ON EXTENSION OF EINSTEIN FIELD EQUATIONS

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Abstract. In this paper I will explore how extension of Einstein field equation can lead to new equation that can be important way of looking at gravity. I do not present solutions to field equations only framework and reasoning behind it. Equation states that matter and gravity field are same thing seen from two point of view. Matter just like gravity field is extended in space-time.

Contents

1. Field equation

In this paper I will present an extension of Einstein field equations [1] [2] [3]. This extension can be reduced to Einstein field equation by contraction and it's form they are equal to:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2}R_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} = \kappa T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} \tag{1.1}$$

First thing that can be deduced from those equation is that they do not have vacuum solutions, if I set energy tensor to zero so does Ricci tensor go to zero so i just flat space-time:

$$R^{\rho}_{\sigma\mu\nu} = 0 \tag{1.2}$$

It seems like an big problem for this model to work but I will explore what if that model really does explain gravity, so there is no true vacuum empty of energy. First off there is need for all space to be filled with matter and this seems wrong with common sense, objects are well located in space, but I will assume that this view is not correct. That leads to key principle of this whole idea:

There is no vacuum empty of energy. Matter field is extended in whole space-time, that extension is responsible for curvature of space-time.

Even more If i set curvature tensor to zero by using all components to have equal index I will get this equation:

$$R^{\sigma}_{\sigma\sigma\sigma} - \frac{1}{2}R_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma} = \kappa T_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma} \tag{1.3}$$

That will reduce to:

$$-\frac{1}{2}R_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma} = \kappa T_{\sigma\kappa}g^{\kappa\sigma}g_{\sigma\sigma} \tag{1.4}$$

$$R_{\sigma\kappa}\delta^{\kappa}_{\sigma} = -2\kappa T_{\sigma\kappa}\delta^{\kappa}_{\sigma} \tag{1.5}$$

$$R_{\sigma\sigma} = -2\kappa T_{\sigma\sigma} \tag{1.6}$$

That means that positive energy momentum tensor diagonal only components will give negative Ricci curvature diagonal elements that means that all geodesic will expand with matter field, so both matter field and space-time curvature expands. For example from point of view of this model if I stand on surface on earth , earth does expand so does space around it, that's why it seems stationary. It would be opposite view than in General Relativity where space will contract not expand to counteract earth expansion.

2. Mathematical details

Now before I will move further into this idea let's put some parts that are key to make this idea work. First of this tensor I did use on left side of equation needs to have same property as Einstein tensor so it has to vanish with respect to covariant derivative, I will start by Einstein tensor and adding only metric tensors and expanding element of Einstein I will get this result [4] [5]:

$$\nabla_\delta \left(R^{\gamma\delta} - \frac{1}{2} R g^{\gamma\delta} \right) = 0 \tag{2.1}$$

$$\nabla_\delta \left(g_{\sigma\gamma} g_{\nu\delta} R^{\gamma\delta} - \frac{1}{2} R g_{\sigma\gamma} g_{\nu\delta} g^{\gamma\delta} \right) = 0 \tag{2.2}$$

$$\nabla_\delta \left(R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} \right) = 0 \tag{2.3}$$

$$\nabla_\delta \left(g_{\mu\alpha} g^{\alpha\rho} g^{\mu\alpha} g_{\alpha\rho} R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} g_{\mu\alpha} g^{\alpha\rho} R_{\mu\kappa} g^{\kappa\mu} g_{\sigma\nu} \right) = 0 \tag{2.4}$$

$$\nabla_\delta \left(\delta^{\rho}_{\mu} \delta^{\mu}_{\rho} R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} \delta^{\rho}_{\mu} R_{\mu\kappa} g^{\kappa\mu} g_{\sigma\nu} \right) = 0 \tag{2.5}$$

$$\nabla_\delta \left(R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} \right) = 0 \tag{2.6}$$

