g = 2 Based on the Center of Mass Coinciding with the Center of Rotation

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Abstract:

This paper is based on a relativistic classical spinning particle described by an internal stress-energy tensor. It is found that if the mechanical momentum is set to zero in the rest frame, then as the size of the particle is reduced and as the center of mass is set to the center of rotation, $g = 2$ is required. In this derivation some of the electromagnetic self-field terms are neglected.

I. Introduction

The g-factor[1] is one if an object's charge distribution is proportional to its mass distribution, neglecting electromagnetic self-field terms. However, for the electron the g-factor is two as predicted by the Dirac equation, and in that regard appears to be a result of relativistic quantum mechanics (for example see Sakauri[2]). However Sakauri[2] also shows a derivation, based on

Feynman, of the non-relativistic Pauli spin equation that also leads to $g = 2$. Levy-Leblond[3] and Greiner[4] also derive $g = 2$ using a linearized version of the Schrodinger equation.

The g-factor is also two for a Kerr-Newman black hole (for example see Misner, Thorne and Wheeler (herein MTW) [5]). Kramers[6] derives the relation $g = 2$ by extending the spin 3-vector to a relativistically invariant complex 3-vector, but as shown by Bargmann, Michel, and Telegdi[7] his argument is questionable. Heslot[8] and Corben[9] also obtain the relation $g = 2$ by using equations of a spinning point particle. Rivas, Aguirregabiria and Hernandez[10] obtain $g = 2$ by analyzing the structure of the spin operator for a point particle.

We start with the equations of an extended particle in flat space-time. We show that for an extended particle with the requirements of zero mechanical momentum in the rest frame and the center of rotation equal to the center of mass, a constraint condition is required. As the size of the particle is reduced, and if we ignore some of the electromagnetic self-field contributions, this constraint condition reduces to the condition $g = 2$.

We set up equations of motion for an extended charged particle using a mechanical stress-energy tensor and a general charge-current distribution. Norvik[11] and Appel and Kiessling[12] use a rigid charge-current distribution, but base the equations of motion on a variational principle rather than a stress-energy tensor.

Dixon[13] and Harte[14] use a mechanical stress-energy tensor and expand the stress-energy tensor and the charge-current distribution in terms of multiple moments. We consider only the total mass, momentum, and first moments of the stress-energy tensor.
If we ignore the self-field terms we find g=2 for a general charge-current distribution, and to include the self-field terms consider a non-relativistic rigid charge distribution.

We denote the mechanical stress-energy tensor by $S^{\alpha\beta}$ and 4-current density by j^{α} so that, following MTW[5] and requiring local energy-momentum conservation

$$S^{\alpha\beta}{}_{,\beta} = F^{\alpha}{}_{\beta}\, j^{\beta} \tag{1}$$

where $F^{\alpha}{}_{\beta}$ is the electromagnetic field tensor, commas represent partial derivatives, and repeated indices indicate a summation. Greek indices represent space-time coordinates, Latin indices represent 3-space coordinates, and a zero index represents time. The metric is defined such that $g_{00} = -1$ and $g_{ij} = \delta_{ij}$, and the speed of light is set equal to one.

Associate a world line with the particle defined by $x^{\alpha}{}_{0}(\tau)$ where τ is the proper time along the world line. To include the angular momentum, again following MTW[5], define

$$M^{\alpha\beta\gamma} = (x^{\alpha} - x^{\alpha}{}_{0}(\tau))S^{\beta\gamma} - (x^{\beta} - x^{\beta}{}_{0}(\tau))S^{\alpha\gamma}$$

which obeys the relation

$$M^{\alpha\beta\gamma}{}_{,\gamma} = (x^{\alpha} - x^{\alpha}{}_{0}(\tau))F^{\beta}{}_{\gamma}\, j^{\gamma} - (x^{\beta} - x^{\beta}{}_{0}(\tau))F^{\alpha}{}_{\gamma}\, j^{\gamma}\,. \tag{2}$$

Eq. (2) is based on Eq. (1) and the fact that $S^{\alpha\beta} = S^{\beta\alpha}$.

