Poincaré dodecahedral space solution of the shape of the Universe

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Abstract

According to a recent theory the Universe could be a dodecahedron. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. In this paper we will prove the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy. The sum of the contributions to the total density parameter at the current time is $\Omega_0=1{,}0139$. This result answers the ancient question of whether space is finite or infinite,while retaining the standard Friedmann–Lemaître foundation for local physics.

Keywords

Density parameter of dark energy , Cosmological parameters , Cosmological constant , Fine-structure constant , Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Fundamental Interactions , Gravitational fine-structure constant

1. Introduction

The Platonic solids have been known since antiquity. It has been suggested that certain carved stone balls created by the late Neolithic people of Scotland represent these shapes; however,these balls have rounded knobs rather than being polyhedral, the numbers of knobs frequently differed from the numbers of vertices of the Platonic solids, there is no ball whose knobs match the 20 vertices of the dodecahedron, and the arrangement of the knobs was not always symmetric. The ancient Greeks studied the Platonic solids extensively. Some sources credit Pythagoras with their discovery. Other evidence suggests that he may have only been familiar with the tetrahedron, cube, and dodecahedron and that the discovery of the octahedron and icosahedron belong to Theaetetus,a contemporary of Plato. In any case, Theaetetus gave a mathematical description of all five and may have been responsible for the first known proof that no other convex regular polyhedra exists. The Platonic solids are prominent in the philosophy of Plato,their namesake. Plato wrote about them in the dialogue Timaeus c. 360 B.C. in which he associated each of the four classical elements (earth, air, water, and fire) with a regular solid. Earth was associated with the cube, air with the octahedron, water with the icosahedron, and fire with the tetrahedron. There was intuitive justification for these associations: the heat of fire feels sharp and stabbing (like little tetrahedra). Air is made of the octahedron; its minuscule components are so smooth that one can barely feel it. Water, the icosahedron, flows out of one's hand when picked up, as if it is made of tiny little balls. By contrast, a highly nonspherical solid, the hexahedron (cube) represents "earth". These clumsy little solids cause dirt to crumble and break when picked up in stark difference to the smooth flow of water. Moreover,the cube being the only regular solid that tessellates Euclidean space was believed to cause the solidity of the Earth.

Of the fifth Platonic solid, the dodecahedron,Plato obscurely remarked,"...the god used for arranging the constellations on the whole heaven". Aristotle added a fifth element, aithēr (aether in Latin, "ether" in English) and postulated that the heavens were made of this element, but he had no interest in matching it with Plato's fifth solid. Euclid completely mathematically described the Platonic solids in the Elements, the last book (Book XIII) of which is devoted to their properties. Propositions 13–17 in Book XIII describe the construction of the tetrahedron, octahedron, cube, icosahedron, and dodecahedron in that order. For each solid Euclid finds the ratio of the diameter of the circumscribed sphere to the edge length. In Proposition 18 he argues that there are no further convex regular polyhedra. Andreas Speiser has advocated the view that the construction of the five regular solids is the chief goal of the deductive system canonized in the Elements. Much of the information in Book XIII is probably derived from the work of Theaetetus [1] .

According to a recent theory the Universe could be a dodecahedron. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos! Plato (c. 427 BC – c. 347 BC) also stated that time had a beginning; it came together with the universe in one instant of creation. A polyhedron bounded by a number of congruent polygonal faces, so that the same number of faces meet at each vertex, and in each face all the sides and angles are equal (i.e. faces are regular polygons) is called a regular polyhedron. One morning the young Werner Heisenberg discovered reading Plato's Timaeus a description of the world with regular polyhedra. Heisenberg could not understand why Plato, being so rational, started to use speculative ideas. But finally he was fascinated by the idea that it could be possible to describe the Universe mathematically. He could not understand why Plato used the Polyhedra as the basic units in his model, but Heisenberg considered that in order to understand the world it is necessary to understand the Physics of the atoms.

Theaetetus (c. 414-367 BC), was a member of Plato's Academy. He was a son of Euphronius of Sounion,student of Theodore of Cyrene. Theaetetus died on his return to Athens after he was wounded at the Battle of Corinth. His friend Plato dedicated one of his dialogues to him. Euclid's elements chapter X and XIII are based on the work of Theaetetus. Hippasus, from Metapontum in Magna Graecia (south Italy), who wrote around 465 BC about a "sphere of 12 pentagons' refers to the dodecahedron. Hippasus performed acoustics Experiments with vessels filled with different amounts of water and with copper discs of different thicknesses [2].

Finite well-proportioned spaces, especially the Poincaré dodecahedron, open something of a Pandora's box for the physics of the early universe. The standard model of cosmology relies in the main on the hypothesis that the early universe underwent a phase of exponential expansion called inflation, which produced density fluctuations on all scales. In the simplest inflationary models, space is supposed to have become immensely larger than the observable universe. Therefore,a positive curvature (i.e. $\Omega>1$), even if weak, implies a finite space and sets strong constraints on inflationary models.

It is possible to build "low scale" inflationary universes in which the inflation phase ends more quickly than it does in general inflationary modes, leading to a detectable space curvature. In other words, even if space is not flat, a multi connected topology does not contradict the general idea of inflation. However, no convincing physical scenario for this has yet been proposed. Perhaps the most fundamental challenge is to link the present-day topology of space to a quantum origin, since general relativity does not allow for topological changes during the course of cosmic evolution. A quantum theory of gravity could allow us to address this problem, but there is currently no indication about how such a unified theory might actually describe the emergence of multiple connected spaces. Data from the European Planck Surveyor, which is scheduled for launch in 2007 will be able to determine Ω with a precision of 1%. A value lower than 1,01 will rule out the Poincaré dodecahedron model, since the size of the corresponding dodecahedron would become greater than the observable universe and would not leave any observable imprint on the microwave background. A value greater than 1,01,on the other hand, would strengthen the models' cosmological pertinence. Whether or not some multiply connected model of space such as the Poincaré dodecahedron is refuted by future astronomical data,cosmic topology will continue to remain at the heart of our understanding about the ultimate structure of our universe.

