A Bipolar Fuzzy Extension of the MULTIMOORA Method

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Abstract. The aim of this paper is to make a proposal for a new extension of the MULTIMOORA method extended to deal with bipolar fuzzy sets. Bipolar fuzzy sets are proposed as an extension of classical fuzzy sets in order to enable solving a particular class of decision-making problems. Unlike other extensions of the fuzzy set of theory, bipolar fuzzy sets introduce a positive membership function, which denotes the satisfaction degree of the element x to the property corresponding to the bipolar-valued fuzzy set, and the negative membership function, which denotes the degree of the satisfaction of the element x to some implicit counter-property corresponding to the bipolar-valued fuzzy set. By using single-valued bipolar fuzzy numbers, the MULTIMOORA method can be more efficient for solving some specific problems whose solving requires assessment and prediction. The suitability of the proposed approach is presented through an example.

Key words: bipolar fuzzy set, single-valued bipolar fuzzy number, MULTIMOORA, MCDM.

1. Introduction

The management of very complex systems is the most complex, and therefore the most difficult task of the managers of today's organizations. The effectiveness of the management and managers of an organization depends to a large extent on the quality of the decisions they make on a daily basis.

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Decision-making and decisions are the core of managerial activities. Bearing in mind the globalization and, therefore, the dynamics of business doing, all of the above-stated have caused business and the decision-making process to become more demanding. Making quality decisions requires an ever more extensive preparation, which also involves the consideration of the different aspects of a decision, for the reason of which the decision-making process becomes considerably formalized. Thus, real problems and situations in real life are characterized by a large number of mostly conflicting criteria, whose strict optimization is generally impossible.

When it is necessary to make a decision on choosing one of several potential solutions to a problem, it is desirable to apply one of the models based on multiple-criteria decision-making methods (MCDM). This most often involves the process of selecting one of several alternative solutions, for which certain goals are set. When MCDM is concerned, Greco *et al.* (2010) point out the fact that it is the study of the methods and procedures aimed at making a proposal for solutions in terms of multiple, often conflicting criteria. Hwang and Yoon (1981) states that MCDM is based on the two basic approaches, i.e. on multiple attribute decision-making (MADM), which implies a choice of courses in the presence of multiple, and often conflicting criteria, i.e. a selection of the best alternative from a finite set of possible alternatives. Unlike MADM, in multiple objective decision-making (MODM), the best alternative is that which is formed with multiple goals, based on the continuous variables of the decision with additional constraints.

So, all the problems of today are, in general, multi-criterial, primarily because problems are mainly related to the achievement of the objectives related to a larger number of, usually conflicting, criteria, which is a great approximation to real tasks in decision-making processes (Das *et al.*, 2012; Zavadskas *et al.*, 2014). The increasing application of the MCDM method to solving various problems has caused an exceptional growth of multi-criteria decision-making as an important field of operational research, especially since 1980 (Aouni *et al.*, 2018; Masri, *et al.*, 2018; Wallenius *et al.*, 2008; Dyer *et al.*, 1992).

Within MADM, some of the methods that have been proposed are: Weighted Sum Model (WSM) (Fishburn, 1967); Simple Additive Weighting (SAW) method (MacCrimon, 1968), Elimination Et Choix Traduisant la REalité (ELECTRE) method (Roy, 1968), DEcision-MAking Trial and Evaluation Laboratory (DEMATEL) method (Gabus and Fontela, 1972), Compromise Programming (CP) method (Zeleny, 1973), Simple Multi Attribute Rating Technique (SMART) (Edwards, 1977), Analytic Hierarchy Process (AHP) method (Saaty, 1978), Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) method (Hwang and Yoon, 1981), Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEE) method (Brans, 1982), Measuring Attractiveness by a Categorical Based Evaluation Technique (MACBETH) (Bana e Costa and Vansnick, 1994), Complex Proportional Assessment of alternatives (COPRAS) method (Zavadskas *et al.*, 1994), Analytic Network Process (ANP) method (Saaty, 1996), Vlse Kriterijumska Optimizacija i kompromisno Resenje (VIKOR) (Opricovic, 1998), Multi-Objective Optimization on basis of Ratio Analysis (MOORA) method (Brauers and Zavadskas, 2006), Additive Ratio ASsessment (ARAS) method (Zavadskas and Turskis,

2010), Multi-Objective Optimization by Ratio Analysis plus the Full Multiplicative Form (MULTIMOORA) method (Brauers and Zavadskas, 2010a), and so on. While within the MODM methods that have been proposed can be stated: Data envelopment analysis (DEA) method (Charnes *et al.*, 1978), Linear Programming (LP) and Nonlinear Programming (NP) (Luenberger and Ye, 1984), Multi-Objective Programming (MOP) technique (Charnes *et al.*, 1989), Multi-Objective Linear Programming (Ecker and Kouada, 1978), and so on.