II. Equations of Motion

Now consider the object to be confined spatially so that a 2-d surface can be taken around it such that $S^{\alpha\beta}$ is zero outside the surface and the world line is inside the surface. Then consider a series of these 2-d surfaces in the rest frames of the object so as to make up a 3-d space-time surface. To make the 3-d surface closed cap the top and bottom with flat 3-d spatial surfaces which include the object and are bounded by the top and bottom 2-d surfaces. Both the top and bottom surfaces are in the rest frame of the object, with the rest frame defined by the world line. The rest of the 3-d surface is outside the object. The bottom surface is at time τ and the top surface at time $\tau+\delta\tau$.

Integrating eq. (1) over the 4-space within this surface, and again following MTW[5], eq. (1) becomes

$$\int d^4xS^{\alpha\beta}{}_{,\beta} = -u_\beta(\tau+\delta\tau)\int_{top} S^{\alpha\beta}d^3\Sigma + u_\beta(\tau)\int_{bottom} S^{\alpha\beta}d^3\Sigma = \int d^4xF^\alpha{}_\beta j^\beta\,. \qquad (3)$$

If we multiply eq. (3) by $u_\alpha(\tau)$ it takes the form

$$-u_\alpha(\tau)u_\beta(\tau+\delta\tau)\int_{top} S^{\alpha\beta}d^3\Sigma + u_\alpha(\tau)u_\beta(\tau)\int_{bottom} S^{\alpha\beta}d^3\Sigma = u_\alpha(\tau)\int d^4xF^\alpha{}_\beta j^\beta \qquad (4)$$

where $u_\alpha = g_{\alpha\beta}u^\beta$ and u^α is the 4-velocity along the world line.

Considering the two rest frames, call the bottom frame the unprimed frame and the top frame the primed frame. Look at the first term in eq. (4) in terms of the primed coordinates, and the second and third terms in terms of the unprimed coordinates, so that eq. (4) becomes

$$u_{\alpha'}(\tau)\int_{top} S^{\alpha'0'}d^3\Sigma + \int_{bottom} S^{00}d^3\Sigma = -\int d^4xF^0{}_\beta j^\beta \qquad (5)$$

Using a Lorentz transformation we then have, to order $\delta\tau$,

$$u_{0'}(\tau) = -1 \qquad u_{i'}(\tau) = -a_i \delta\tau$$

where a_i is the acceleration along the world line in the bottom rest frame. Equation (5) then becomes

$$-\int_{top} S^{0'0'} d^3\Sigma - a_i \delta\tau \int_{top} S^{i'0'} d^3\Sigma + \int_{bottom} S^{00} d^3\Sigma = -\int d^4x F^0{}_\beta j^\beta . \qquad (6)$$

Again using a Lorentz transformation, the top surface can be defined by $t = (1 + a_i r^i)\delta\tau$ where we have set $r^i = x^i - x^i{}_0(\tau)$. The 4-space integral on the left of eq. (6) thus takes the form

$$\int d^4x F^0{}_\beta j^\beta = \delta\tau \int_{bottom} (1 + a_i r^i) F^0{}_\beta j^\beta d^3\Sigma$$

to order $\delta\tau$. Dividing by $\delta\tau$ and taking the limit as $\delta\tau$ goes to zero, eq. (6) becomes

$$\frac{dm_0}{d\tau} + a_i p^i = \int (1 + a_k r^k) F^0{}_\beta j^\beta d^3\Sigma \qquad (7)$$

where

$$m_0 = \int S^{00} d^3\Sigma \qquad p^i = \int S^{i0} d^3\Sigma .$$

In deriving eq. (7) we are assuming that the spatial integrals on the top and bottom slices are different by the order of $\delta\tau$.