Surprisingly,not all small-volume universes suppress the large-scale fluctuations. In 2003 J.-P. Luminet in [3] proved that the long-wavelength modes tend to be relatively lowered only in a special family of finite, multi connected spaces that are called "well-proportioned spaces" because they have a similar extent in all three dimensions. More specifically,we discovered that the best candidate to fit the observed power spectrum is a well-proportioned space called the Poincaré dodecahedral space. This space may be represented by a polyhedron with 12 pentagonal faces, with opposite faces being "glued" together after a twist of 36°. This is the only consistent way to obtain a spherical (i.e. positively curved) space from a dodecahedron: if the twist was 108°, for example, we would end up with a radically different hyperbolic space. The Poincaré dodecahedral space is essentially a multiply connected variant of a simply connected hypersphere, although its volume is 120 times smaller.A rocket leaving the dodecahedron through a given face immediately re-enters through the opposite face,and light propagates such that any observer whose line-of-sight intercepts one face has the illusion of seeing a slightly rotated copy of their own dodecahedron. This means that some photons from the cosmic microwave background, for example, would appear twice in the sky.

The power spectrum associated with the Poincaré dodecahedral space is different from that of a flat space because the fluctuations in the cosmic microwave background will change as a function of their wavelengths. In other words, due to a cut-off in space corresponding to the size of the dodecahedron, one expects fewer fluctuations at large angular scales than in an infinite flat space, but at small angular scales one must recover the same pattern as in the flat infinite space. In order to calculate the power spectrum we varied the mass-energy density of the dodecahedral universe and computed the quadrupole and the octopole modes relative to the WMAP data. To our delight,we found a small interval of values over which both these modes matched the observations perfectly. Moreover,the best fit occurred in the range $1,01<\Omega<1,02$,which sits comfortably with the observed value.

The Poincaré dodecahedral space therefore accounts for the lack of large-scale fluctuations in the microwave background and also for the slight positive curvature of space inferred from WMAP and other observations. Moreover, given the observed values of the mass-energy densities and of the expansion rate of the universe, the size of the dodecahedral universe can be calculated. We found that the smallest dimension of the Poincaré dodecahedron space is 43 billion light-years, compared with 53 billion light-years for the "horizon radius" of the observable universe. Moreover,the volume of this universe is about 20% smaller than the volume of the observable universe. (There is a common misconception that the horizon radius of a flat universe is 13,7 billion light-years, since that is the age of the universe multiplied by the speed of light. However, the horizon radius is actually much larger because photons from the horizon that are reaching us now have had to cross a much larger distance due to the expansion of the universe.) If physical space is indeed smaller than the observable universe, some points on the map of the cosmic microwave background will have several copies. As first shown by Neil Cornish of Montana State University and co-workers in 1998,these ghost images would appear as pairs of so-called matched circles in the cosmic microwave background where the temperature fluctuations should be the same. This "lensing" effect,which can be precisely calculated, is thus purely attributable to the topology of the universe. Due to its 12-sided regular shape, the Poincaré dodecahedral model actually predicts six pairs of diametrically opposite matched circles with an angular radius of 10-50°,depending on the precise values of cosmological parameters such as the mass-energy density.

2. Dimensionless unification of the fundamental interactions

In [4] we presented exact and approximate expressions between the Archimedes constant π,the golden ratio φ,the Euler's number e and the imaginary number i. We propose in [5] and [6] the exact formula for the fine-structure constant α with the golden angle,the relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}=137{,}035999164...$$

Also we propose in [7] , [8] and [9] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\pi\cdot\ln2=137{,}035999078...$$

We propose in [10] the exact mathematical expression for the proton to electron mass ratio:

$$\mu^{32}=\varphi^{-42}\cdot F5^{160}\cdot L5^{47}\cdot L19^{40/19}=1836{,}15267343...$$

$$7\cdot\mu^{3}=165^{3}\cdot\ln^{11}10 \Rightarrow \mu=1836{,}15267392...$$

$$\mu=6\cdot\pi^{5}+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}=1836{,}15267343...$$

Also in [10] was presented the exact mathematical expressions that connects the proton to electron mass ratio μ and the fine-structure constant α:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42)$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231$$

$$\mu-807\cdot\alpha=1.205\cdot\pi-518\cdot\varphi-411\cdot e$$

Also in [11] was presented the unity formula that connects the fine-structure constant and the proton to electron mass ratio. It was explained that $\mu\cdot\alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2\cdot10^{2}\cdot\cos(\mu\cdot\alpha^{-1})+13^{2}=0$$

The exponential form of this equation is:

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^{2}=0$$

Also this unity formula can also be written in the form:

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i$$

In [12] ,[13] , [14] , [15] , [16] , [17] it presented the dimensionless unification of the fundamental interactions. We calculated the unity formulas that connect the coupling constants of the fundamental forces. The dimensionless unification of the strong nuclear and the weak nuclear interactions:

$$e\cdot\alpha_s=10^7\cdot\alpha_w$$

$$\alpha_s^2=i^{2i}\cdot10^7\cdot\alpha_w$$

The dimensionless dimensionless unification of the strong nuclear and electromagnetic interactions:

$$\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot i^{2i}$$

The dimensionless dimensionless unification of the weak nuclear and electromagnetic interactions:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot e\cdot i^{2i}$$

The dimensionless unification of the strong nuclear,the weak nuclear and electromagnetic interactions:

$$10^7\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha})=2\cdot\alpha_s$$

The dimensionless unification of the gravitational and the electromagnetic interactions:

$$4\cdot e^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=1$$

$$16\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=(e^{i/\alpha}+e^{-i/\alpha})^2$$

The dimensionless unification of the strong nuclear,the gravitational and the electromagnetic interactions:

$$4\cdot\alpha_s^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=i^{4i}$$

$$\alpha^2\cdot(e^{i/\alpha}+e^{-i/\alpha})\cdot\alpha_s^4\cdot\alpha_G\cdot N_A^2=i^{8i}$$

The dimensionless unification of of the weak nuclear,the gravitational and the electromagnetic interactions:

$$4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2=i^{4i}\cdot e^2$$

$$10^{14}\cdot\alpha^2\cdot(e^{i/\alpha}+e^{-i/\alpha})^2\cdot\alpha_w^2\cdot\alpha_G\cdot N_A^2=i^{8i}$$

The dimensionless unification of the strong nuclear,the weak nuclear,the gravitational and the electromagnetic interactions:

$$\alpha_s^2=4\cdot10^{14}\cdot\alpha_w^2\cdot\alpha^2\cdot\alpha_G\cdot N_A^2$$

$$8\cdot10^7\cdot N_A^2\cdot\alpha_w\cdot\alpha^2\cdot\alpha_G=\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha})$$

We found the formula for the Gravitational constant:

$$G=\alpha_s^2\left(2\,10^7\alpha_w\alpha N_A\right)^{-2}\frac{\hbar c}{m_e^2}$$

2. Unification of atomic physics and cosmology

In [18] and [19] resulting in the dimensionless unification of atomic physics and cosmology. In the context of cosmology the cosmological constant is a homogeneous energy density that causes the expansion of the universe to accelerate. Originally proposed early in the development of general relativity in order to allow a static universe solution it was subsequently abandoned when the universe was found to be expanding. Now the cosmological constant is invoked to explain the observed acceleration of the expansion of the universe. The cosmological constant is the simplest realization of dark energy,which is the more generic name given to the unknown cause of the acceleration of the universe. Its existence is also predicted by quantum physics,where it enters as a form of vacuum energy,although the magnitude predicted by quantum theory does not match that observed in cosmology.