The MULTIMOORA method (Brauers and Zavadskas, 2010b) is an important MCDM method that has been applied so far to solve the most diverse problems in the field of economics, management, etc. Basically, the MULTIMOORA method consists of the well-known MOORA method (Brauers and Zavadskas, 2006) and the method of multi-object optimization (the Full Multiplicative Form of Multiple Objects method). Thus, Brauers and Zavadskas (2010a) proposed the updating of the MOORA method by adding a multi-object optimization method which involves maximizing and minimizing useful multiplicative functions (Lazauskas *et al.*, 2015).

As noted above, the MULTIMOORA method was applied in order to solve a variety of problems, such as: using MULTIMOORA for ranking and selecting the best performance appraisal method (Maghsoodi *et al.*, 2018), project critical path selection (Dorfeshan *et al.*, 2018), the selection of the optimal mining method (Liang *et al.*, 2018), pharmacological therapy selection (Eghbali-Zarch *et al.*, 2018), ICT hardware selection (Adali and Işik, 2017), industrial robot selection (Karande *et al.*, 2016), a CNC machine tool evaluation (Sahu *et al.*, 2016), personnel selection (Karabasevic *et al.*, 2015; Baležentis *et al.*, 2012), the economy (Baležentis and Zeng, 2013; Brauers and Zavadskas, 2011a, 2010b; Brauers and Ginevičius, 2010), and so on.

However, most decisions made in the real world are made in an environment in which goals and constraints cannot be precisely expressed due to their complexity; therefore, a problem cannot be displayed exactly in crisp numbers (Bellman and Zadeh, 1970). For such problems, characterized by uncertainty and indeterminacy, it is more appropriate to use values expressed in intervals instead of concrete (crisp) values. In this case, the existing, ordinary MCDM methods are expanded by using the extensions based on fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov, 1986), and neutrosophic sets (Smarandache, 1999). Accordingly, in order to allow a much wider use of the MULTIMOORA method, some extensions of the MULTIMOORA method have been proposed, some of which are as follows: Brauers *et al.* (2011) proposed a fuzzy extension of the MULTIMOORA method; Baležentis and Zeng (2013) proposed an IVFN extension of the MULTIMOORA method; Baležentis *et al.* (2014) also proposed an IFN extension of the MULTIMOORA method; Stanujkic *et al.* (2015) proposed an extension of the MULTIMOORA method based on the use of interval-valued triangular fuzzy numbers; Zavadskas *et al.* (2015) proposed an IVIF-based extension of the MULTIMOORA method; Hafezalkotob *et al.* (2016) proposed an extension of the MULTIMOORA method based on the use of interval numbers; Stanujkic *et al.* (2017a) proposed a neutrosophic extension of the MULTIMOORA method, and so on.

In addition to the aforementioned extensions of the fuzzy set theory, Zhang (1994) introduced the concept of bipolar fuzzy sets and proposed the usage of the two membership

functions that represent membership to a set and membership to a complementary set, thus providing an efficient approach to solving a widely larger number of complex decision-making problems.

Despite an advantage that can be achieved by using bipolar fuzzy logic, they are significantly less used for solving MCDM problems compared to other fuzzy logic extensions. The following examples can be mentioned as some of the really rare usages of BFS for solving MCDM problems: Alghamdi *et al.* (2018) and Akram and Arshad (2018) proposed bipolar fuzzy extensions of TOPSIS and ELECTRE I methods; while Han *et al.* (2018) provide a comprehensive mathematical approach based on the TOPSIS method for improving the accuracy of bipolar disorder clinical diagnosis.

It is also important to note that these are current researches. In addition, the bipolar logic has been considerably used in the neutrosophic set theory, where Uluçay *et al.* (2018), Pramanik *et al.* (018) and Tian *et al.* (2016) can be cited as some of the current researches.

