MODEL OF SPACE-TIME’ STRUCTURE

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Summary

Present paper considers a structure model of discrete space-time, the properties of which are determined by the substrate. As a substrate a 2-dimensional plane under no matter is chosen, the surfaces of which are space and anti-space.

A new concept called spacetron, considered as a spatial element of the substrate, which is the basis for the subsequent formation of 4-dimensional space-time, is proposed. Upper-level spacetrons are a hexagonal 2-dimensional packing, with spacetrons of other levels (first, second, etc.) placed between them, followed by their identification with electron-positron pairs, neutrinos and electromagnetic field quanta. It is shown how the connectivity of space-time is determined by completely filling the substrate space with spacestrons of different sizes, which allows us to consider the resulting 4-dimensional space-time as a quasi-continuous medium. It is shown that in the case of contact interaction of space-time substrate spacestrons, all of them can be given a spherical shape by partitioning the upper level spacestrons into linked objects, called a loveton, which is a three-dimensional object, and an anti-loveton, which is an anti-space object.

Within the framework of the computer model the mechanism of the appearance of matter in space-time is considered. The appearance of matter takes place under the influence of energy perturbations of space-time structural elements, causing the appearance of both free lovetons (anti-lovetons) and other material objects.

Based on the analysis of the geometric position and dimensions of the substrate features responsible for the formation of elementary particles and the electromagnetic field quantum spectrum, the estimated masses of a number of elementary particles are determined and the known particle masses are assigned to them. The masses of free lovetons and neutrinos are evaluated on the basis of geometric relations between the sizes of particles on the basis of electron masses. At least three varieties of neutrinos, which can be regarded as types of high-energy γ-quanta, are identified.

The results of the work are valid and reliable because they are based on known approaches of relativity theory and quantum physics, as well as on the application of approximations adequate to the phenomena under study.

Key words: space, substrate, spacestron, loveton, anti-space, mass, electron, neutrino, electromagnetic field quantum

INTRODUCTION

The theory of relativity and quantum mechanics are considered to be the basis of the modern physical picture of the world, and are directly related to changes in the fundamental concepts of space, time and matter [1]. Within the framework of Einstein's general theory of relativity [2], it is believed that the structure of space depends on the distribution of masses of material objects, and the properties of space are determined by its geometry. As a rule, the space-time in question is considered as a continuum. At the same time, in a number of works [3], space and time have come to be perceived as attributes of matter, determined by its relations and interactions. However, nowadays, both in physics and in philosophy, it has been established that space is also an entity independent of material bodies, as it can exist empty, in the absence of physical bodies [4].

In the general theory of relativity, when considering 4-dimensional space-time, it is not accepted to take into account possible quantum effects. All known attempts to construct a model of both only continuous or only discrete space and time have encountered contradictions, the solution of which has not been found so far. It testifies rather to a certain tendency to the unity of continuous and discrete space-time and consequently speaks not about wrongness of space and time continuity concepts, but about necessity of synthesis of the last ones with discrete space-time concepts [5]. Clarification of space-time essence at the present time is one of the most important tasks of fundamental physics.

In philosophy, it is customary to define space as a form of matter being, which expresses the extent, structure, order of coexistence and ranking of material objects [6]. It should be noted that a fundamentally important problem in the study of space-time is the relation of space and time to matter. There are two concepts on this question: substantive and relational [7]. The substantive concept assumes that space is an independent entity, as if a special kind of substance, existing along with time, substance and physical fields. According to the relational concept, space and time are not independent entities, but are formed by the interaction of material objects [8]. The philosophical approach in the development of various ideas about space-time and various aspects of this line of research can be found in Vyaltsev's monograph [9].