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. It is commonly believed that the cosmological constant problem can only be solved ultimately in a unified theory of quantum gravity and the standard model of electroweak and strong interactions,which is still absent so far. But connecting vacuum energy to the cosmological constant is not straightforward. Based on their observations of supernovas,astronomers estimate that dark energy should have a small and sedate value,just enough to push everything in the universe apart over billions of years. Yet when scientists try to calculate the amount of energy that should arise from virtual particle motion,they come up with a result that's 120 orders of magnitude greater than what the supernova data suggest. The cosmological constant has the same effect as an intrinsic energy density of the vacuum,ρ_{vac} and an associated pressure. In this context,it is commonly moved onto the right-hand side of the equation,and defined with a proportionality factor of $\Lambda=8\cdot\pi\cdot\rho_{vac}$ where unit conventions of general relativity are used (otherwise factors of G and c would also appear,i.e:

$$\Lambda = 8\pi\rho_{vac}\frac{G}{c^4} = \kappa\rho_{vac}$$

where κ is Einstein's rescaled version of the gravitational constant G. The cosmological constant has been introduced in gravitational field equations by Einstein in 1917 in order to satisfy Mach's principle of the relativity of inertia. Then it was demonstrated by Cartan in 1922 that the Einstein field tensor including a cosmological constant Λ:

$$\mathrm{E}_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}$$

,is the most general tensor in Riemannian geometry having null divergence like the energy momentum tensor $T_{\mu\nu}$. This theorem has set the general form of Einstein’s gravitational field equations as $E_{\mu\nu}=\kappa\cdot T_{\mu\nu}$ and established from first principles the existence of Λ as an unvarying true constant. The cosmological constant problem dates back to the realization that it is equivalent to a vacuum energy density. One of the main consequences in cosmology of a positive cosmological constant is an acceleration of the expansion of the universe. Such an acceleration has been first detected in 1981 in the Hubble diagram of infrared elliptical galaxies,yielding a positive value close to the presently measured one,but with still large uncertainties. Accurate measurements of the acceleration of the expansion since 20 years have reinforced the problem. The cosmological constant Λ,as it appears in Einstein’s equations,is a curvature. As such,besides being an energy density,it is also the inverse of the square of an invariant cosmic length L.

In the early-mid 20th century Dirac and Zel’dovich were among the first scientists to suggest an intimate connection between cosmology and atomic physics. Though a revolutionary proposal for its time,Dirac’s Large Number Hypothesis (1937) adopted a standard assumption of the non-existence of the cosmological constant term $\Lambda=0$. Zel'dovich insight (1968) was to realize that a small but nonzero cosmological term $\Lambda>0$ allowed the present day radius of the Universe to be identified with the de Sitter radius which removed the need for time dependence in the fundamental couplings. Thus,he obtained the formula:

$$\Lambda = \frac{m_p^6 G^2}{\hbar^6}$$

where m is a mass scale characterizing the relative strengths of the gravitational and electromagnetic interactions,which he identified with the proton mass m_p. Laurent Nottale in which,instead,suggests the identification $m=m_e/\alpha$. He assumed that the cosmological constant Λ is the sum of a general-relativistic term and of the quantum,scale-varying,gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of Λ is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha\frac{m_{pl}}{m_e} = \left(\frac{L}{l_{pl}}\right)^{\frac{1}{3}}$$

The cosmological constant Λ has the dimension of an inverse length squared. The cosmological constant is the inverse of the square of a length L:

$$L = \sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d = \sqrt{3}L$$

So the de Sitter radius and the cosmological constant are related through a simple equation:

$$R_d = \sqrt{\frac{3}{\Lambda}}$$

From this equation resulting the expressions for the gravitational fine structure constant α_g:

$$\alpha\frac{m_{pl}}{m_e} = \left(l_{pl}\sqrt{\Lambda}\right)^{-\frac{1}{3}}$$

$$\alpha_g = l_{pl}\sqrt{\Lambda}$$

$$\alpha_g = \sqrt{\frac{G\hbar\Lambda}{c^3}}$$

So the cosmological constant Λ equals:

$$\Lambda = \alpha_g^2 l_{pl}^{-2}$$

$$\Lambda = \frac{l_{pl}^4}{r_e^6}$$

$$\Lambda = \alpha_g^2\frac{c^3}{G\hbar}$$

$$\Lambda = \frac{G}{\hbar^4}\left(\frac{m_e}{a}\right)^6$$

From the expression of the gravitational fine structure constant resulting the dimensionless unification of the atomic physics and the cosmology:

$$\alpha_g=(2\cdot e\cdot \alpha^2\cdot N_A)^{-3}$$

$$l_{pl}^2\cdot\Lambda=(2\cdot e\cdot \alpha^2\cdot N_A)^{-6} \quad (1)$$

Now we will use the unity formulas of the dimensionless unification of atomic physics and cosmology to find the equations of the cosmological constant. For the cosmological constant equals:

$$\Lambda = \left(2e\alpha^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \quad (2)$$

From the expression of the gravitational fine structure constant resulting the dimensionless unification of atomic physics and cosmology:

$$l_{pl}^2\cdot\Lambda=i^{12i}\cdot(2\cdot\alpha_s\cdot\alpha^2\cdot N_A)^{-6} \quad (3)$$

For the cosmological constant equals:

$$\Lambda = i^{12i}\left(2\alpha_s a^2 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{4}$$