Therefore, in order to enable a wider use of the MULTIMOORA method for solving even a wider range of problems, a bipolar extension of the MULTIMOORA method is proposed in this paper. Accordingly, the paper is structured as follows: in Section 1, the introductory considerations are presented. In Section 2, some basic definitions regarding bipolar fuzzy sets are given. In Section 3, the ordinary MULTIMOORA method is presented, whereas in Section 4, an extension of the MULTIMOORA method based on single-valued bipolar fuzzy numbers is proposed. In Section 5, a numerical example is demonstrated, and finally, the conclusions are given at the end of the paper.

2. The Basic Elements of a Bipolar Fuzzy Set

DEFINITION 1 (*Fuzzy set*, Zadeh, 1965). Let X be a nonempty set, with a generic element in X denoted by x. Then, a fuzzy set A in X is a set of ordered pairs:

$$A = \left\{ \langle x, \mu_A(x) \rangle \middle| x \in X \right\}, \tag{1}$$

where the membership function $\mu_A(x)$ denotes the degree of the membership of the element x to the set A, and $\mu_A(x) \in [0, 1]$.

DEFINITION 2 (*Bipolar fuzzy set*, Lee, 2000). Let X be a nonempty set. Then, a bipolar fuzzy set (BFS) is defined as:

$$A = \left\{ \langle x, \mu_A^+(x), \nu_A^-(x) \rangle \middle| x \in X \right\}, \tag{2}$$

where: the positive membership function $\mu_A^+(x)$ denotes the satisfaction degree of the element x to the property corresponding to the bipolar-valued fuzzy set, and the negative membership function $\nu_A^-(x)$ denotes the degree of the satisfaction of the element x to some implicit counter-property corresponding to the bipolar-valued fuzzy set, respectively; $\mu_A^+(x) : X \to [0, 1]$ and $\nu_A^-(x) : X \to [-1, 0]$.

DEFINITION 3. A single-valued bipolar fuzzy number (SVBFN) $a = \langle a^+, a^- \rangle$ is a special bipolar fuzzy set on the real number set R, whose positive membership and negative membership function are as follows:

$$\mu^+(x) = \begin{cases} 1 & x = a^+, \\ 0 & \text{otherwise,} \end{cases} \tag{3}$$

$$v^-(x) = \begin{cases} 1 & x = a^-, \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

respectively.

DEFINITION 4. Let $a_1 = \langle a_1^+, a_1^- \rangle$ and $a_1 = \langle a_2^+, a_2^- \rangle$ be two SVBFNs, and $\lambda > 0$. Then, the basic operations for these numbers are defined as shown below:

$$a_1 + a_2 = \langle a_1^+ + a_2^+ - a_1^+ a_2^+, -a_1^- a_2^- \rangle, \tag{5}$$

$$a_1 \cdot a_2 = \langle a_1^+ a_2^+, -\left(-a_1^- - a_2^- - a_1^- a_2^- \right) \rangle, \tag{6}$$

$$\lambda a_1 = \langle 1 - \left(1 - a_1^+\right)^\lambda, -\left(-a_1^- \right)^\lambda \rangle, \tag{7}$$

$$a_1^\lambda = \langle \left(a_1^+\right)^\lambda, -\left(1 - \left(1 - \left(-a_1^- \right)\right)^\lambda\right) \rangle. \tag{8}$$

DEFINITION 5. Let $a = \langle a^+, a^- \rangle$ be an SVBFN. Then, the score function $s_{(a)}$ is as follows:

$$s_a = \left(1 + a^+ + a^-\right)/2. \tag{9}$$

DEFINITION 6. Let a_1 and a_2 be two SVBFNs. Then, $a_1 > a_2$ if $s_{a_1} > s_{a_2}$.