To describe space-time in terms of quantum physics a number of models have been proposed too, in particular - string theory and loop gravitation [10, 11]. At the same time string theory has shown that space-time is not a mathematical abstraction, necessary to describe the state of material objects, but an entity which participates in interactions with objects located in space, while these objects, in turn, change the parameters of space-time [12, 13]. However, recognizing that elementary particles may eventually be the manifestation of the dynamics of some more fundamental objects, in most cases, continues to consider space-time as separate from the rest of the matter, as something external, in which it is customary to place matter. An important advantage of string theory is that there are no free parameters that need to be adjusted to ensure agreement, as, for example, is required in the Standard Model [13].

The most developed and fruitful application of discrete space-time has turned out to be the construction of the theory of loop quantum gravitation, where the assumption of one-dimensional physical excitations of space-time on Planck scales is put forward [14]. According to this theory, space-time, in contrast to its representation in the theory of relativity, is discrete. Space-time is considered to consist of quantum cells, which are connected to each other, and by bundles of knots and entanglements form elementary particles. Thus ribbon structures, for example in the Bilson-Thompson model [15], are represented as entities consisting of the same matter as space-time itself, indicating belonging to the unity of nature of space-time and matter. From the point of view of discrete space-time, the latter can also be represented in the form of a crystal lattice, and both the speed of light and Planck's constant have been theorized using this approach [16].

Of particular interest is David Bohm's idea[17] of imagining the universe as a holographic structure in which the hidden order of the new deep reality is analogous to the order of a hologram which needs to be deciphered and deployed in space as a three-dimensional image. According to Mark Van Raamsdonk [18] the infinite Anti-de-Sitter space-time (AdS- space-time) has an infinity boundary [18]. To represent it, it is customary to use a distorted length scale allowing compressing infinite distance into finite one. The boundary is similar to the surface of a continuous cylinder. In the latter case, the boundary has two dimensions: spatial (the cylinder guide) and time (the cylinder form). The two-dimensional boundary contains an implicative order, because the image encoded in interference patterns is a latent completeness rolled up in space. The hologram, projected by the two-dimensional plane, already has an explicative order, because it is an unfolded and visible version of the image.

David Bohm's view that space is real and there are many processes going on in it, as in the matter moving in it, was developed in his ideas of an implicative ocean of energy. Matter does not exist independently of this ocean, from so-called empty space, it is itself part of space. Matter is inseparable from this ocean of energy and acts as a pattern of excitation. As Bohm [17] states: "This pattern of excitation is relatively autonomous and produces relatively stable and repetitive visible projections in our three-dimensional explicative order".

The proposed approach was continued in the work of Gerard 't Hooft [19]. His main idea is called the holographic principle. According to this principle, whatever happens in the domain that we use as space, can be represented as taking place on the surface surrounding that space. What is more, a description of the world that exists on this boundary is not a quantum theory, but a deterministic theory, which he is certain, will replace the quantum concept. A similar idea has also been proposed by Louis Crane [20] in the context of background-independent approaches to quantum gravity. Crane suggested that quantum mechanics is not a static description of the system, but a record of information that one subsystem of the universe can have near the other through their interaction. He further suggested that there is a quantum mechanical description associated with each way the universe is divided into two parts. Quantum states do not live in one part or the other of space-time, but at the boundary between them.

An attempt to unify space-time and matter in the framework of five-dimensional space was the mathematical model of Theodor Kaluza [21], which contained the hypothesis of five-dimensionality of physical space. Kaluza supposed that his theory points to the possibility of interpreting gravitation and electricity as manifestations of some universal field. A supplement to Kaluza's hypothesis, was the formulation of the conditions for the secrecy of the fifth dimension, expressed by Oskar Klein [22]. He suggested that the structure of the physical space can contain both "extended" and "contracted" dimensions. In this case additional (to three visible) dimensions are convolved to small dimensions.