From the expression of the gravitational fine structure constant resulting the dimensionless unification of atomic physics and cosmology:

$$l_{pl}^2 \cdot \Lambda = i^{12i} \cdot e^6 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot a^3 \cdot N_A)^{-6} \tag{5}$$

For the cosmological constant equals:

$$\Lambda = i^{12i} e^6 \left(2\,10^7 \alpha_w a^3 N_A\right)^{-6}\frac{c^3}{G\hbar} \tag{6}$$

From the expression of the gravitational fine structure constant resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42}\left(\frac{\alpha_G a_w^2}{e^2\alpha_s^2\alpha^2}\right)^3$$

$$l_{pl}^2\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3 \tag{7}$$

$$e^6 \cdot \alpha_s^6 \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{8}$$

For the cosmological constant equals:

$$\Lambda = 10^{42}\left(\frac{\alpha_G \alpha_w^2}{e^2\alpha_s^2\alpha^2}\right)^3\frac{c^3}{G\hbar} \tag{9}$$

From the expression of the gravitational fine structure constant resulting the dimensionless unification of atomic physics and cosmology:

$$\alpha_g^2 = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3$$

$$l_{pl}^2\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{10}$$

$$\alpha_s^{12} \cdot \alpha^6 \cdot l_{pl}^2 \cdot \Lambda = 10^{42} \cdot i^{12i} \cdot \alpha_G^3 \cdot \alpha_w^6 \tag{11}$$

This unity formula is a simple analogy between atomic physics and cosmology. For the cosmological constant equals:

$$\Lambda = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3\frac{c^3}{G\hbar} \tag{12}$$

The Equation of the Universe is:

$$\frac{\Lambda G\hbar}{c^3} = 10^{42} i^{12i}\left(\frac{\alpha_G \alpha_w^2}{\alpha^2\alpha_s^4}\right)^3 \tag{13}$$

4. Solution for the density parameter of baryonic matter

The Hubble constant H_0 is one of the most important numbers in cosmology because it is required to estimate the size and age of the universe. This number indicates the rate at which the universe is expanding. The Hubble constant can be used to determine the inherent brightness and masses of stars in nearby galaxies,examine the same properties in more distant galaxies and galaxy clusters,infer the amount of dark matter in the universe,and obtain the scale size of distant clusters as far as clusters test for theoretical cosmological models. In 1929,American astronomer Edwin Hubble announced his discovery that galaxies,in all directions,seemed to be moving away from us and have greater displacement for attenuated galaxies. However,the true value for H_0 is very complicated. Astronomers need two measurements:

a) First, spectroscopic observations reveal the redshift of the galaxy, showing its radial velocity.

b) The second measurement, the most difficult value, is the exact distance of the galaxy from Earth.

The unit of the Hubble constant is 1 km/s/Mpc. The 2018 CODATA recommended value of the Hubble constant is $H_0=67{,}66\pm0{,}42$ (km/s)/Mpc$=(2{,}1927664\pm0{,}0136)\times10^{-18}$ s^{-1}. Hubble length or Hubble distance is a unit of distance in cosmology,defined as the speed of light multiplied by Hubble time $L_H=c\cdot H_0^{-1}$. This distance is equivalent to 4,550 million parsecs,or 14,4 billion light-years,13,8 billion years. Hubble's distance would be the distance between the Earth and the galaxies currently falling away from us at the speed of light,as shown by the substitution $r=c\cdot H_0^{-1}$ in the equation for Hubble's law,$u=H_0\cdot r$.

The critical density is the average density of matter required for the Universe to just halt its expansion,but only after an infinite time. A Universe with a critical density is said to be flat. In his theory of general relativity, Einstein demonstrated that the gravitational effect of matter is to curve the surrounding space. In a Universe full of matter, both its overall geometry and its fate are controlled by the density of the matter within it. If the density of matter in the Universe is high (a closed Universe),self-gravity slows the expansion until it halts, and ultimately re-collapses. In a closed Universe, locally parallel light rays converge at some extremely distant point. This is referred to as spherical geometry. If the density of matter in the Universe is low (an open Universe),self-gravity is insufficient to stop the expansion, and the Universe continues to expand forever (albeit at an ever decreasing rate). In an open Universe, locally parallel light rays ultimately diverge. This is referred to as hyperbolic geometry. Balanced on a knife edge between Universes with high and low densities of matter, there exists a Universe where parallel light rays remain parallel. This is referred to as a flat geometry,and the density is called the critical density. In a critical density Universe, the expansion is halted only after an infinite time. To date,the critical density is estimated to be approximately five atoms per cubic meter,whereas the average density of ordinary matter in the Universe is believed to be 0,2–0,25 atoms per cubic meter. A much greater density comes from the unidentified dark matter; both ordinary and dark matter contribute in favor of contraction of the universe. However,the largest part comes from so-called dark energy, which accounts for the cosmological constant term. Although the total density is equal to the critical density the dark energy does not lead to contraction of the universe but rather may accelerate its expansion. Therefore,the universe will likely expand forever. An expression for the critical density is found by assuming Λ to be zero and setting the normalized spatial curvature,k,equal to zero. When the substitutions are applied to the first of the Friedmann equations we find:

$$\rho_c=\frac{3H_0^3}{8\pi G}$$

It should be noted that this value changes over time. The critical density changes with cosmological time,but the energy density due to the cosmological constant remains unchanged throughout the history of the universe. The amount of dark energy increases as the universe grows,while the amount of matter does not. The density parameter Ω is defined as the ratio of the actual density ρ to the critical density ρ_c of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe,when they are equal,the geometry of the universe is flat (Euclidean). The galaxies we see in all directions are moving away from the Earth,as evidenced by their red shifts. Hubble's law describes this expansion. Remarkably,study of the expansion rate has shown that the universe is very close to the critical density that would cause it to expand forever. The density parameter Ω is defined as the ratio of the average density of matter and energy in the Universe ρ to the critical density ρ_c of the Friedmann universe. The relation between the actual density and the critical density determines the overall geometry of the universe;when they are equal,the geometry of the universe is flat (Euclidean). In earlier models,which did not include a cosmological constant term, critical density was initially defined as the watershed point between an expanding and a contracting Universe. The density parameter is given by:

$$\Omega_0 = \frac{\rho}{\rho_c}$$

where ρ is the actual density of the Universe and ρ_c the critical density. Although current research suggests that Ω_0 is very close to 1,it is still of great importance to know whether Ω_0 is slightly greater than 1,less than 1,or exactly equal to 1,as this reveals the ultimate fate of the Universe. If Ω_0 is less than 1,the Universe is open and will continue to expand forever. If Ω_0 is greater than 1,the Universe is closed and this will eventually halt its expansion and recollapse. If Ω_0 is exactly equal to 1 then the Universe is flat and contains enough matter to halt the expansion but not enough to recollapse it. It is important to note that the ρ used in the calculation of Ω_0 is the total mass/energy density of the Universe. In other words,it is the sum of a number of different components including both normal and dark matter as well as the dark energy suggested by recent observations. We can therefore write:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_\Lambda$$

$$\Omega_0=\Omega_m+\Omega_\Lambda$$

$$\Omega_0=\Omega_B+\Omega_{D+\Lambda}$$

where:
Ω_B is the density parameter for normal baryonic matter,
Ω_D is the density parameter for dark matter,
Ω_Λ is the density parameter for dark energy,
Ω_m is the sum of the density parameter for normal baryonic matter and the density parameter for dark matter,
$\Omega_{D+\Lambda}$ is the sum of the density parameter for the density parameter for dark matter and the density parameter for dark energy.

The sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=1{,}02\pm0{,}02$$

Current observations suggest that we live in a dark energy dominated Universe with $\Omega_\Lambda=0{,}73$,$\Omega_D=0{,}23$ and $\Omega_B=0{,}04$. To the accuracy of current cosmological observations,this means that we live in a flat,$\Omega_0=1$ Universe. Instead of the cosmological constant Λ itself,cosmologists often refer to the ratio between the energy density due to the cosmological constant and the critical density of the universe,the peak point of a density sufficient to prevent the universe from expanding forever,at one level of the universe is the ratio between the energy of the universe due to the cosmological constant Λ and the critical density of the universe,that is what we would call the fraction of the universe consisting of dark energy [20]. By definition, baryonic matter should only include matter composed of baryons. In other words, it should include protons, neutrons and all the objects composed of them (i.e. atomic nuclei), but exclude things such as electrons and neutrinos which are actually leptons. In astronomy, however, the term 'baryonic matter' is used more loosely, since on astronomical scales, protons and neutrons are always accompanied by electrons. Astronomers therefore use the term 'baryonic' to refer to all objects made of normal atomic matter, essentially ignoring the presence of electrons which, after all, represent only ~0,0005 of the mass. Neutrinos, on the other hand, are considered non-baryonic by astronomers. Another slight oddity in the usage of the term baryonic matter in astronomy is that black holes are included as baryonic matter. While the matter from which black holes form is mainly baryonic matter, once swallowed by the black hole, this distinction is lost. For example, a theoretical black hole constructed purely out of photons is indistinguishable from one made from normal baryonic matter. This is often referred to as the 'black holes have no hair' theorem which simply states that black holes do not have properties such as baryonic or non-baryonic. Objects in the Universe composed of baryonic matter include Clouds of cold gas, Planets, Comets and asteroids, Stars, Neutron stars and Black holes.

The assessment of baryonic matter at the current time was assessed by WMAP to be $\Omega_B=0{,}044\pm0{,}004$. From the dimensionless unification of the fundamental interactions the density parameter for normal baryonic matter is:

$$\Omega_B=e^{-\pi} \tag{14}$$

$$\Omega_B=i^{2i} \tag{15}$$

$$\Omega_B=0{,}0432 \tag{16}$$

$$\Omega_B=4{,}32\% \tag{17}$$

From Euler's identity for the density parameter of baryonic matter apply:

$$\Omega_B^{i}+1=0 \tag{18}$$

$$\Omega_B^{i}=i^2 \tag{19}$$

$$\Omega_B^{2i}=1 \tag{20}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_B=e^{-1}\cdot\alpha_s \tag{21}$$

$$\Omega_B=\alpha_w^{-1}\cdot\alpha_s^{2}\cdot10^{-7} \tag{22}$$

$$\Omega_B=2^{-1}\cdot\alpha_s\cdot(e^{i/\alpha}+e^{-i/\alpha}) \tag{23}$$

$$\Omega_B=2\cdot N_A\cdot\alpha_s\cdot\alpha\cdot\alpha_G^{1/2} \tag{24}$$

$$\Omega_B=2^{-1}\cdot e^{-1}\cdot10^{7}\cdot\alpha_w\cdot(e^{i/\alpha}+e^{-i/\alpha}) \tag{25}$$

$$\Omega_B=2\cdot10^{7}\cdot N_A\cdot e^{-1}\cdot\alpha_w\cdot\alpha\cdot\alpha_G^{1/2} \tag{26}$$

$$\Omega_B=10^{-7}\cdot\alpha_g^{1/3}\cdot\alpha_s^{2}\cdot\alpha\cdot\alpha_w^{-1}\cdot\alpha_G^{-1/2} \tag{27}$$

5. Solution for the density parameter of dark energy

In physical cosmology and astronomy, dark energy is an unknown form of energy that affects the universe on the largest scales. The first observational evidence for its existence came from measurements of supernovas, which showed that the universe does not expand at a constant rate; rather, the universe's expansion is accelerating. Understanding the universe's evolution requires knowledge of its starting conditions and composition. Before these observations, scientists thought that all forms of matter and energy in the universe would only cause the expansion to slow down over time. Measurements of the cosmic microwave background (CMB) suggest the universe began in a hot Big Bang, from which general relativity explains its evolution and the subsequent large-scale motion. Without introducing a new form of energy, there was no way to explain how scientists could measure an accelerating universe. Since the 1990s, dark energy has been the most accepted premise to account for the accelerated expansion. As of 2021, there are active areas of cosmology research to understand the fundamental nature of dark energy.