DEFINITION 7. Let $a_1 = \langle a_1^+, a_1^- \rangle$ and $a_1 = \langle a_2^+, a_2^- \rangle$ be two SVBFNs. The Hamming distance between a_1 and a_2 is as follows:

$$d_H(a_1, a_2) = \frac{1}{2}\left(\left|a_1^+ - a_2^+\right| + \left|a_1^- - a_2^-\right|\right). \tag{10}$$

DEFINITION 8. Let $a_j = \langle a_j^+, a_j^- \rangle$ be a collection of SVBFNs. The bipolar weighted average operator (A_w) of the n dimensions is a mapping as follows:

$$A_w(a_1, a_2, \ldots, a_n) = \sum_{j=1}^n w_j a_j$$

$$= \left(1 - \prod_{j=1}^n \left(1 - a_j^+\right)^{w_j}, -\left(1 - \prod_{j=1}^n \left(1 - \left(-a_j^- \right)\right)^{w_j}\right)\right), \tag{11}$$

where: w_j is the element j of the weighting vector, $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

DEFINITION 9. Let $a_j = \langle a_j^+, a_j^- \rangle$ be a collection of SVBFNs. The bipolar weighted geometric operator (G_w) of the n dimensions is a mapping $G_w : Q_n \rightarrow Q$ as follows:

$$G_w(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} a_j^{w_j} = \left(\prod_{j=1}^{n} \left(a_j^+\right)^{w_j}, - \prod_{j=1}^{n} \left(- a_j^-\right)^{w_j} \right). \tag{12}$$

3. The MULTIMOORA Method

Compared to the other MCDM methods, the MULTIMOORA method is characteristic because it combines three approaches, namely: the Ratio System (RS) Approach, the Reference Point (RP) Approach and the Full Multiplicative Form (FMF) Approach, in order to select the most appropriate alternative.

In addition, this method does not calculate and does not use the overall significance for ranking alternatives and selecting the most appropriate one. Instead of using an overall parameter for ranking alternatives, the final ranking order of the considered alternatives, as well as the selection of the most appropriate alternative, is based on the use of the theory of dominance.

For an MCDM problem that includes the m alternatives that should be evaluated on the basis of the n criteria, the computational procedure of the MULTIMOORA can be expressed as follows:

Step 1. *Construct a decision matrix and determine the weights of criteria.*

Step 2. *Calculate a normalized decision matrix*, as follows:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \tag{13}$$

where: r_{ij} denotes the normalized performance of the alternative i with respect to the criterion j, and x_{ij} denotes the performance of the alternative i to the criterion j.

Step 3. *Calculate the overall significance of each alternative*, as follows:

$$y_i = \sum_{j \in \Omega_{\max}} w_j r_{ij} - \sum_{j \in \Omega_{\min}} w_j r_{ij}, \tag{14}$$

where: y_i denotes the overall importance of the alternative i, $\Omega_{\max}$ and $\Omega_{\min}$ denote the sets of the benefit cost criteria, respectively.

Step 4. *Determine the reference point*, as follows:

$$r^* = \{r_1^*, r_2^*, \ldots, r_n^*\} = \left\{ \left(\max_i r_{ij} \,\middle|\, j \in \Omega_{\max} \right), \left(\min_i r_{ij} \,\middle|\, j \in \Omega_{\min} \right) \right\}. \tag{15}$$

Step 5. *Determine the maximal distance between each alternative and the reference point*, as follows:

$$d_i^{\max} = \max_j \left(w_j \left| r_j^* - r_{ij} \right| \right), \tag{16}$$

where: $d_i^{\max}$ denotes the maximal distance of the alternative i to the reference point.

Step 6. *Determine the overall utility of each alternative*, as follows:

$$u_i = \frac{\prod_{j \in \Omega_{\max}} w_j r_{ij}}{\prod_{j \in \Omega_{\min}} w_j r_{ij}}, \tag{17}$$

where: u_i denotes the overall utility of the alternative i.

In particular case, when evaluation is made only on the basis of benefit criteria, Eq. (17) is as follows:

$$u_i = \prod_{j \in \Omega_{\max}} w_j r_{ij}. \tag{18}$$

Step 7. *Determine the final ranking order of the considered alternatives and select the most appropriate one*. In this step, the considered alternatives are ranked based on their:

- overall significance,
- maximal distance to the reference point, and
- overall utility.

As a result of these rankings, the three different ranking lists are formed, representing the rankings based on the RS approach, the RP approach and the FMF approach of the MULTIMOORA method.

The final ranking of the alternatives is based on the dominance theory, i.e. the alternative with the highest number of appearances in the first positions on all ranking lists is the best-ranked alternative.