When analysing these theories of space-time structure, it is easy to find associations that allow space to be seen as some entity formed on the basis of another fundamental object. In other words, the emergence of space-time points to its secondary nature. In this case, the structure of space-time is defined by constructing it on a substrate [23]. The substrate must exist on the basis of something else. Consequently, the substrate is a form of state of matter, which is considered not manifested and, therefore, being in a passive state. Thus in [24] space-time is considered as a phenomenon appearing in a special sub-quantum medium preceding it. At the same time matter cannot fill up space in any way, as it is a form of existence of space itself [25]. In John Wheeler's quantum geometrodynamics [26] space-time also arises. But with Wheeler, space is fundamental and space-time arises from it. However, in his opinion, a 4-metric arises from a 3-metric defined on a 3-dimensional smooth manifold which acts as a substrate in Wheeler-DeWitt's quantum gravity theory [27].

Concluding the analysis of space-time and matter interrelations it is possible to quote Einstein [28]: "We come to a strange conclusion: it now begins to seem to us that space plays a primary role, while matter must be derived from space, so to speak, at the next stage. Space absorbs matter. We have always regarded matter as primary and space as secondary. Space, figuratively speaking, is now taking revenge and 'eating' matter. We can add to this statement that it is not even fully understood what matter consists of and how it arises. For this reason, the aim of the presented paper is to develop a model of discrete 4-dimensional space-time and matter that could accurately reflect the nature of the real world.

1. RESEARCH OBJECTIVE

The main components of the proposed model are space, time and matter. As an object of study, in this case, we can consider real physical space-time, including its substrate.

At statement of the problem we will proceed from a number of known facts, as well as certain conditions and restrictions:

1. Space-time is discrete and consists of elements which may differ in size and between which there is a certain relation organized on the basis of redistribution of quanta of energy.

2. At research of space-time structure issues related to permeability of matter, as well as charge dependence of elementary particles are excluded from consideration.

3. The construction of the model of space-time structure is carried out without taking into account dynamic effects.

4. On the basis of the chosen substrate, as which space can be considered if there is no matter, it is possible to place its elements completely filling "voids" in order to form a quasi-continuous space.

The main task facing us is to choose the right structure of a 4-dimensional space substrate and then analyze the placement of spatial elements on it. Detailed formulation of the formulated problem allows us to give a logically consistent description of the model of space-time and matter as a whole. Moreover, it admits a variant in which matter, consisting of matter and fields forming our world, is not an independent physical reality, but a specific structure included in the space-time substance itself.

2. MODELING

In our modeling, we will assume that a possible way of combining space-time and matter into a single object is to consider a representation of space in which its geometric properties are defined according to the choice of substrate.

2.1 PRIMARY SPACE (SUBSTRATE)

First, let us pay attention to the fact that the space-time model of general relativity theory allows a limiting transition to the case in which there is no matter, i.e. to the presence of empty space. Such solutions of the Einstein equations [29] are obtained when the energy-momentum tensor in the considered region of space is identically equal to zero. In this case, the curved space-time of general relativity transforms to the flat space of the special theory of relativity - the Minkowski space.

Let's assume that the empty space must have no more than three dimensions. In other words, we should choose either 2-dimensional or 3-dimensional space as a substrate. Immediately note that it is undesirable to use one-dimensional space due to the lack of possibility of obtaining unambiguous relations between its elements. On the basis of this solution to the Einstein equations, let us choose a 2-dimensional plane as a substrate [30]. We assume that the surfaces of this plane can contain two layers, one referring to space and the other to anti-space. This choice of the substrate as a given plane is schematically illustrated in figure 1, which shows a flat space that is infinitely extended (only part of it is shown).

Figure 1 Layout of a part (region) of primary space (on a cross section of this plane two layers can be seen, one referring to space and the other to anti-space)

2.2 "FILLING IN" THE SUBSTRATE

Let us assume that the basis of the space chosen as the substrate is discrete elements into which the entire 2-dimensional plane provided by them is divided. As such discrete elements we choose disks of different sizes, each of which, hereinafter referred to as a spacetron (Fig. 2).

Figure 2 Graphical representation of the spacestron
(shown with tilt for ease of presentation of the spatial layers)

The spacestron introduced as a substrate element will be defined as a massless object with a boundary between the spatial and anti-spatial layers inside, with interaction with other spacestrons by contacting them with each other.