Then multiplying Eq. (3) by unit spatial vectors in the bottom rest frame, a similar calculation yields the equation

$$\frac{dp^i}{d\tau} + m_0 a^i = \int (1 + a_k r^k) F^i{}_\beta j^\beta d^3\Sigma . \qquad (8)$$

Using similar methods with the rotation eq. (2) we obtain

$$\frac{dm^i}{d\tau} - p^i + a_j L^{ij} = \int (1 + a_k r^k) r^i F^0{}_\beta j^\beta d^3\Sigma \qquad (9)$$

$$\frac{dL^{ij}}{d\tau} + a^j m^i - a^i m^j = \int (1 + a_k r^k) \{ r^i F^j{}_\beta j^\beta - r^j F^i{}_\beta j^\beta \} d^3\Sigma \qquad (10)$$

where

$$m^i = \int r^i S^{00} d^3\Sigma$$

and

$$L^{ij} = \int (r^i S^{j0} - r^j S^{i0}) d^3\Sigma$$

A derivation of eq. (9) is given in the appendix since this equation is needed for the g = 2 derivation.

Thus, given a particular world line $x^i{}_0(\tau)$ and 4-current distribution j^β, there are four equations (7), (8), (9), and (10) for the four rest frame relations, the mass m_0, momentum p^i, angular momentum L^{ij}, and mass moment m^i, with the acceleration being determined by the world line. Given the 4-current distribution j^α, the field tensor $F^\alpha{}_\beta$ can be determined by Maxwell's equations.

To impose a restriction on the world line, require that the rest frame mechanical momentum p^i be zero. Some authors, for example Garcia and Uson[15], impose this restriction, but it is not necessarily the case. For a stationary situation in which the mechanical and electromagnetic stress-energy tensors do not change in time, the total mechanical and electromagnetic momentum must be zero in the rest frame (for example see Hnizdo[16, 17]). In this case, if the electromagnetic momentum is not zero neither can the mechanical momentum be zero. However, here we take the electromagnetic momentum to be zero in the rest frame of a stationary situation.

Taking the mechanical momentum to be zero, eqs. (7) and (8) become

$$\frac{dm_0}{d\tau} = \int (1 + a_k r^k) F^0{}_\beta j^\beta d^3\Sigma \tag{11}$$

$$m_0 a^i = \int (1 + a_k r^k) F^i{}_\beta j^\beta d^3\Sigma \tag{12}$$

Eqs. (11) and (12) taken together can be written in a relativistic form in the rest frame as

$$\frac{d}{d\tau}(m_0 u^\alpha) = u^\alpha \frac{dm_0}{d\tau} + m_0 \frac{du^\alpha}{d\tau} = \delta^\alpha{}_0 \frac{dm_0}{d\tau} + \delta^\alpha{}_i m_0 a^i$$

$$= \delta^\alpha{}_0 \int (1 + a_k r^k) F^0{}_\beta j^\beta d^3\Sigma + \delta^\alpha{}_i \int (1 + a_k r^k) F^i{}_\beta j^\beta d^3\Sigma$$

$$= \int (1 + a_k r^k) F^\alpha{}_\beta j^\beta d^3\Sigma$$

Kaup[18] comes up with a similar equation based on an equation by Dixon[19]. He also uses a mechanical stress-energy tensor, but defines the particle 4-momentum as $-\int S^{\alpha\beta} u_\beta d^3\Sigma$ rather than $m_0 u^\alpha$.

We can put another restriction on the world line such that it goes through the center of mass so that $m^i = 0$ at some point, but by eq. (9) it will not necessarily stay on the center of mass. If we require that the center of mass coincide with the world line, we need $\frac{dm^i}{d\tau} = 0$. Eq. (9) then becomes the constraint equation

$$a_j L^{ij} = \int (1 + a_k r^k) r^i F^0{}_\beta j^\beta d^3\Sigma \qquad (13)$$

and eq. (10) for the angular momentum becomes

$$\frac{dL^{ij}}{d\tau} = \int (1 + a_k r^k)\{r^i F^j{}_\beta j^\beta - r^j F^i{}_\beta j^\beta\} d^3\Sigma \tag{14}$$

There are possible definitions of the center of mass other than m^i / m_0 in the rest frame (see for example Pryce[20]), but we take that as the definition here.

III. g = 2 ignoring Self-Field Contributions

Now ignore the self-field terms, and assume that the charge distribution is such that there is no net current in the rest frame, that is $\int j^i d^3\Sigma = 0$.

