ibid.* 250)<sup>13</sup>.

$$I = 1 - \frac{r_{eq}}{\bar{r}} \quad (2)$$

$I$  - Inequality;  $r_{eq}$  - Per capita income that would generate equivalent welfare if distributed equally;  $\bar{r}$  - Average income of the current distribution.

Therefore, the characterization of the *equally distributed equivalent income* is:

$$r_{eq} = \bar{r} * (1 - I) \quad (3)$$

Atkinson introduced a coefficient  $e$  for inequality aversion. If the inequality aversion coefficient  $e$  is equal to 1, inequality is the ratio of the geometric mean to the arithmetic mean of income.

$$e = 1 \rightarrow r_{eq} = \left( \prod i_i \right)^{\frac{1}{n}} \rightarrow I = 1 - \frac{(\prod r_j)^{\frac{1}{n}}}{\bar{r}} \quad (4)$$

That is, if  $e$  is equal to 1, then the mean equivalent income  $r_{eq}$  is the geometric mean of incomes.

Atkinson pointed out that *any convex function* would be valid to transform the income distribution into a collective utility function providing an equal ranking of the different distributions, as long as the Lorenz curves do not cross. The problem arose because in many distributions the curves crossed so contradictory results appeared; some inequality indexes considered some situations better than others while others considered the opposite.

In 2010, Alkire & Foster (2010) continued this perspective. Their objective was to relate the utility generated by the income of a group of individuals and the distribution of said income. They asserted that the diminishing marginal utility of income implies that the maximum utility of the group is achieved in the situation of maximum distribution but they introduced the

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<sup>13</sup> Atkinson also drew a parallel between income inequality and the theory of decision-making under uncertainty, where the equally distributed equivalent level of income is the analogue of the certainty equivalent (*ibid.* 251)



diminishing marginality of utility by means of a logarithmic weighting that assigns lower values to the increases in the distribution when it presents higher values.

They asserted inequality in different dimensions is a measure of "percentage loss in potential human development or welfare" (Alkire and Foster, 2010:6). It is equivalent to considering that Income Concentration implies a reduction in the total utility that it would be possible to create with that Income (or in other words, it is a measure of inefficiency in the creation of total utility) and the optimal Income concentration state of society is the one maximizing the total utility created with the available income in the system.

Therefore, from the above contributions, we can consider that *the concentration of wealth or income implies a percentage of wasted utility*. If we discount this percentage, the resulting value is a measure of collective utility, which, divided among all individuals, gives the *equivalent per capita income*.

### 2.3 THE AXIOMATIZATION OF THE EQUIVALENT PER CAPITA INCOME

The review has shown different conceptualizations of both *inequality* and *equivalent per capita income*<sup>14</sup>, which poses an obstacle to the characterization of both values in relation to different human groups. To progress on this issue several authors have proposed formal conditions that should be met by the indexes, enabling us to assess the suitability of each formula. A formal framework was proposed by Schwartz and Winship (1979: 6ff) who compiled from earlier authors some prerequisites and four axioms that should be fulfilled by inequality indexes<sup>15</sup>:

- Prerequisites/Shared properties of all indexes:
  - They are zero when income is equally distributed and positive otherwise, approaching 1 as the concentration increases<sup>16</sup>.
  - They are impartial (anonymous); the value of the index does not depend on who possesses what income.
- **Axiom 01. The principle of transfers (Pigou, 1912; Dalton, 1920).** Inequality is reduced if we transfer income from a richer to a poorer person. The transfer should not be so large that the receiver becomes richer than the donor.
- **Axiom 02. Population Symmetry (Dalton, 1920).** If two populations are equal in size and the income is identically distributed, the inequality of each population is identical

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<sup>14</sup> The very mathematical definition links the two concepts (equivalent income and inequality) implying that each concept has as many possible mathematical characterisations as the other.

<sup>15</sup> Neves & Perez-Duarte (2019: 10) have summarized the main properties that appear in the literature as basic requirements for an inequality indicator, being approximately coincident with those below.

<sup>16</sup> If we consider the definition of equivalent per capita income, it is clear that the value of inequality is 0 when the equalised per capita income is equal to the median income and approaches 1 as income is concentrated in a single individual. Two limiting values to the inequality coefficients are thus erected.

and equal to the inequality of the combined population. This is, the indicator remains unchanged if the distribution is replicated a finite number of times.

- **Axiom 3: Scale Invariance (Atkinson, 1970).** If we increase every individual's income by the same proportion income inequality remains unchanged. In other words, the total income to be divided has no bearing on the degree of inequality; it is only the relative share each person receives that is important in determining inequality.