It means that like Einstein tensor it does vanish so gravity field is conserved. Next important part is that this equation has only equal amount of unknowns only in four dimensions it means that it will only work correctly in four dimensional space-time. And at last I can prove that this equation reduces to Einstein field equations I do it by contraction of all indexes from μ to ρ:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2} R_{\mu\kappa} g^{\kappa\rho} g_{\sigma\nu} = \kappa T_{\sigma\kappa} g^{\kappa\rho} g_{\mu\nu} \tag{2.7}$$

$$R^{\rho}_{\sigma\rho\nu} - \frac{1}{2} R_{\rho\kappa} g^{\kappa\rho} g_{\sigma\nu} = \kappa T_{\sigma\kappa} g^{\kappa\rho} g_{\rho\nu} \tag{2.8}$$

$$R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} = \kappa T_{\sigma\kappa} \delta^{\kappa}_{\nu} \tag{2.9}$$

$$R_{\sigma\nu} - \frac{1}{2} R g_{\sigma\nu} = \kappa T_{\sigma\nu} \tag{2.10}$$

Same procedure can be using two metric tensor that will lead to delta that will change index from μ to ρ and even without writing this it comes from equation before as this is just reversing that equation. But there it's done opposite way I change ρ to index μ both are equivalent.

3. Gravity field

I did state in section Field Equation that gravity field is equal to curvature of space-time. It means that both are same phenomenon just seen from two points of view. It can be easy proven by transformation of field equation. First i start by change in Ricci tensor to energy tensor [6] so I will get:

$$R^{\rho}_{\sigma\mu\nu} - \frac{1}{2}\kappa\left(T_{\mu\kappa} - \frac{1}{2}Tg_{\mu\kappa}\right)g^{\kappa\rho}g_{\sigma\nu} = \kappa T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} \tag{3.1}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}\left(T_{\mu\kappa} - \frac{1}{2}Tg_{\mu\kappa}\right)g^{\kappa\rho}g_{\sigma\nu}\right] \tag{3.2}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} - \frac{1}{4}Tg_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu}\right] \tag{3.3}$$

$$R^{\rho}_{\sigma\mu\nu} = \kappa\left[T_{\sigma\kappa}g^{\kappa\rho}g_{\mu\nu} + \frac{1}{2}T_{\mu\kappa}g^{\kappa\rho}g_{\sigma\nu} - \frac{1}{4}T\delta^{\rho}_{\mu}g_{\sigma\nu}\right] \tag{3.4}$$

From it follows that curvature always has energy, so everywhere where there is curvature present there is energy present. Both can't be separated so in truth they are same phenomenon seen from two point of view, we do observe matter and gravity field but there is not say to be same thing, here they have to be same thing in order for this equation to work. This gives a clear definition of gravity field. Gravity field is curvature of space-time, from it follows that gravity field has energy at each point of space-time that is equal to matter itself. So there is matter field in every point of space-time and that matter field gives rise to curvature thus gravity. It means that if there is gravity present energy-momentum tensor is non zero at each point of space-time as follows from equation 3.4. That being said in first section of this paper, matter extends in whole space-time. So gravity field is same as extended matter field and gravity field has energy in form of matter. By extension of Einstein field equation result is more simple statement about gravity field, in Einstein field equation there is no energy tensor for gravity here matter extended in all space is a energy tensor of gravity. But in truth that energy tensor is just a extension of matter field, so there is equality between matter and spacetime curvature. Gravity field is represented by matter and matter is represented by spacetime curvature, so there is no vacuum solutions.

4. Measurement and summary

Measurement is key property of any physics theory. Quantum theory has proven that outcome of measurement has to be statistical in nature, it lead to so called measurement problem in quantum physics. Key question is what physical process does cause collapse of wave function of system?

I will assume that gravity itself is responsible for collapse of wave function. All systems will tend to become concentrated by gravity itself, object focus at some region of space is the cause of wave function collapse. Same with all properties that look wave like - they are caused by un-focusing of object field.

This view if is true would be explanation for quantum mechanics problem of measurement and it does come from field itself or rather it's ability to be focus and un-focus in some region of space. Going back to wave function collapse, flow of let's say electron field (where I assume all fields not only gravity field) always focus at some volume of space, but it can be un-focus that will look like spreading of wave function of that electron. When measurement is done electron does focus in smaller volume of space and it leads to being localized in space. This is view I do present on measurement process that does not need quantum physics to explain it's effects. Direct calculations that means solving field equation that I do not present here are needed.

This view on measurement could be wrong and in truth quantum physical view of statistical nature of measurement could be true as it seems now, but this idea is in opposition to this view and it could be hard to prove it right but it's still idea worth considering in future.

References

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