4. An Extension of the MULTIMOORA Method Based on Single-Valued Bipolar Fuzzy Numbers

For an MCDM problem involving m alternatives and n criteria and K decision-makers, whereby the performances of the alternatives are expressed by using SVBFNs, the calculation procedure of the extended MULTIMOORA method can be expressed as follows:

Step 1. *Evaluate the alternatives in relation to the evaluation criteria*, and do that for each DM. In this step, each DM evaluates the alternatives and forms an evaluation matrix.

In order to provide an easier evaluation, the following Likert scale, shown in Table 1, is proposed for evaluating alternatives in relation to the evaluation criteria.

Table 1
Nine-point Likert scale for expressing degree of satisfaction.

Satisfaction level	Numerical value
Neutral/without attitude	0
Extremely low	1
Very low	2
Low	3
Medium low	4
Medium	5
Medium high	6
High	7
Very high	8
Extremely high	9
Absolute	10

However, the respondents should be introduced that the values listed in Table 1 are only approximative and that they can use any value from the interval $[0, 10]$ and $[-10, 0]$.

After forming initial decision-making matrix, obtained responses should be divided by 10 in order to transform it into the allowed interval $[-1, 1]$. This approach for evaluating alternatives is proposed to avoid the use of vector normalization procedure, used in the ordinary MULTIMOORA method.

Step 2. *Determine the importance of the evaluation criteria*, and do that for each DM. In this step, each DM determines the weights of the criteria by using one of several existing methods for determining the weights of criteria.

Step 3. *Determine the group decision matrix.* In order to transform individual into group preferences, individual evaluation matrices are transformed into group one by applying Eq. (11).

Step 4. *Determine the group weights of the criteria.* In order to transform individual into group preferences with respect to the weights of criteria, the group weights of criteria can be determined as follows:

$$w_j = \sum_{k=1}^{K} w_j^k \tag{19}$$

where: w_j denotes the weight of the criterion j, and w_j^k denotes the weight of the criterion j obtained from the DM k.

After calculating the group evaluation matrix and the group weights of the criteria, all the necessary prerequisites for applying all the three approaches integrated in the MULTIMOORA method are obtained. Based on the approach proposed by Stanujkic *et al.* (2017b), the remainder steps of the proposed approach are as follows:

Step 5. *Determine the significance of the evaluated alternatives based on the RS approach.* This step can be explained through the following sub-steps:

Step 5.1. *Determine the impact of the benefit and cost criteria to the importance of each alternative*, as follows:

$$Y_i^+ = \left(1 - \prod_{j \in \Omega_{max}}^{n} (1 - r_{ij})^{w_j}, -\left(1 - \prod_{j \in \Omega_{max}}^{n} \left(1 - (-r_{ij})\right)^{w_j}\right)\right), \tag{20}$$

$$Y_i^- = \left(1 - \prod_{j \in \Omega_{min}}^{n} (1 - r_{ij})^{w_j}, -\left(1 - \prod_{j \in \Omega_{min}}^{n} \left(1 - (-r_{ij})\right)^{w_j}\right)\right), \tag{21}$$

where: Y_i^+ and Y_i^- denote the importance of the alternative i obtained on the basis of the benefit and cost criteria, respectively; Y_i^+ and Y_i^- are SVBFNs.

It is evident that A_w operator is used to calculate the impact of the benefit and cost criteria.

Step. 5.2. *Transform Y_i^+ and Y_i^- into crisp values by using the Score Function*, as follows:

$$y_i^+ = s\left(Y_i^+\right), \tag{22}$$

$$y_i^- = s\left(Y_i^-\right). \tag{23}$$

Step 5.3. *Calculate the overall importance for each alternative*, as follows:

$$y_i = y_i^+ - y_i^-. \tag{24}$$

Step 6. *Determine the significance of the evaluated alternatives based on the RP approach.* This step can be explained through the following sub-steps:

Step 6.1. *Determine the reference point.* The coordinates on the bipolar fuzzy reference point $r^* = \{r_1^*, r_2^*, \ldots, r_n^*\}$ can be determined as follows:

$$r^* = \left\{ \left(\left\langle \max_i r_{ij}, \min_i r_{ij} \right\rangle \middle| j \in \Omega_{max} \right), \left(\left\langle \min_i r_{ij}, \max_i r_{ij} \right\rangle \middle| j \in \Omega_{min} \right) \right\} \tag{25}$$

where: r_j^* denotes the coordinate j of the reference point.