**The three axioms above imply the Lorenz criterion:** if when representing two distributions, the curves do not intersect, the outermost one implies a more unequal distribution of income. The problem arises because often the Lorenz curves intersect. In such cases, two or more formulas may satisfy the above axioms, while differently arranging the distributions (Atkinson 1970). Thus, a fourth axiom is put in place:

- **Ax.04. Principle of diminishing transfers.** A measure of inequality should take into account the differential impact of transfers between different points in the distribution. Consider two persons with incomes of  $X$  and  $Y$ , with  $X$  less than  $Y$ . The principle of diminishing transfers states that the reduction in inequality attributable to a transfer from the person with income  $X$  to another person with income  $X - C$  (where  $0 < C < X$ ) is greater than the reduction attributable to an equal transfer from the person with income  $Y$  to someone with income  $Y - C$ .

However, the author stated that the above four properties are not sufficient to single out a measure, so other properties may be added to ensure the uniqueness of the measure.

## 2.4 RECAP

The goal of this text is to propose a characterization of *equivalent income or wealth* that enables consistently measuring the existing remuneration or wealth gap between two groups of individuals. It has been shown that the highly unequal income distribution within groups hinders their characterization, thus their comparison with other groups. To move forward, we resort to economic theory, which has proposed the concept of *equivalent average income or wealth*. As formulations harmonic and geometric means have been proposed.

Additionally, from a collective utility perspective, most authors highlight that income concentration implies a reduction in the creation of collective utility. Although there is no agreement on the exact measurement of this reduction, most authors have related it to the concentration percentage. Thus, the *per capita* or average utility generated should be

obtained by multiplying the total wealth or income by its distribution (complement of its concentration) and dividing it by the number of people<sup>17</sup>.

In the following section, a system of axioms is proposed, against which the different formulas used to characterize the groups' income or wealth can be contrasted.

### 3 A CONSISTENT AXIOMATIC FRAMEWORK FOR EQUALLY DISTRIBUTED EQUIVALENT INCOME

#### 3.1 AXIOM SYSTEM

We have reviewed several axioms proposed for inequality indices. However, our objective is a formula for measuring *equally distributed equivalent income* or wealth (henceforth, *equivalent income*) and, therefore, the system of axioms that we must posit refers to the formal conditions that *equivalent income* must satisfy.

Since there is a mathematical relationship between equivalent income and inequality, to advance towards our objective we can adapt the above axioms for the inequality indexes, substituting in the equation that relates both concepts. In addition, it is necessary to consider that the wealth and even the income) of an individual can be zero or even negative if his/her debts exceed the value of his/her assets (or income). Thus, the index must be able to provide an *equivalent income or wealth* value if there are zero or negative wealth/income values<sup>18</sup>.

Specifically, we propose four general conditions and six axioms (own proposal, based on Dalton, 1920; Aigner & Heins, 1967; Atkinson 1970; and Neves & Perez-Duarte, 2019) that the quantitative characterization of Equivalent Income or Wealth ( $R_{eq}$ ,  $W_{eq}$ ) must fulfil:

General conditions:

01.  $R_{eq}$ ,  $W_{eq}$  are computable for any set of  $r, w \in \mathbb{R}$  (this is, individuals' income or wealth can be zero or negative).
02.  $R_{eq}$ ,  $W_{eq} \in \mathbb{R}$  (equivalent income or wealth can be also zero or negative).
03. In the absence of variation in total wealth or income, the index is inversely correlated with inequality: if inequality increases, the values of  $R_{eq}$ ,  $W_{eq}$  decrease; if inequality decreases, the values of  $R_{eq}$ ,  $W_{eq}$  increase.
04. If inequality is zero (the wealth or income of all individuals is equal), then  $R_{eq}$ = average income and  $W_{eq}$ = average wealth.

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<sup>17</sup> Moreover, if we take into account that it is widely accepted income concentration implies an underlying probability distribution, this calculation procedure *de facto* implies an estimation of the *expected collective utility*, enabling us to rank different society states in terms of expected collective utility, the most accepted criterion for *rational* collective decision making in choice theory.

<sup>18</sup> Noteworthy, in the EU an average of 5% of households have negative or zero net wealth, ranging between 1% of and 14% of households in each country (Neves & Perez-Duarte, 2019:18). Chancel *et al* (2022) have found the top 50% of Chileans have negative wealth. An index which fails to compute zero or negative values provides an incorrect assessment of the current socio-economic reality, which is behind major socio-political conflicts.

Axioms:

- **Ax.1: Monotonicity (Aigner & Heins, 1967):** if any individual's wealth or income increases (or decreases), all the others being unchanged, the equivalent wealth or income increases (or decreases).
- **Ax.2: Population symmetry (Dalton, 1920):** the equivalent wealth or income remains unchanged if the distribution is replicated a finite number of times.
- **Ax.3: Principle of transfers (Pigou, 1912; Dalton, 1920):** if a transfer of value  $t > 0$  is made from an individual with wealth or income  $y_i$  to another with wealth or income  $y_j$  such that  $y_i - t > y_j + t$ , equivalent wealth or income increases. In other words, a transfer that takes from the rich and gives to the poor, but the rich remain richer than the poor, preserves the value of the mean wealth or income and causes inequality to decrease, thereby *increasing equivalent wealth or income*<sup>19</sup>.
- **Ax.4: Scale invariance (Atkinson, 1970):** if wealth or income is multiplied by a factor  $k$ , the resulting equivalent wealth or income is the initial wealth or income multiplied by that factor  $k$ .
- **Ax.5: Additive invariance (Atkinson 1970).** If an amount  $k$  is added (or subtracted) to every individual wealth or income, the equivalent wealth or income is increased (or reduced) by  $k$ .