We divide the spacestrons, depending on their position and size, into separate levels. We assign the largest spacetrons to the top level, the first level to those with a slightly smaller size, the second level to those with a smaller size, and so on. The upper level is the main one and represents a set of spacetrons forming a dense hexagonal packing of elements in the form of disks. The first level consists of spacestrons located in between the upper level spacestrons in such a way that they must necessarily touch at least three upper level spacestrons. Subsequent smaller spacestrons, in a similar manner, sequentially fill all remaining unoccupied "voids" between the already placed higher level spacestrons (Fig. 3).

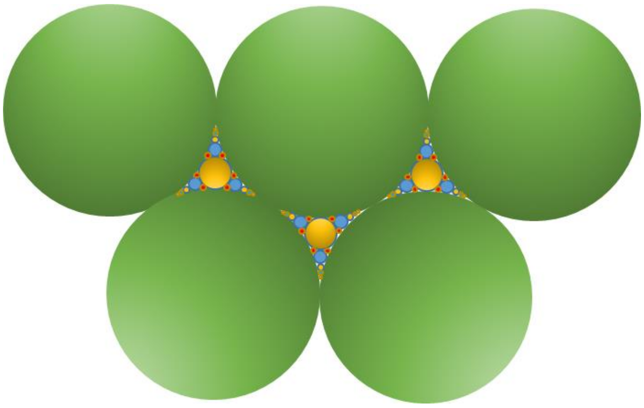

Figure 3 Schematic spacestrons arrangement forming a quasi-continuous medium (shown without anti-space).

Consistently dividing the substrate into spacestrons of lower levels, we can lead the situation to an almost complete filling of the entire plane forming the substrate of the space-time being formed.

Thus, we introduce a principally new model of the structure of the space-time substrate which, on the one hand, is discrete in its structure and, on the other hand, due to a complete filling of the provided region on the substrate, it can be considered as a quasi-continuous medium.

2.3 THE MANIFESTATION OF SPACE-TIME

Between all the objects making up the various levels of spacestrons, we can make identification according to real physical entities. Thus, we will a priori define the first-level spacestrons as virtual electron-positron pairs, while the spacestrons from the second to the fourth level will be regarded as neutrino-antineutrino pairs. The rest of the set of spacestrons allows us to attribute them to the elements generating quanta of the electromagnetic field.

Each of the upper-level spacestrons, in turn, can be subdivided into two independent objects, one of which we will call a loveton, represented as an element of space, and the other as an anti-loveton, already an element of anti-space. It is the appearance of lovetons as elements of space-time, which are already arranged in explicative order, that transforms the 2-dimensional plane into 4-dimensional space-time, and the lovetons themselves, together with anti-lovetons, form a dense cubic lattice [31], making space-time discrete with a unit cell size comparable to the size of lovetons (anti-lovetons) themselves (Fig. 4). In the bound state, the lovetons together with anti-lovetons form structural units of space-time consisting of contacting loveton-anti-loveton pairs. It is such loveton-anti-loveton pairs that are the basis for manifestation of the holographic space-time component.

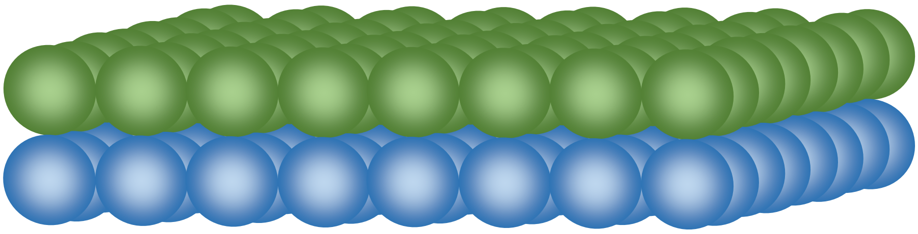

Figure 4 Layout of a 4-dimensional space-time domain
(in 3-dimensional representation without the holographic component)

It is more correct to consider that in our representation of space-time structure, being in a state with no matter and, accordingly, must be stationary, since in this case the space is absolutely empty. However, it is possible [32] that in such space-time state too virtual particles already appear, which, in turn, can turn into real particles and antiparticles if there is some sufficient energy interaction.