The fraction of the effective mass of the universe attributed to dark energy or the cosmological constant is $\Omega_\Lambda=0{,}73\pm0{,}04$. With 73% of the influence on the expansion of the universe in this era,dark energy is viewed as the dominant influence on that expansion. The previous history of the big bang is viewed as being at first radiation dominated, then matter dominated,and now having passed into the era where dark energy is the dominant influence. The density parameter for dark energy is defined as:

$$\Omega_\Lambda=\frac{\Lambda c^2}{3\mathrm{H}_0^2}$$

The cosmological constant is the inverse of the square of a length L:

$$L=\sqrt{\Lambda^{-1}}$$

For the de Sitter radius equals:

$$R_d=\sqrt{3}L$$

So for the density parameter for dark energy apply:

$$\Omega_\Lambda=\frac{c^2}{R_d^2\mathrm{H}_0^2}$$

The Hubble length or Hubble distance is a unit of distance in cosmology, defined as:

$$L_H = c \cdot H_0^{-1}$$

the speed of light multiplied by the Hubble time. It is equivalent to 4,420 million parsecs or 14.4 billion light years. (The numerical value of the Hubble length in light years is, by definition, equal to that of the Hubble time in years.) The Hubble distance would be the distance between the Earth and the galaxies which are currently receding from us at the speed of light, as can be seen by substituting $D = c \cdot H_0^{-1}$ into the equation for Hubble's law, $\upsilon = H_0^{-1} \cdot D$. So for the density parameter for dark energy apply:

$$\Omega_\Lambda = \frac{L_H^2}{R_d^2}$$

$$\Omega_\Lambda = \left(\frac{L_H}{R_d}\right)^2$$

From the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot e^{-1} \tag{28}$$

$$\Omega_\Lambda = 0{,}73576 \tag{29}$$

$$\Omega_\Lambda = 73{,}57\% \tag{30}$$

So from expression (28) apply:

$$2 \cdot R_d^2 = e \cdot L_H^2 \tag{31}$$

Also from the dimensionless unification of the fundamental interactions the density parameter for dark energy is:

$$\Omega_\Lambda = 2 \cdot \cos a^{-1} \tag{32}$$

So apply the expression:

$$\cos \alpha^{-1} = \frac{\Omega_\Lambda}{2} \tag{33}$$

So the beautiful equation for the density parameter for dark energy is:

$$\Omega_\Lambda = e^{i/a} + e^{-i/a} \tag{34}$$

The figure 1 shows the geometric representation of the density parameter for dark energy.

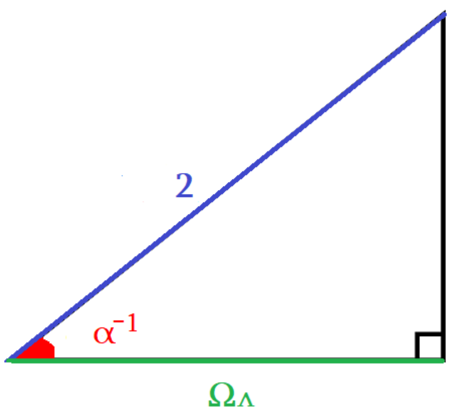

Figure 1. *Geometric representation of the the density parameter for the dark energy*

So from the expression (28) apply:

$$\cos\alpha^{-1} = \frac{L_{\mathrm{H}}^2}{2R_d^2} \tag{35}$$

The figure 2 shows the geometric representation of the relationship between the de Sitter radius and the Hubble length.

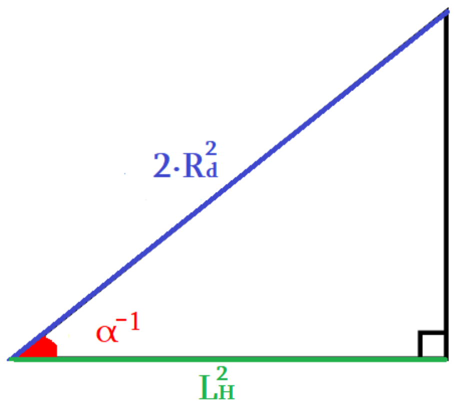

Figure 2. *Geometric representation of the relationship between the de Sitter radius and the Hubble length*

From the dimensionless unification of the fundamental interactions for the density parameter of dark energy apply:

$$\Omega_\Lambda = 2 \cdot 10^{-7} \alpha_s \cdot \alpha_w^{-1} \tag{36}$$

$$\Omega_\Lambda = 2 \cdot i^{2i} \cdot \alpha_s^{-1} \tag{37}$$

$$\Omega_\Lambda = 2 \cdot e \cdot 10^{-7} \cdot i^{2i} \cdot \alpha_w^{-1} \tag{38}$$

$$\Omega_\Lambda = 2 \cdot 10^{-7} \cdot \alpha_s \cdot \alpha_w^{-1} \tag{39}$$

$$\Omega_\Lambda = 4 \cdot \alpha \cdot \alpha_G^{1/2} \cdot N_A \tag{40}$$

$$\Omega_\Lambda = i^{8i} \cdot \alpha^{-2} \cdot \alpha_s^{-4} \cdot \alpha_G^{-1} \cdot N_A^{-2} \tag{41}$$

$$\Omega_\Lambda = 10^7 \cdot i^{4i} \cdot \alpha^{-1} \cdot \alpha_w^{-1} \cdot \alpha_G^{-1/2} \cdot N_A^{-1} \tag{42}$$

$$\Omega_\Lambda = 8 \cdot 10^7 \cdot N_A^2 \cdot \alpha_w \cdot \alpha^2 \cdot \alpha_G \cdot \alpha_s^{-1} \cdot \alpha_w \tag{43}$$

6. Solution for the density parameter of dark matter

Dark matter is a hypothetical form of matter thought to account for approximately 85% of the matter in the universe. Dark matter is called "dark" because it does not appear to interact with the electromagnetic field, which means it does not absorb, reflect, or emit electromagnetic radiation and is, therefore, difficult to detect. Various astrophysical observations – including gravitational effects which cannot be explained by currently accepted theories of gravity unless more matter is present than can be seen – imply dark matter's presence. For this reason, most experts think that dark matter is abundant in the universe and has had a strong influence on its structure and evolution. The primary evidence for dark matter comes from calculations showing that many galaxies would behave quite differently if they did not contain a large amount of unseen matter. Some galaxies would not have formed at all and others would not move as they currently do. Other lines of evidence include observations in gravitational lensing and the cosmic microwave background, along with astronomical observations of the observable universe's current structure, the formation and evolution of galaxies, mass location during galactic collisions, and the motion of galaxies within galaxy clusters. The figure 3 shows the geometric representation of the relationship between the density parameter of dark and baryonic matter.