### 3.2 AXIOM FULFILMENT

The three formulas usually used to characterize the equivalent wealth/income of groups before calculating the gap do not fulfil many of the above conditions. Specifically:

**Table 1.** Compliance with Axioms and General Conditions

	GC	Ax.01	Ax.02	Ax.03	Ax.04	Ax.05
Arithmetic Mean	N(1)	Y	Y	N	Y	Y
Median	Y	N	Y	N	Y	Y
Geometric Mean	N	N(2)	Y	Y	N(2)	N

Source: Own elaboration

(1) Does not comply with GC 03

(2) It does not comply if there are zero or negative values.

Thus, it is necessary to develop a formula to characterize the equivalent income more consistently. A first approach could be to build on the very definition of equivalent income linked to inequality, which makes it possible to propose a formula consisting of multiplying the average wealth or income by the complement of some inequality index (Gini coefficient, Pietra, Atkinson, or Theil indices...). This formula would fulfil the axioms to the same extent to

<sup>19</sup> This axiom implies a weak principle of diminishing marginality of income/wealth: if  $y_i > y_j$  then an increase  $k$  in individual  $j$ 's wealth/income increases equivalent wealth/income to a greater extent than the same increase  $k$  in individual  $i$ 's wealth/income.

which the formula used to characterize inequality satisfies the equivalent axioms. However, the issue remains that currently used inequality indexes fail to compute equal to zero or negative values.

Therefore, a computationally simple alternative formula for equivalent per capita wealth ( $W_{eq}$ ) or Income ( $R_{eq}$ ) is explained below:

$$W_{eq}; R_{eq} = \left[ [y_{max} - y_{min}] * \frac{1}{n} * \sum_{i=1}^n \frac{y_i - y_{min}}{y_{max} - y_{min}} * k_i \right] + y_{min} \quad (5)$$

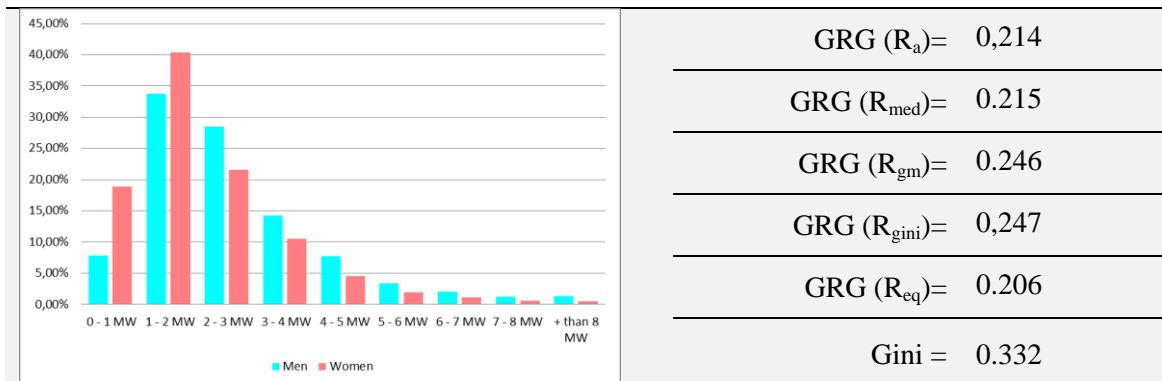
Being the  $k_i$  coefficient calculated as:

$$k_i = 1 + \frac{1}{n} * \sum_{i=1}^n \frac{y_i - y_{min}}{y_{max} - y_{min}} - \frac{y_i - y_{min}}{y_{max} - y_{min}} \quad (6)$$

This formula satisfies all the above general conditions and axioms. Let us now review the results obtained for several samples of income or wealth distributions.

#### 4 REVIEW OF REAL SAMPLES

The review has shown different properties of the formulas. However, the question remains as to whether the formulas provide significantly different or practically equal measures for real distributions. To shed light on this issue we evaluate several real income and wealth distributions using the Arithmetic Mean, the Median, the Geometric Mean, the Mean multiplied by the Gini Coefficient (Dalton, 1920), and the proposed formula. For example, the remuneration structure of Spanish workers in 2020 shows a high bias towards lower incomes. We calculate the Gender Remuneration Gap (GRG) using the five formulas:



**Image 05.** Structure of workers' remuneration in Spain 2020 (expressed as the number of times the Minimum Wage was earned). Source: Own elaboration using: workers distribution according to MW, Mean Income ( $R_a$ ) and Median Income ( $R_{med}$ ) from 'Encuesta de Estructura Salarial 2020', INE (<http://www.ine.es>). Geometric Mean ( $R_{gm}$ ), Income by Gini Coefficient ( $R_{gini}$ ) and Equivalent Income ( $R_{eq}$ ) have been calculated using simplified workers distribution. Therefore some minor discrepancy is expected compared to micro-data for the last three values.