Take the external fields to be of the same order as the acceleration, so that, ignoring second order acceleration effects, eqs. (12) and (13) take the form

$$m_0 a^i = \int F^i{}_0 j^0 d^3\Sigma = qF^i{}_0 \tag{15}$$

$$a_j L^{ij} = \int r^i F^0{}_j j^j d^3\Sigma = F^0{}_j \int r^i j^j d^3\Sigma \tag{16}$$

where we have ignored the self-field terms and have taken the external fields to be constant over the size of the particle. Multiplying eq. (15) by δ_{ji} and noting that $\delta_{ji} F^i{}_0 = F^0{}_j$, we can substitute eq. (15) into eq. (16) to obtain

$$\frac{q}{m_0} F^0{}_j L^{ij} = F^0{}_j \int r^i j^j d^3\Sigma \tag{17}$$

Then noting that $L^{ij} = \varepsilon^{ij}{}_k L^k$, where L^k is the angular momentum, for arbitrary $F^0{}_j$ we need

$$\frac{q}{m_0}\varepsilon^{ij}{}_k L^k = \int r^i j^j d^3\Sigma \qquad (18)$$

Multiplying eq. (18) by $\varepsilon^{k'}{}_{ij}$ it becomes

$$\frac{q}{m_0}\varepsilon^{k'}{}_{ij}\varepsilon^{ij}{}_k L^k = 2\frac{q}{m_0}\delta^{k'}{}_k L^k = \varepsilon^{k'}{}_{ij}\int r^i j^j d^3\Sigma \qquad (19)$$

Then noting that the magnetic moment is $\mu^{k'} = \frac{1}{2}\varepsilon^{k'}{}_{ij}\int r^i j^j d^3\Sigma$, eq. (19) takes the form

$$\frac{q}{m_0} L^i = \mu^i \qquad (20)$$

where we have replaced the index k' by i. This is the relation for $g = 2$.

IV. Non-Relativistic Rigid Charge Distribution and Self-Field Contribution

To see the effect of the constraint eq. (13) using a specific current distribution, consider a non-relativistic rigid current distribution of the form

$$j^i(x^i) = \sigma(r)(v^i + \varepsilon^i{}_{jk}\omega^j r^k)$$

where $\sigma(r)$ is the charge distribution, v^i the velocity and ω^j the angular velocity. In the rest frame the charge and current distributions are

$$j^0(x^i) = \sigma(r)$$

$$j^i(x^i) = \sigma(r)\varepsilon^i{}_{jk}\omega^j r^k$$

with the rest frame derivatives

$$\frac{dj^i}{dt} = \sigma(r)a^i$$

$$\frac{d^2 j^i}{dt^2} = -\varepsilon^i{}_{jk}(x^k a^m \frac{\partial\sigma(r)}{\partial x^m} + \sigma(r)a^k)\omega^j$$

and $\frac{d^n j^i}{dt^n} = 0$ for n greater than 2.

We have set $x^\beta{}_0$ equal to zero, $r = |r^i|$, and have neglected non-linear terms in the acceleration. We have also neglected derivatives of the acceleration and derivatives of the angular velocity.

If we insert the rigid charge and current distributions into eqs. (12) and (13), they become

$$m_0 a^i = \int \sigma(r)(1 + a_m r^m)\{F^i{}_0 + F^i{}_j\varepsilon^j{}_{kl}\omega^k r^l\}d^3\Sigma \tag{21}$$

$$a_j L^{ij} = \int \sigma(r)(1 + a_m r^m)r^i F^0{}_k\varepsilon^k{}_{jl}\omega^j r^l d^3\Sigma. \tag{22}$$

To include the self-field terms, consider the self-fields of the particle. From Jackson[21] and Crisp[22] for a general charge distribution, the electric and magnetic fields take the form

$$E^i = \int j^0(x^i{}')\frac{R^i}{R^3}d^3\Sigma'$$

$$-\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+2)}\frac{\partial^{n+1}}{\partial t^{n+1}}\int R^{n-1}[-(n-1)\frac{R^i}{R^2}j_k(x^i{}',t)R^k + (n+1)j^i(x^i{}',t)]d^3\Sigma' \qquad (23)$$

$$B^i = -\int \varepsilon^i{}_{jk}\frac{R^j}{R^3}j^k(x^i{}')d^3\Sigma' + \varepsilon^i{}_{jk}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+2)}\frac{\partial^{n+2}}{\partial t^{n+2}}\int R^{n-1}R^j j^k(x^i{}',t)d^3\Sigma' \qquad (24)$$

with $R^i = r^i - r^i{}'$, and $R = |R^i|$.