Step 6.2. *Determine the maximum distance from each alternative to all the coordinates of the reference point.* The maximum distance of each alternative to the reference point can be determined as follows:

$$d_{ij}^{\max} = d_{\max}\left(r_{ij}, r_j^*\right) w_j, \tag{26}$$

where $d_{ij}^{\max}$ denotes the maximum distance of the alternative i to the criterion j determined by Eq. (10).

Step 6.3. *Determine the maximum distance of each alternative*, as follows:

$$d_i^{\max} = \max_j d_{ij}^{\max}, \tag{27}$$

where $d_i^{\max}$ denotes the maximum distance of the alternative i.

Step 7. *Determine the significance of the evaluated alternatives based on* the FMF.

This step can be explained through the following sub-steps:

Step 7.1. *Calculate the utility obtained based on the benefit U_i^+ and cost U_i^- criteria*, for each alternative, as follows:

$$U_i^+ = \left(\prod_{j \in \Omega_{max}}^{n} (r_{ij})^{w_j}, - \prod_{j \in \Omega_{max}}^{n} (-r_{ij})^{w_j} \right), \tag{28}$$

$$U_i^- = \left(\prod_{j \in \Omega_{min}}^{n} (r_{ij})^{w_j}, - \prod_{j \in \Omega_{min}}^{n} (-r_{ij})^{w_j} \right), \tag{29}$$

where U_i^+ and U_i^- are SVBFNs.

Step 7.2. *Transform U_i^+ and U_i^- into crisp values by using the Score Function*, as follows:

$$u_i^+ = s\left(U_i^+\right), \tag{30}$$

$$u_i^- = s\left(U_i^-\right). \tag{31}$$

Step 7.3. *Determine the overall utility for each alternative*, as follows:

$$u_i = \frac{u_i^+}{u_i^-}. \tag{32}$$

In the case when evaluation is made only on the basis of benefit criteria, Eq. (32) is as follows:

$$u_i = u_i^+. \tag{33}$$

Step 8. *Determine the final ranking order of the alternatives*. The final ranking order of the alternatives can be determined as in the case of the ordinary MULTIMOORA method, i.e. based on the dominance theory (Brauers and Zavadskas, 2011b).

In this stage, the alternatives are ranked based on their overall importance, maximum distance to the reference point and overall utility. As a result of that, three ranking lists are formed.

Based on these ranking lists, the final ranking list of the alternatives is formed on the basis of the theory of dominance, i.e. the alternative with the largest number of appearances on the first position in the three ranking lists is the most acceptable.

5. A Numerical Example

In this section, a numerical example of purchasing rental space is considered in order to explain the proposed approach in detail.

Table 2
The ratings obtained from the first of the three DMs.

	C_1		C_2		C_3		C_4		C_5	
	a^+	a^-	a^+	a^-	a^+	a^-	a^+	a^-	a^+	a^-
A_1	7	−2	7	−3	5	−1	7	−5	8	−1
A_2	4	−1	5	−2	4	−2	4	−6	7	−1
A_3	7	−1	3	−1	2	0	2	−1	7	−2
A_4	9	−1	4	−1	3	0	3	−1	6	−1

Table 3
The ratings obtained from the first of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	⟨0.70, −0.20⟩	⟨0.70, −0.30⟩	⟨0.50, −0.10⟩	⟨0.70, −0.50⟩	⟨0.80, −0.10⟩
A_2	⟨0.40, −0.10⟩	⟨0.50, −0.20⟩	⟨0.40, −0.20⟩	⟨0.40, −0.60⟩	⟨0.70, −0.10⟩
A_3	⟨0.70, −0.10⟩	⟨0.30, −0.10⟩	⟨0.20, 0.00⟩	⟨0.20, −0.10⟩	⟨0.70, −0.20⟩
A_4	⟨0.90, −0.10⟩	⟨0.40, −0.10⟩	⟨0.30, 0.00⟩	⟨0.30, −0.10⟩	⟨0.60, −0.10⟩