This high resemblance between the results provided by the five characterizations is confirmed when we review the period 2008-2020:

**Table 2.** GRG in Spain 2008-2020

	GRG( $R_a$ )	GRG( $R_{med}$ )	GRG( $R_{gm}$ )	GRG( $R_{gini}$ )	GRG( $R_{eq}$ )	Gini Coef.
<b>2020</b>	0,187	0,185	0,227	0,233	0,187	0,319
<b>2019</b>	0,195	0,197	0,227	0,231	0,188	0,319
<b>2018</b>	0,214	0,215	0,246	0,247	0,206	0,332
<b>2017</b>	0,219	0,216	0,243	0,244	0,207	0,329
<b>2016</b>	0,223	0,222	0,248	0,249	0,211	0,333
<b>2015</b>	0,229	0,226	0,260	0,262	0,221	0,333
<b>2014</b>	0,233	0,221	0,258	0,260	0,220	0,334
<b>2013</b>	0,240	0,230	0,264	0,265	0,229	0,337
<b>2012</b>	0,239	0,229	0,263	0,263	0,228	0,332
<b>2011</b>	0,230	0,218	0,252	0,251	0,217	0,324
<b>2010</b>	0,225	0,220	0,250	0,250	0,215	0,323
<b>2009</b>	0,220	0,207	0,245	0,243	0,207	0,319
<b>2008</b>	0,219	0,208	0,245	0,243	0,208	0,315

Source: Own calculation building on same data as the above image. The standard deviation between the values provided by the four formulas is reduced (0,019) while the average Pearson correlation is high (0,955). A significant 0,72 Pearson correlation is found between GRG average value and the Gini Coefficient, suggesting a strong link between both parameters: the higher the inequality, the higher the Gap.

However, while the review of a large population shows a high resemblance between the GRG according to these five formulations when reviewing specific distributions, some relevant discrepancies are detected. For example, let us review the Members of Spain Parliament compensation:

**Table 3.** GRG in Members of Spanish Parliament Compensation, 2015

		GRG					Gini
		GRG( $R_a$ )	GRG( $R_{med}$ )	GRG( $R_{gm}$ )	GRG( $R_{gini}$ )	GRG( $R_{eq}$ )	
MPs Remuneration, Spain 2015		3.73%	0.00%	3.39%	2.62%	3.88%	0,062

Source: Own calculation building on data from Medialab, Madrid City Council, 2015 (<https://www.medialab-prado.es/>). Remuneration received by the MPs from other sources than Parliament is not accounted for. The inequality of remuneration was very low (Gini = 0.062).

The above example shows again high agreement between most formulations –  $R_a$ ,  $R_{gm}$  and  $R_{eq}$ - Remuneration Gap assessment. Yet, while the small difference in the result provided by  $R_{med}$  is apparently meaningless, it actually is not. It shows  $R_{med}$  fails to detect the existing Gender discrimination in Spain Congress in 2015 when highest paid positions were occupied mostly by men. This challenges the suitability of the median for characterizing the income of the groups. Another interesting issue in the above table is that the much lower inequality of MPs'

compensation (as measured by the Gini coefficient) compared to the whole set of Spain workers correlates with a much smaller GRG.

To provide more contrast, we review below the Gender/Ethnic Remuneration Gap (GRG, ERG) in three recent Forbes lists:

**Table 4.** Ethnic and Gender Remuneration Gaps in three Forbes' Lists year 2018

		$R_a$	$R_{med}$	$R_{gm}$	$R_{gini}$	$R_{eq}$	Gini
100 Best Paid Athletes (3)	ERG	16,11%	5,85%	11,11%	6,80%	9,94%	0,23
100 Best Paid Celebrities (2)	GRG	-17,92%	-9,61%	-12,81%	-9,35%	-10,34%	0,21
	ERG	-3,44%	-9,64%	-3,97%	-4,46%	-2,90%	
10 Best Paid Authors (4)	GRG	20,37%	35,14%	22,16%	51,22%	28,27%	0,39

Source: Own elaboration building on the following data from [www.forbes.com](http://www.forbes.com). Notes:

- (1) Race has been estimated based on the visual information available.
- (2) For this set of people Remuneration Gap favours women over men and non-whites over whites. Hence, the negative values of the gaps.
- (3) We do not calculate the GRG since there is only 1 female (Serena Williams in position #63).
- (4) There are no non-whites; therefore we do not calculate the ERG.