Using eqs. (23) and (24) in the right side of eqs. (21) and (22) and using the time derivatives of the current density, we find

$$\int \sigma(r)(1+a_m r^m)\{F^i{}_0 + F^i{}_j\varepsilon^j{}_{kl}\omega^k r^l\}d^3\Sigma$$

$$= -\frac{1}{2}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{a^i}{R} + \frac{2}{3}\omega^i a_m\omega^m\frac{x_j x^j{}'}{R}\} \qquad (25)$$

$$\int \sigma(r)(1+a_m r^m)r^i F^0{}_k\varepsilon^k{}_{jl}\omega^j r^l d^3\Sigma$$

$$= \frac{1}{6}\varepsilon^i{}_{jk}\omega^j a^k\int dv\sigma(r)\int dv'\sigma(r')\{\frac{x_m x^m{}' + x_m x^m}{R}\} \qquad (26)$$

where we have used $F^i{}_0 = E^i$, $F^i{}_j = \varepsilon^i{}_{jk}B^k$, the spherical symmetry of the charge distribution, and have neglected non-linear terms in the acceleration. Since we are not including the derivatives of the acceleration, eqs. (25) and (26) do not include the radiation reaction terms.

Using eqs. (25) and (26) in eqs. (21) and (22) and including the external fields, eqs. (21) and (22) become

$$(m_0 + \frac{1}{2}\int dv\sigma(r)\int dv'\sigma(r')\frac{1}{R})a^i = -\frac{1}{3}\int dv\sigma(r)\int dv'\sigma(r')\{\omega^i a_m \omega^m \frac{x_j x^{j\,\prime}}{R}\}$$

$$+\int \sigma(r)(1+a_m r^m)\{F^i{}_0 + F^i{}_j \varepsilon^j{}_{kl}\omega^k r^l\}d^3\Sigma \qquad (27)$$

$$a_j \varepsilon^{ij}{}_k \omega^k \{I_0 + \frac{2}{3}\int dv\sigma(r)\int dv'\sigma(r')\frac{x_m x^{m\,\prime}}{R}\} = -\frac{1}{6}\varepsilon^i{}_{jk}\omega^k a^j \int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^{m\,\prime} + x_m x^m}{R}\}$$

$$+\int \sigma(r)(1+a_m r^m)r^i F^0{}_k \varepsilon^k{}_{jl}\omega^j r^l d^3\Sigma \qquad (28)$$

where now $F^i{}_0$ and $F^i{}_j$ are the external fields, and we have set $L^{ij} = I_0 \varepsilon^{ij}{}_k \omega^k$, I_0 as the mechanical moment of inertia. From Crisp[22] the electromagnetic contribution to the moment of inertia of a spherically symmetric rotating object is

$$I_{em} = \frac{2}{3}\int dv\sigma(r)\int dv'\sigma(r')\frac{x_m x^{m\,\prime}}{R}$$

and rotational electromagnetic contribution to the mass is $\frac{1}{2}I_{em}\omega^2$. From Jackson[21] the electrostatic contribution to the mass is $\frac{1}{2}\int dv\sigma(r)\int dv'\sigma(r')\frac{1}{R}$. The total electromagnetic contribution to the mass is then

$$m_{em} = \frac{1}{2}(\int dv\sigma(r)\int dv'\sigma(r')\frac{1}{R} + I_{em}\omega^2)$$

which we obtain for the case of a^i in the same direction as ω^i, but not in general. We also obtain the electromagnetic moment of inertia plus the term

$$\frac{1}{6}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^m{}' + x_m x^m}{R}\}.$$

which is zero in the case of a spherically symmetric charged shell since
in that case we have

$$\int dv\sigma(r)\int dv'\sigma(r')\frac{x_m x^m}{R} = 3\int dv\sigma(r)\int dv'\sigma(r')\frac{x_m x^m{}'}{R}$$