Table 4
The ratings obtained from the second of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	⟨0.70, −0.20⟩	⟨0.70, −0.50⟩	⟨0.40, −0.20⟩	⟨0.70, −0.50⟩	⟨0.80, −0.10⟩
A_2	⟨0.60, −0.10⟩	⟨0.40, −0.60⟩	⟨0.40, −0.20⟩	⟨0.40, −0.60⟩	⟨0.80, −0.10⟩
A_3	⟨0.80, −0.10⟩	⟨0.20, −0.10⟩	⟨0.20, −0.10⟩	⟨0.20, −0.10⟩	⟨0.70, −0.10⟩
A_4	⟨0.90, −0.10⟩	⟨0.30, −0.10⟩	⟨0.30, −0.10⟩	⟨0.30, −0.10⟩	⟨0.60, −0.10⟩

Suppose that a company is planning to start its sales business in a new location, and therefore is looking for a new sales building. After the initial consideration of the available alternatives, four alternatives have been identified as suitable. For this reason, a team of three decision-makers (DMs) was formed with the aim of evaluating suitable alternatives based on the following criteria:

- C_1 – Rental space quality;
- C_2 – Rental space adequacy;
- C_3 – Location quality;
- C_4 – Location distance from the city centre, and
- C_5 – Rental price.

As previously reasoned, in this evaluation the ratings of the alternatives in relation to the criteria are expressed by using SVBFNs.

The ratings obtained from the first of the three DMs are shown in Table 2, as the points of the Likert scale, whereas in Table 3, they are shown in the form of SVBFNs.

The ratings obtained from the second and the third of the three DMs are accounted for in Table 4 and Table 5.

The group decision matrix, calculated by applying Eq. (11), is presented in Table 6.

Table 5
The ratings obtained from the third of the three DMs, in the form of SVBFNs.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.60, -0.10 \rangle$	$\langle 0.90, -0.20 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.80, -0.10 \rangle$
A_2	$\langle 0.40, -0.60 \rangle$	$\langle 0.40, -0.60 \rangle$	$\langle 1.00, -0.40 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.80, -0.10 \rangle$
A_3	$\langle 0.20, -0.10 \rangle$	$\langle 0.90, -0.40 \rangle$	$\langle 0.80, -0.30 \rangle$	$\langle 0.70, -0.10 \rangle$	$\langle 0.70, -0.10 \rangle$
A_4	$\langle 0.30, -0.10 \rangle$	$\langle 1.00, -0.30 \rangle$	$\langle 0.80, -0.20 \rangle$	$\langle 0.80, -0.10 \rangle$	$\langle 0.60, -0.10 \rangle$

Table 6
The group decision-making matrix.

	C_1	C_2	C_3	C_4	C_5
A_1	$\langle 0.67, -0.16 \rangle$	$\langle 0.79, -0.32 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.80, -0.10 \rangle$
A_2	$\langle 0.47, -0.18 \rangle$	$\langle 0.43, -0.42 \rangle$	$\langle 1.00, -0.26 \rangle$	$\langle 1.00, 0.00 \rangle$	$\langle 0.77, -0.10 \rangle$
A_3	$\langle 0.64, -0.10 \rangle$	$\langle 0.61, -0.16 \rangle$	$\langle 0.49, 0.00 \rangle$	$\langle 0.41, -0.10 \rangle$	$\langle 0.70, -0.13 \rangle$
A_4	$\langle 0.81, -0.10 \rangle$	$\langle 1.00, -0.15 \rangle$	$\langle 0.53, 0.00 \rangle$	$\langle 0.53, -0.10 \rangle$	$\langle 0.60, -0.10 \rangle$

Table 7
The weights of the criteria obtained from the first of the three
DMs.

	s_j	k_j	q_j	w_j
C_1		1	1	0.19
C_2	1.2	0.80	1.25	0.23
C_3	0.9	1.10	1.14	0.21
C_4	0.7	1.30	0.87	0.16
C_5	1.2	0.80	1.09	0.20
		5.00	5.35	1.00

Table 8
The group criteria weights.

	w_j^1	w_j^2	w_j^3	w_j
C_1	0.19	0.17	0.19	0.18
C_2	0.23	0.24	0.23	0.24
C_3	0.21	0.22	0.21	0.21
C_4	0.16	0.17	0.16	0.16
C_5	0.20	0.21	0.20	0.21
				1.00

The weights obtained from the first of the three DMs by applying the PIPRECIA method (Stanujkic *et al.*, 2017b) are accounted for in Table 7, while the group weights of the criteria, calculated by applying Eq. (19), are shown in Table 8.