The above table confirms that the five characterizations of the equivalent income can lead to different results, with the median remuneration providing the most divergent values for calculating the Gaps. However, Average, Geometric Mean, and Equivalent Income provide not so divergent Remuneration Gap values. So... do these three formulations provide appreciably divergent assessments in real-world examples? The answer is yes. Let us show it with several examples. Firstly, we review the GRG in 22 Administration Boards of IBEX-35<sup>20</sup> companies, which show important differences depending on which characterization of the groups we use:

**Table 5.** Gender Remuneration Gap in 22 IBEX-35 companies' Administration Boards, 2016

	BRG					Gini
	$R_a$	$R_{med}$	$R_{gm}$	$R_{gini}$	$R_{eq}$	
Abengoa	0.053	-0.597	-0.239	-0,365	-0,104	0.534
Abertis Infraestructuras	0.878	0.034	0.721	0,730	0,742	0.689
Acerinox	0.577	0.184	0.312	0,211	0,328	0.500
Actividades de Construcción y Servicios, ACS	0.826	-0.064	0.277	0,386	0,727	0.746
AENA	0.673	0.095	0.421	0,359	0,417	0.611
Banco de Sabadell	0.841	-0.090	0.451	0,481	0,628	0.720

<sup>20</sup> IBEX-35 is the group of 35 highest-value companies of the Spanish stock market.

Banco Santander	0.080	0.653	0.612	0,421	0,570	0.704
Banco Popular	0.935	0.000	0.567	0,664	0,883	0.840
Bankia	0.625	0.000	0.304	0,222	0,313	0.516
Endesa	0.792	0.115	0.587	0,542	0,619	0.561
Ferrovial	0.931	0.058	0.408	0,636	0,822	0.817
FCC	1.000	1.000	1.000	1.000	1.000	0.684
Gamesa Corporación Tecnológica	0.547	0.056	0.105	-0,075	0,184	0.545
Gas Natural SDG	0.544	-0.150	0.134	-0,011	0,304	0.599
Grifols	0.741	0.468	0.612	0,567	0,642	0.492
Inditex	0.909	0.527	0.607	0,624	0,669	0.775
Mapfre	0.832	0.324	0.569	0,626	0,750	0.661
OHL	0.836	-0.349	0.169	0,476	0,641	0.756
Red Electrica Corporación, REE	0.472	0.000	0.399	0,303	0,374	0.331
Repsol	0.672	0.201	0.400	0,276	0,412	0.545
Sacyr	0.844	-0.114	0.326	0,368	0,414	0.739
Telefonica	0.702	0.242	0.377	0,309	0,537	0.648
Whole set of 22 ABs members	0.591	0.238	-0.085	0,560	0,746	0.775

Sources and Notes: Own elaboration with data extracted from the CNMV website for 2016 (<https://www.cnmv.es>). Only 22 Administration Boards have been reviewed, since many companies have issued protected data, thus difficulting its analysis. Total gross annual compensation for each person has been accounted for (therefore, important differences may arise if considering hourly or daily compensation). Reviewed Board sizes (excluding non-personal members) range from 7 to 24 members. Negative GRG values indicate GRG favouring women. The whole set of 22 Boards members comprises 347 directors/advisors (22 companies; 266 men and 59 women). Negative values of GRG indicate that it favours women.

We see the values provided by the formulas are often considerably different; both within companies' Administration Boards and for the whole set of 347 Boards' members, and may even point to the existence of opposite sign discrimination. This confirms that *the five characterizations of the groups can lead to very different assessments of the existence, size and sign of the Remuneration Gap*. Again, we find a high correlation between formulations' discrepancies and the high inequality of remuneration structure (Gini=0,775). To obtain further proof of these different assessments of the Gaps, let us review the Gender and Ethnic Wealth Gap in the group composed of the 150 wealthiest people in the USA according to Forbes:

**Table 6.** Gender and Ethnic Wealth Gap in 2018 Forbes' 400 List for the USA

	$W_a$	$W_{med}$	$W_{gm}$	$W_{gini}$	$W_{eq}$	Gini
The 150 wealthiest of the USA	GWG	0,357	0,139	0,208	0,152	0,369
	EWG	0,553	0,029	0,333	0,186	0,487



Source and Notes: Own elaboration building on the data from <https://www.forbes.com/forbes-400/#495dad867e2f>. Only the 150 wealthiest positions have been considered (implying 152 people, due to three persons tied in position 150). When more than one person is listed (e.g., Matilda & Family; John & Wife...) the gender and race of the person in the first place has been taken into account. Race has been estimated based on visual information. The total members are 126 males, 24 females; 145 whites, and 7 non-whites. The variation in the size of the average Gap highly correlates with the increase in overall wealth inequality (Gini).

Again, we see the values provided by the formulas are considerably different.