3. RESULTS AND DISCUSSION

To confirm the results of the performed simulation of the space-time structure, we first define the ratio of the space-time radii for both particles and quanta of the electromagnetic field. For this purpose we will connect the mass of an *n*-particle m_n with its dimension r_n in 4-dimensional space [31]

$$m_n = \rho\frac{\pi^2}{2}{r_n}^4 \tag{1}$$

where ρ is the particle density.

Here the lack of particle density data does not allow us to use formula (1) directly to calculate particle masses, but at the same time we can use the results of calculations of the ratio of the masses of the *i*-th and *j*-th particles

$$m_i = m_j(r_i/r_j)^4 \tag{2}$$

On the basis of the above mentioned approach let us try to obtain estimates of the masses of the particles in question. For this purpose, as has been suggested earlier, we shall assume that the first level spacetron is an electron-positron pair. As the basis we take the mass of the electron (positron), which has a value of about 0.511 meV. Now, using Descartes' formula [33] for the particle curvature one can determine the ratio of the electron radius re and the loveton radius r_L: r_e/r_L = 0.1547 (here we take the loveton radius to be unity). In this case the loveton has a mass according to formula (2) equal to m_L = 892.18 meV. The obtained value of the loveton mass is quite close to the experimental value of the proton mass, which is a value of about 938 meV [34]. It is instructive to recall that the proton is considered a compound particle; hence, the mass of the loveton must be somewhat less than the proton mass, which is the case in our case.

Similarly, for the largest neutrino (let's denote it as neutrino-2, according to its level) we obtain a value of about 7.9 eV. The value obtained is of nearly the same order of magnitude as is known for the upper bound of the neutrino mass estimate, which is close to 1.1 eV according to present data [34]. Let’s pay attention to the existence of two more spacetron levels with radii close to neutrino-2 for which one can similarly calculate their masses, obtaining values of 0.68 eV and 0.11 eV, respectively. These neutrinos can be designated neutrino-3 and neutrino-4.

In the analysis of lower level spacestrons, which we will also consider as space elements, there may appear particles which are transitional between neutrinos and γ- particles. Such particles have a Compton wavelength intermediate between neutrino-4 and γ- particles. Let's pay attention to the number of possible particles of a given level. As the number of particles of a given radius increases according to the formula: $N = 3^n$, where *n* is the level number, the number of these particles will already exceed the value of a few hundred with respect to the lovetons and electrons, and their masses are estimated to be two orders of magnitude smaller than the masses of the previously considered neutrinos. Consequently, these particles can already be considered more related to γ- particles than to neutrinos. Of course, it must be taken into account that the masses of the loveton and neutrinos obtained from the proposed geometrical considerations, as well as those similarly determined for the electromagnetic field quanta, may represent only approximate values due to the presence of any additional physical properties, such as the elasticity property.

All next levels of spacestrons, as it has been mentioned before, we will refer to the particles of the electromagnetic field spectrum. If we take into account that γ-particles, like neutrinos, have a large penetrating power, it is possible to consider the possibility that neutrinos can be specific high-energy γ-quanta, devoid of the electric component [35]. Thus, let us compare the value of the top value of the Compton wavelength of γ-particles with the value of the spacestrons of the fifth and further levels. At present, the experimentally determined maximum energy of γ-quanta is 1.42 PeV, which is a value of the order of $2\cdot10^{-20}$ m [36], for the upper limit of the Compton wavelength [36]. To estimate the masses of quanta of electromagnetic fields we can use the expression for the mass of the *i*-th quantum m_k of the electromagnetic field

$$m_k = h/c\lambda_k \tag{3}$$

where h is Planck's constant, λ is the Compton wavelength, c is the speed of light.
All the results of calculations of the masses of the considered elementary particles and gamma-quanta have been summarized in one general table (Table 1).