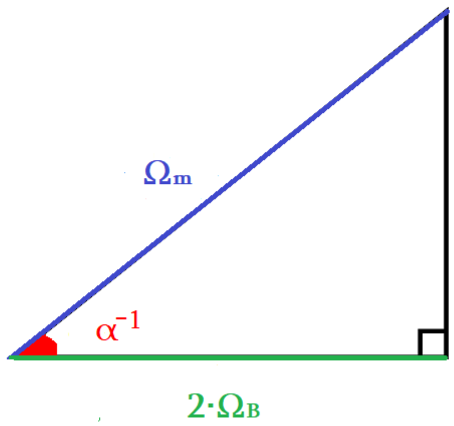

Figure 3. *Geometric representation of the relationship between the density parameter of dark and baryonic matter.*

Current observations suggest that we live in a dark energy dominated Universe with density parameters for dark matter $\Omega_D=0{,}23$. From the dimensionless unification of the fundamental interactions the density parameter for dark matter is:

$$\Omega_D=2\cdot e^{1-n} \tag{44}$$

$$\Omega_D=2\cdot e\cdot i^{2i} \tag{45}$$

$$\Omega_D=0{,}2349 \tag{46}$$

$$\Omega_D=23{,}49\% \tag{47}$$

From the dimensionless unification of the fundamental interactions for the density parameter for normal baryonic matter apply:

$$\Omega_D=2\cdot a_s \tag{48}$$

$$\Omega_D=2\cdot 10^7\cdot e^{-1}\cdot a_w \tag{49}$$

$$\Omega_D=2\cdot (i^{2i}\cdot 10^7\cdot a_w)^{1/2} \tag{50}$$

$$\Omega_D=4\cdot i^{2i}\cdot (e^{i/a}+e^{-i/a})^{-1} \tag{51}$$

$$\Omega_D=10^7\cdot a_w\cdot (e^{i/a}+e^{-i/a}) \tag{52}$$

$$\Omega_D=4\cdot 10^7\cdot a_w\cdot a\cdot a_G^{1/2}\cdot N_A \tag{53}$$

$$\Omega_D=16\cdot 10^7\cdot N_A^2\cdot a_w\cdot a^2\cdot a_G\cdot (e^{i/a}+e^{-i/a})^{-1} \tag{54}$$

The relationship between the density parameter of dark matter and baryonic matter is:

$$\Omega_m=2\cdot e\cdot \Omega_B \tag{55}$$

The relationship between the density parameter of dark energy,dark matter and baryonic matter is:

$$\Omega_m\cdot \Omega_\Lambda=4\cdot \Omega_B \tag{56}$$

7. Poincaré dodecahedral space

The shape of the universe,in physical cosmology, is the local and global geometry of the universe. The local features of the geometry of the universe are primarily described by its curvature,whereas the topology of the universe describes general global properties of its shape as a continuous object. The spatial curvature is defined by general relativity, which describes how spacetime is curved due to the effect of gravity. The spatial topology cannot be determined from its curvature, due to the fact that there exist locally indistinguishable spaces that may be endowed with different topological invariants. Cosmologists distinguish between the observable universe and the entire universe, the former being a ball-shaped portion of the latter that can, in principle, be accessible by astronomical observations. Assuming the cosmological principle, the observable universe is similar from all contemporary vantage points, which allows cosmologists to discuss properties of the entire universe with only information from studying their observable universe. The main discussion in this context is whether the universe is finite, like the observable universe, or infinite. Several potential topological and geometric properties of the universe need to be identified. Its topological characterization remains an open problem. Some of these properties are Boundedness (whether the universe is finite or infinite),Flatness (zero curvature), hyperbolic (negative curvature), or spherical (positive curvature) and Connectivity: how the universe is put together as a manifold, i.e., a simply connected space or a multiply connected space.

There are certain logical connections among these properties. For example, a universe with positive curvature is necessarily finite. Although it is usually assumed in the literature that a flat or negatively curved universe is infinite, this need not be the case if the topology is not the trivial one. For example, a multiply connected space may be flat and finite, as illustrated by the three-torus. Yet, in the case of simply connected spaces, flatness implies infinitude.

From the dimensionless unification of the fundamental interactions the sum of the contributions to the total density parameter Ω_0 at the current time is:

$$\Omega_0=\Omega_B+\Omega_D+\Omega_\Lambda$$

$$\Omega_0=e^{-\pi}+2\cdot e^{1-\pi}+2\cdot e^{-1} \tag{57}$$

$$\Omega_0=1{,}0139 \tag{58}$$

A positively curved universe is described by elliptic geometry, and can be thought of as a three-dimensional hypersphere,or some other spherical 3-manifold (such as the Poincaré dodecahedral space),all of which are quotients of the 3-sphere. Poincaré dodecahedral space is a positively curved space,colloquially described as "soccer ball-shaped",as it is the quotient of the 3-sphere by the binary icosahedral group,which is very close to icosahedral symmetry,the symmetry of a soccer ball. This was proposed by Jean-Pierre Luminet and colleagues in 2003 and an optimal orientation on the sky for the model was estimated in 2008. When the universe expands sufficiently,the cosmological constant L becomes more important than the energy density of matter in determining the fate of the universe. If $\Lambda>0$ there will be an approximately exponential expansion. This seems to be happening now in our universe.

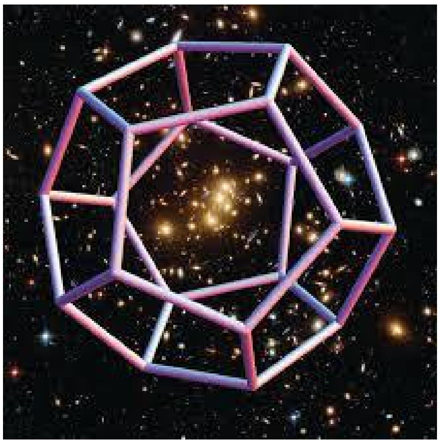

Figure 4. *The shape of the universe is Poincaré dodecahedral space.*

In [21] J.-P. Luminet, J. Weeks, A. Riazuelo, R. Lehoucq and J.-P. Uzan presents a simple geometrical model of a finite, positively curved space, the Poincaré dodecahedral space – which accounts for WMAP's observations with no fine-tuning required. Circle searching (Cornish,Spergel and Starkman,1998) may confirm the model's topological predictions,while upcoming Planck Surveyor data may confirm its predicted density of:

$$\Omega_0\simeq1{,}013>1$$

If confirmed,the model will answer the ancient question of whether space is finite or infinite,while retaining the standard Friedmann-Lemaˆıtre foundation for local physics. The Poincaré dodecahedral space is a dodecahedral block of space with opposite faces abstractly glued together, so objects passing out of the dodecahedron across any face return from the opposite face. Light travels across the faces in the same way, so if we sit inside the dodecahedron and look outward across a face, our line of sight re-enters the dodecahedron from the opposite face. We have the illusion of looking into an adjacent copy of the dodecahedron. If we take the original dodecahedral block of space not as a Euclidean dodecahedron (with edge angles ≈ 117°) but as a spherical dodecahedron (with edge angles exactly 120°),then adjacent images of the dodecahedron fit together snugly to tile the hypersphere, analogously to the way adjacent images of spherical pentagons (with perfect 120° angles) fit snugly to tile an ordinary sphere. Thus the Poincar´e space is a positively curved space, with a multiply connected topology whose volume is 120 times smaller than that of the simply connected hypersphere (for a given curvature radius).