- Building on the Median Wealth, the size of the gap is between approx. 40% (GWG) and approx. 5% (EWG) of the size calculated using Average Wealth.
- Building on the Geometric Mean Wealth, the size of both gaps is approx. 60% of the size calculated using Average Wealth.

Thus, the above tables confirm that the three currently used characterizations of the groups ( $W_a$ ,  $W_{med}$  and  $W_{gm}$ ), or even the average by the Gini coefficient, can lead to very different assessments of the existence and size of the Gap in real-world situations, posing an actual challenge for its assessment or even compliance with legal thresholds. To shed more light on these discrepancies, it is interesting to review the correlations between the Remuneration and Wealth values provided by these five formulas for the cases reviewed in these last tables:

**Table 7.** Correlation between revised formulations and Gini Coefficient

	$R_a/W_a$	$R_{med}/W_{med}$	$R_{gm}/W_{gm}$	$R_{gini}/W_{gini}$	$R_{eq}/W_{eq}$	Gini
$R_a/W_a$	-	0.237	0.685	0.743	0.850	0.768
$R_{med}/W_{med}$		-	0.681	0.625	0.500	0.143
$R_{gm}/W_{gm}$			-	0.814	0.771	0.468
$R_{gini}/W_{gini}$				-	0.917	0.590
$R_{eq}/W_{eq}$					-	0.793

Source: Own elaboration building on data of above tables 2-5 (Table 1 is excluded since it is calculated based on simplified intervals). Discrepancies between formulas increase when the inequality of the distribution increases. Additionally, there is a high correlation between the value of the average Gap calculated using the five characterizations and the Gini coefficient for the whole distribution. This suggests that in the presence of greater inequality, the gap tends to be higher.

The small correlation between GRG calculated using  $R_a/W_a$  and  $R_{med}/W_{med}$  (0.237) confirms Average and Median do not measure the same phenomenon and challenges the belief that both formulations are interchangeable to measure GRG. On the other hand, the high correlation between GRG calculated using Equivalent  $R_{eq}/W_{eq}$  and Average  $R_a/W_a$  (0.850) as well as using Equivalent  $R_{eq}/W_{eq}$  and Geometric  $R_{gm}/W_{gm}$  (0.771) support the validity of herein proposed formula to replace such formulations.

Furthermore, the importance of the inequality of the distribution, and the almost nil correlation ( $R=0.143$ ) between  $R_{med}/W_{med}$  and Gini coefficient, sustains the inadequacy of the Median for calculating the GRG. The higher sensitivity of  $R_a/W_a$  and  $R_{eq}/W_{eq}$  to the inequality of the distribution, suggests these two formulas have higher validity for modelling the

significance of the distribution into the GRG characterization while from said formulas, only  $R_{eq}/W_{eq}$  provides consistent values in all cases and is able to compute negative values.

## 5 CONCLUSIONS

Wealth and income gaps have long been recognized as indicators of the degree to which different societies meet the ideal of providing equal access to opportunities for all their members. However, the widely accepted importance of this issue contrasts with the fact that we do not know how to consistently characterize the income or wealth of the groups; which in turn implies we do not know how to consistently measure the gaps.

In order to make progress on this issue the concept of *equivalent per capita income* (Dalton, 1920; Atkinson, 1970) has been recovered. Its mathematical link with inequality characterization has enabled us to draft a system of logically equivalent axioms to those of inequality indexes. Since the currently used formulas to characterize income or wealth do not meet these axioms, an alternative formula has been explained.

Subsequently, several examples of real economic distributions have been evaluated, comparing the results provided by this formula, the three most currently used formulas (Mean, Median, and Geometric Mean), and a fourth formula derived from Dalton (1920, mean by the Gini coefficient). The review has confirmed that the Mean, Median, and Geometric Mean currently used to characterize each group of workers to then calculate the Remuneration or Wealth Gap can lead to contradictory, sometimes even opposed, evaluations of the Gaps<sup>21</sup>.

This implies that both the (gender, ethnic and disability) analyses international organizations are currently undertaking for assessing the dimension of the Gap and the regression analyses that seek to determine the causes of the Gaps, could be drawing partly incorrect conclusions since they build on (sometimes) incorrect characterizations of groups' income.

This is of utmost importance at a time when companies<sup>22</sup> and institutions' Gender Pay Gap reporting is becoming mandatory in an increasing number of countries, and it is expected that Ethnicity and Disability Gaps reporting will also become mandatory in forthcoming regulations. These regulations use formulas whose inconsistency has been shown, challenging their validity for regulatory purposes. On the other hand, the formal and factual consistency of the herein-explained formula has been shown, suggesting its suitability for characterizing the Remuneration and Wealth Gap both for research and regulatory purposes.

Likewise, herein proposed characterization can be used to evaluate other Gaps such as the Education Remuneration Gap (i.e., the Remuneration Gap between college graduates and high school graduates) or emerging gaps, such as the one detected between workers who kept their

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<sup>21</sup> The fact that detected discrepancies are higher in the small-size environments, allows suggesting errors produced by incorrectness of central tendency measures currently used may be more important in analyses of firms, being perhaps the reason why some contradictory results have been found (see Heinze, 2010).