On the basis of the ratings from Table 6 and the weights from Table 8, the overall significance, the maximum distance to the reference point and the overall utility are calculated for each alternative in the next step.

The overall significances, accounted for in Table 9, are calculated by applying Eqs. (20)–(24).

Table 9
The overall significances of the considered alternatives.

	Y_i^+	Y_i^-	y_i^+	y_i^-	y_i	Rank
A_1	$\langle 1.00, -0.11 \rangle$	$\langle 1.00, -0.02 \rangle$	0.94	0.99	-0.05	3
A_2	$\langle 1.00, -0.20 \rangle$	$\langle 1.00, -0.02 \rangle$	0.90	0.99	-0.09	4
A_3	$\langle 0.42, -0.06 \rangle$	$\langle 0.30, -0.05 \rangle$	0.68	0.63	0.05	2
A_4	$\langle 1.00, -0.06 \rangle$	$\langle 0.29, -0.04 \rangle$	0.97	0.62	0.35	1

Table 10
The reference points.

	Ω_{max}		Ω_{min}	
	r^+	r^-	r^+	r^-
r^*	1.00	-0.20	0.29	-0.02

Table 11
The ratings of the alternatives obtained based on the reference point approach.

	C_1	C_2	C_3	C_4	C_5	d_i^{max}	Rank
A_1	0.08	0.16	0.13	0.29	0.10	0.08	4
A_2	0.17	0.28	0.00	0.29	0.09	0.00	1
A_3	0.13	0.33	0.38	0.05	0.06	0.05	3
A_4	0.04	0.14	0.36	0.11	0.00	0.00	1

Table 12
The overall utility of the considered alternatives.

	U_i^+	U_i^-	u_i^+	u_i^-	u_i	Rank	
A_1	$\langle 1.00, -0.11 \rangle$	$\langle 1.00, -0.02 \rangle$	0.94	0.99	-0.05	3
A_2	$\langle 1.00, -0.20 \rangle$	$\langle 1.00, -0.02 \rangle$	0.90	0.99	-0.09	4
A_3	$\langle 0.42, -0.06 \rangle$	$\langle 0.30, -0.05 \rangle$	0.68	0.63	0.05	2
A_4	$\langle 1.00, -0.06 \rangle$	$\langle 0.29, -0.04 \rangle$	0.97	0.62	0.35	1

After that, the reference point shown in Table 10 is determined by applying Eq. (25).

The maximum distances to the reference point accounted for in Table 11 are determined by applying Eq. (26) and Eq. (27).

The overall utility shown in Table 12 is calculated by applying Eqs. (28)–(32).

Finally, on the basis of the ranking orders shown in Tables 9, 11 and 12, the most appropriate alternative is determined by means of the theory of dominance, as is shown in Table 13.

As can be seen from Table 12, the most appropriate alternative is the alternative denoted as A_4.

Table 13
The final ranking order of the considered alternatives.

	RS	RP	FMP	Final rank
A_1	3	4	3	3
A_2	4	1	4	4
A_3	2	3	2	2
A_4	1	1	1	1

6. Conclusions

The bipolar fuzzy sets introduced two membership functions, namely the membership function to a set and the membership function to a complementary set.

On the other hand, the MULTIMOORA method is an efficient and already proven multiple-criteria decision-making method, which has been used for solving a number of different decision-making problems so far.

Therefore, an extension of the MULTIMOORA method enabling the use of single-valued bipolar fuzzy numbers is proposed in this article. The usability and efficiency of the proposed extension is successfully demonstrated on the example of the problem of the best location selection.

In the literature, numerous extensions of the MULTIMOORA methods have been proposed with the aim to adapt it for the use of grey system theory, fuzzy set theory, as well as various extensions of fuzzy set theory. Some extensions that enable the use of neutrosophic sets are also proposed. The mentioned extensions aim to exploit the specificities of particular sets for solving certain types of decision-making problems, and thus enable more efficient decision making.

Because of the specificity that bipolar fuzzy sets provide, the proposed expanded MULTIMOORA method can be expected to be acceptable for solving a particular class of complex decision-making problems.

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