The Poincar´e dodecahedral space's power spectrum depends strongly on the assumed mass-energy density parameter Ω_0. The octopole term (ℓ=3) matches WMAP's octupole best when $1,010<\Omega_0<1,014$. Encouragingly, in the subinterval $1,012<\Omega_0<1,014$ the quadrupole (ℓ=2) also matches the WMAP value. More encouragingly still, this subinterval agrees well with observations, falling comfortably within WMAP's best fit range of $\Omega_0=1,02\pm0,02$. The excellent agreement with WMAP's results is all the more striking because the Poincaré dodecahedral space offers no free parameters in its construction. The Poincar´e space is rigid, meaning that geometrical considerations require a completely regular dodecahedron. By contrast, a 3-torus, which is nominally made by gluing opposite faces of a cube but may be freely deformed to any parallelepiped, has six degrees of freedom in its geometrical construction. Furthermore, the Poincar´e space is globally homogeneous, meaning that its geometry-and therefore its power spectrum - looks statistically the same to all observers within it. By contrast a typical finite space looks different to observers sitting at different locations. Confirmation of a positively curved universe ($\Omega_0>1$) would require revisions to current theories of inflation, but the jury is still out on how severe those changes would be. Some researchers argue that positive curvature would not disrupt the overall mechanism and effects of inflation, but only limit the factor by which space expands during the inflationary epoch to about a factor of ten. Others claim that such models require fine-tuning and are less natural than the infinite flat space model. Having accounted for the weak observed quadrupole, the Poincaré dodecahedral space will face two more experimental tests in the next few years:

The Cornish-Spergel-Starkman circles–in–the–sky method predicts temperature correlations along matching circles in small multi connected spaces such as this one. When $\Omega_0\approx1,013$ the horizon radius is about 0,38 in units of the curvature radius, while the dodecahedron's inradius and outradius are 0,31 and 0,39, respectively, in the same units; as a result, the volume of the physical space is only 83% the volume of the horizon sphere. In this case the horizon sphere self intersects in six pairs of circles of angular radius about 35°, making the dodecahedral space a good candidate for circle detection if technical problems (galactic foreground removal, integrated Sachs-Wolfe effect, Doppler effect of plasma motion) can be overcome. Indeed the Poincaré dodecahedral space makes circle searching easier than in the general case, because the six pairs of matching circles must a priori lie in a symmetrical pattern like the faces of a dodecahedron, thus allowing the searcher to slightly relax the noise tolerances without increasing the danger of a false positive. The Poincaré dodecahedral space predicts $\Omega_0\approx1,013>1$. The upcoming Planck surveyor data (or possibly even the existing WMAP data in conjunction with other data sets) should determine Ω_0 to within 1%. Finding $\Omega_0<1,01$ would refute the Poincaré space as a cosmological model, while $\Omega_0>1,01$ would provide strong evidence in its favor.

8. The ratio of the dark energy density to the Planck energy density

R. Adler in [22] calculated the energy ratio in cosmology,the ratio of the dark energy density to the Planck energy density. Atomic physics has two characteristic energies,the rest energy of the electron E_e,and the binding energy of the hydrogen atom E_H. The rest energy of the electron E_e is defined as:

$$\mathrm{E}_e = m_e c^2$$

The binding energy of the hydrogen atom E_H is defined as:

$$\mathrm{E}_{\mathrm{H}} = \frac{m_e e^4}{2\hbar^2}$$

Their ratio is equal to half the square of the fine-structure constant:

$$\frac{\mathrm{E}_{\mathrm{H}}}{\mathrm{E}_e} = \frac{\alpha^2}{2}$$

Cosmology also has two characteristic energy scales,the Planck energy density ρ_{pl},and the density of the dark energy ρ_Λ. The Planck energy density is defined as:

$$\rho_{pl} = \frac{E_{pl}}{l_{pl}} = \frac{c^7}{\hbar G^2}$$

To obtain an expression for the dark energy density in terms of the cosmological constant we recall that the cosmological term in the general relativity field equations may be interpreted as a fluid energy momentum tensor of the dark energy according to so the dark energy density ρ_Λ is given by:

$$\rho_\Lambda = \frac{\Lambda c^4}{8\pi G}$$

The ratio of the energy densities is thus the extremely small quantity:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{\alpha_g^2}{8\pi}$$

The expression that connects the gravitational fine-structure constant α_g with the golden ratio φ and the Euler's number e is:

$$\alpha_g = \frac{4e}{3\sqrt{3\varphi^5}} \times 10^{-60} = 1,886837 \times 10^{-61} \tag{59}$$

From this expression for the ratio of the dark energy density to the Planck energy density apply:

$$\frac{\rho_\Lambda}{\rho_{pl}} = \frac{2e^2\varphi^{-5}}{3^3\pi\varphi^5} \times 10^{-120} \tag{60}$$

9. Conclusions

According to a recent theory the Universe could be a dodecahedron. We proved the shape of the Universe is Poincaré dodecahedral space. From the dimensionless unification of the fundamental interactions will propose a possible solution for the density parameter of baryonic matter,dark matter and dark energy:

$$\Omega_B = e^{-\pi} = i^{2i} = 0,0432 = 4,32\%$$

$$\Omega_\Lambda = 2 \cdot e^{-1} = 0,73576 = 73,57\%$$

$$\Omega_D = 2 \cdot e^{1-\pi} = 2 \cdot e \cdot i^{2i} = 0,2349 = 23,49\%$$

The sum of the contributions to the total density parameter at the current time is $\Omega_0 = 1,0139$. It is surprising that Plato used a dodecahedron as the quintessence to describe the cosmos. These results prove that the weather space is finite.

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