<sup>22</sup> It is usually mandatory for companies with more than 250 employees. In Spain, companies with more than 50 employees must report their remuneration structure, setting the limit value at 25% of the gap referred to as both the average and median value (RD 902/2020. Art. 6 and 7).

jobs during the Great Recession (2008-2010) and those who lost it and subsequently found a new job with lower remuneration, or the gap which is currently being detected between workers who kept their jobs and those who lost them as a result of the Covid-19 crisis.

Finally, the high correlation detected (0.83) between the Remuneration and Wealth Gender and Ethnic Gaps between subgroups and the Income/Wealth Concentration in the whole group (as measured by the Gini Coefficient) points out to the moderation of inequality as one of the most effective strategies to reduce remuneration gaps.

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## 6 ANNEXES

### ANNEX 1: AN INDEX TO MEASURE INEQUALITY

A formulation has been explained that makes it possible to calculate the average equivalent income or wealth ( $R_{eq}$ ,  $W_{eq}$ ) for a group of individuals. Said parameter, is defined in relation to both the mean income or wealth and the inequality ( $I_r$ ;  $I_w$ ) in its distribution within the group (Dalton, 1920; Atkinson, 1970):

$$I_R = 1 - \frac{1}{\bar{r}} * r_{eq}; I_W = 1 - \frac{1}{\bar{w}} * w_{eq} \quad (7)$$

Therefore, by substituting in the above equations herein proposed *equivalent remuneration or wealth* formula, we obtain an *Index to measure inequality*.

$$\text{income} \quad I_R = 1 - \frac{\sum_{i=1}^n [(r_i - r_{min}) * k_i]}{n * \bar{r}} * r_{min} \quad (8)$$

$$\text{wealth} \quad I_W = 1 - \frac{\sum_{i=1}^n [(w_i - w_{min}) * k_i]}{n * \bar{w}} * w_{min} \quad (9)$$

Being the coefficient  $k_i$  calculated as:

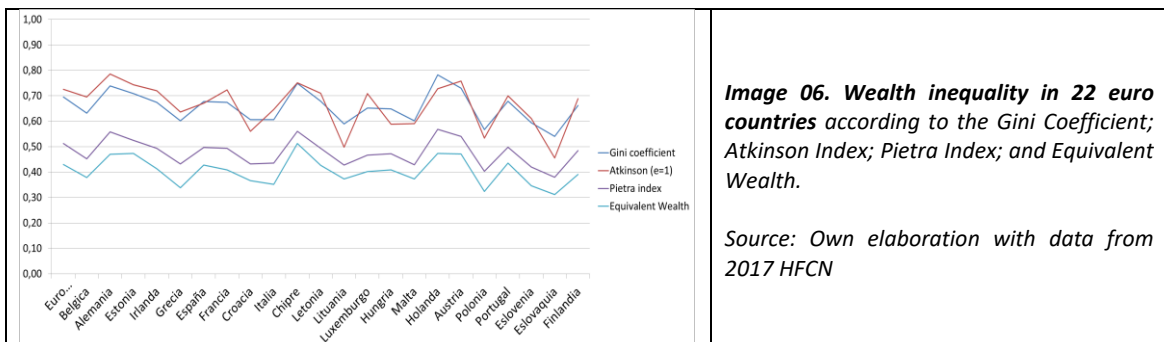
$$\text{income} \quad k_i = 1 + \frac{1}{n} * \left[ \sum_{i=1}^n \frac{r_i - r_{min}}{r_{max} - r_{min}} \right] - r_i \quad (10)$$

$$\text{wealth} \quad k_i = 1 + \frac{1}{n} * \left[ \sum_{i=1}^n \frac{w_i - w_{min}}{w_{max} - w_{min}} \right] - w_i \quad (11)$$

This Inequality index is consistent with the above axiomatic, which is not met by any other index. Specifically:

- Ax.00\_ Limits and Computability (General Conditions)
  - The maximum value of the index is the average remuneration or wealth, which is reached when income or wealth is equally distributed
  - The index can compute negative wealth or income values
- Ax.01\_ Population Principle
- Ax.02\_ Principle of Transfers
- Ax.03\_ Scale Invariance

To provide some empirical test we review the Wealth Inequality data in the 2017 Household Finance and Consumption Network (HFCN) survey for 22 countries plus the Eurozone.



Likewise, we observe high correlations with the inequality measurement using equivalent wealth and the indexes used by the ECB, being:

**Table 8.** Correlation between concentration indexes

	Equivalent Wealth	Gini Coefficient	Atkinson index (e=1)	Theil index	Pietra Index
Equivalent Wealth	-	0,958	0,806	0,929	0,967
Gini Coefficient		-	0,872	0,855	0,995
Atkinson index (e=1)			-	0,758	0,871
Theil index				-	0,859
Pietra Index					-

Notes: The equivalent wealth has been calculated with simplified data so, if calculated with microdata, the actual correlation with the other indexes is expected to be higher. As an illustration, the correlation with the Gini coefficient calculated with the simplified data increases to 0.971.