Table 1.

Masses of elementary particles and gamma rays quantum

# level	Particle	Ratio of particle radii, r_n/r_e	Estimated particle mass, meV	Experimental particle mass, meV	Number of contacts, *N*
0	Loveton	1.0	892.18	938.3	
1	Electron	0.1547	0.511	0.511	3
2	Neutrino-2	0.0628	$7.94\cdot10^{-6}$		9
3	Neutrino -3	0.0340	$0.68\cdot10^{-6}$	$<1.1\cdot10^{-6}$	18
4	Neutrino-4	0.0213	$0.11\cdot10^{-6}$		81
5	Neutrino-5 / Gamma-5	0.0109	$7,25\cdot10^{-9}$		243
6	Gamma-6	0.0037	$9.25\cdot10^{-11}$	$<0.7\cdot10^{-11}$	729
7	Gamma-7	0.0018	$5.61\cdot10^{-12}$		2187
8	Gamma-8	0.0006	$7.81\cdot10^{-14}$		6561

* The number of contacts is undetermined for the top-level loveton.

From the results of the study one can state a number of corollaries which allow one to compare them with the currently known facts. First, the number of existing types of neutrinos associated with leptons is probably randomly close to the number of neutrino variants proposed in this model. Secondly, the nature of the birth and annihilation of elementary particles, in the framework of this model, can be seen as a breaking of the bond between the loveton-anti-loveton or electron-positron pair. Third, considering that the dimensions of the loveton and electron differ only about six times, perhaps their real linear dimensions are small enough to be comparable to the Planck length and not to the proton size. Fourth, the existing Standard Model is possibly responsible for a deeper level of electromagnetic interaction. This is indicated by the presence of fractional values of charges in it and also by the existence of the confinement phenomenon which does not allow for the decay of lovetons and electrons.

4. CONCLUSION

In the present paper a model of space-time structure is considered, representing it as a quasi-continuous medium, whose properties are determined by the presence of primary space (substrate). A flat 2-dimensional space in the absence of matter was chosen as a substrate. The proposed model made it possible, by partitioning the substrate structure into separate spatial elements (spacestrons), to both obtain a set of lovetons and identify the elements composing them with the elementary particle masses and electromagnetic field quanta.

The main results of the work are as follows:

1. The presented model of the structure of quasi-continuous space-time is based on the hypothesis about the presence of a 2-dimensional substrate of space in no matter.

2. It is shown that in the case of contact interaction all elements of space can be given either disk (spacestrons) or spherical (lovetons) shape.

3. New notions, such as spacestrons and lovetons, are proposed. Upper-level spacetrons can be represented in the form of a hexagonal packing, followed by the placement between them of spacetrons of other levels having different dimensions and relating to the possibility of their identification with the electron-positron pairs, neutrinos and quanta that make up the electromagnetic field.

4. Based on the analysis of the geometric positions and dimensions of the substrate elements responsible for the formation of elementary particles and the electromagnetic field quanta spectrum, were determined the estimated masses of a number of elementary particles, which correspond to the known masses of particles.

5. The existence of at least three varieties of neutrinos is revealed.

The obtained results allow us to hope to consider the whole spectrum of masses of subnuclear particles and electromagnetic field quanta, based on the energy interaction of such elements as lovetons and electrons, which are the basis of the structure of space-time. This concept can be confirmed in that on the basis of this concept it is possible to see in a new way the nature of elementary particles and, as will be shown later, to determine their masses with sufficiently high accuracy.

The elucidation of the essence of space-time is one of the most important problems of fundamental physics. The solution to this problem will, in the future, help to determine ways to solve some of the problems of physics and give the key to a new physical theory.

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