Therefore, the formal consistency of this Inequality Index correlates to its empirical consistency, enabling its use to characterize the economic inequality of groups or societies. However, it should be noted that the values obtained from reviewed samples are approximately half those obtained with the Gini coefficient, which should be taken into account when assigning meaning to these measures. *A value obtained with this coefficient represents much greater inequality than the same value obtained with the Gini Coefficient.*

Besides, the review shows the importance of Wealth Concentration, which has been largely forgotten in inequality analyses, and whose average value for the sample of countries reviewed is twice that of Income inequality, an issue which could be behind social unrest in the last decades, both in Europe and in Latin America (Chacel *et al*, 2022).

## **ANNEX 2. A COMPLEMENTARY CONCEPT: THE APPROPRIABLE SURPLUS GAP**

Dalton (1920) already referred to the fact that the relevant remuneration for generating welfare is that which is above the subsistence level. Subsistence would be a precondition for welfare, but it is not welfare. Besides, the additive invariance of equivalent income (Atkinson, 1970) implies that if we subtract a positive and equal amount from all incomes, inequality increases<sup>23</sup>. This is confirmed by substituting in the revised equations:

$$r_{eq} < \bar{r} \rightarrow \frac{r_{eq} - b}{\bar{r} - b} < \frac{r_{eq}}{\bar{r}} \rightarrow \quad (12)$$

<sup>23</sup> Dalton already asserted the opposite but equivalent transformation, this is, that if an equal quantity is added to every individual inequality was reduced. Only in the exceptional situation where income is distributed equally, both by adding and subtracting the same amount to all individuals does inequality remain unchanged.

$$\rightarrow 1 - \frac{r_{eq}}{\bar{r}} > 1 - \frac{r_{eq} - b}{\bar{r} - b} \rightarrow I_1 > I_2 \quad (13)$$

Therefore, if we take into account that the income that generates welfare (that which each person has available to "invest" in his or her welfare) and allows access to opportunities is that which exceeds the subsistence level, then to assess the real equality or inequality in a society in access to opportunities (but also to assess the opportunities gap between groups), what is relevant is *the inequality obtained once the cost of subsistence has been deduced*. We designate this concept as the *Appropriable Surplus Gap*.

The analysis of the data from remuneration structure in Spain (2008-2020) considering the cost of living equal to the Public Multiple Effect Income Indicator (IPREM)<sup>24</sup> shows that the Gender Appropriable Surplus Gap may be approx. 1.5 times the Gender Remuneration Gap.

**Table 9.** Comparison between Remuneration Gap and Appropriable Surplus Gap

YEAR	MINIMUM WAGE	COST OF LIVING (IPREM)	GENDER GAP		GINI COEFFICIENT	
			REMUNERATION GAP	APPROPRIABLE SURPLUS GAP	REMUNERATION GAP	APPROPRIABLE SURPLUS GAP
2020	13,300.00 €	7,519.59 €	0.187	0.267	0.319	0.455
2019	12,600.00 €	7,519.59 €	0.188	0.272	0.319	0.460
2018	10,302.60 €	7,519.59 €	0.206	0.304	0.332	0.486
2017	9,907.80 €	7,519.59 €	0.207	0.308	0.329	0.484
2016	9,172.80 €	7,455.14 €	0.211	0.317	0.333	0.493
2015	9,080.40 €	7,455.14 €	0.221	0.332	0.333	0.494
2014	9,034.20 €	7,455.14 €	0.220	0.332	0.334	0.498
2013	9,034.20 €	7,455.14 €	0.229	0.346	0.337	0.505
2012	8,979.60 €	7,455.14 €	0.228	0.345	0.332	0.496
2011	8,979.60 €	7,455.14 €	0.217	0.326	0.324	0.483
2010	8,866.20 €	7,455.14 €	0.215	0.325	0.323	0.483
2009	8,736.00 €	7,381.33 €	0.207	0.315	0.319	0.478
2008	8,400.00 €	7,236.60 €	0.208	0.318	0.315	0.473

Source: Own Elaboration. Note: for the lowest remuneration level negative values are obtained (i.e., the obtained remuneration is below the subsistence level), implying these situations cannot be modelled using the geometric mean.

Assuming the stronger relation between the *surplus remuneration* and the actual access to opportunities implies that the real gap and inequality in access to opportunities would be approximately 1.5 times higher than indicated by the actual *remuneration*. The relevance of this variation suggests this figure should be paid attention in forthcoming studies.

<sup>24</sup> IPREM is an index used in Spain as a reference for the granting of subsidies and social aid.