IV. g=2 Calculation

Now reduce the size of the particle and take the external fields to be slowly varying so that they can be considered as constant over the size of the particle. Also take the electric and magnetic fields to be of the same order as the acceleration, ignoring second order acceleration terms and using the spherical symmetry of the charge. eqs. (27) and (28) then become

$$ma^i = qF^i{}_0 - \frac{1}{3}\omega^i a_m \omega^m \int dv\sigma(r)\int dv'\sigma(r')\frac{x_j x^j{}'}{R} \qquad (29)$$

$$a_j \varepsilon^{ij}{}_k \omega^k \{I + \frac{1}{6}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^m{}' + x_m x^m}{R}\}\} = F^0{}_k \int \sigma(r) r^i \varepsilon^k{}_{jl}\omega^j r^l d^3\Sigma \qquad (30)$$

where now

$$m = m_0 + \frac{1}{2}\int dv\sigma(r)\int dv'\sigma(r')\frac{1}{R}$$

$$I = I_0 + \frac{2}{3}\int dv\sigma(r)\int dv'\sigma(r')\frac{x_m x^m{}'}{R}$$

while noting that

$$q = \int j^0 d^3\Sigma = \int \sigma(r) d^3\Sigma$$

If we now multiply eq. (29) by $\varepsilon_{ijk}\omega^k$ we obtain $a^i \varepsilon_{ijk}\omega^k = \frac{q}{m}F^i{}_0 \varepsilon_{ijk}\omega^k$, which when substituted into eq. (30) yields

$$\frac{q}{m}F^j{}_0 \varepsilon_{jik}\omega^k \{I + \frac{1}{6}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^m{}' + x_m x^m}{R}\}\} = -F^0{}_k \int \sigma(r) r^i \varepsilon^k{}_{jl}\omega^j r^l d^3\Sigma$$

Now $F^0{}_k = \delta_{kj}F^j{}_0$, so for arbitrary $F^j{}_0$ and ω^k this relation becomes

$$\frac{q}{m}\varepsilon_{jik}\{I+\frac{1}{6}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^m{}'+x_m x^m}{R}\}\}=-\int\sigma(r)r^i\varepsilon_{jkl}r^l d^3\Sigma$$

$$=-\frac{1}{3}\varepsilon_{jkl}\delta^{il}\int\sigma(r)r^2d^3\Sigma=\frac{1}{3}\varepsilon_{jik}\int\sigma(r)r^2d^3\Sigma$$

again using the spherical symmetry of the charge distribution. Thus

$$\frac{q}{m}\{I+\frac{1}{6}\int dv\sigma(r)\int dv'\sigma(r')\{\frac{-3x_m x^m{}'+x_m x^m}{R}\}\}=\frac{1}{3}\int\sigma(r)r^2d^3\Sigma \tag{31}$$

If we have a spherical shell of charge this becomes

$$\frac{q}{m}I=\frac{1}{3}\int\sigma(r)r^2d^3\Sigma \tag{32}$$

For other charge distributions if the mechanical angular momentum is much greater than the electromagnetic angular momentum we can approximate I by I_0 in the eq. (32). Also, if the angular velocity is small, m will approximate the total electromagnetic and mechanical mass. In this case, multiplying eq. (32) by ω^i yields

$$\frac{q}{m}I\omega^i=\frac{q}{m}L^i=\frac{1}{3}\omega^i\int\sigma(r)r^2d^3\Sigma.$$

where L^i is the angular momentum. The magnetic moment is

$$\mu^i = \frac{1}{2}\varepsilon^i{}_{jk}\int r^j j^k d^3\Sigma = \frac{1}{3}\omega^i\int \sigma(r) r^2 d^3\Sigma$$

using our relation for the current and the spherical symmetry of the charge distribution. So we have

$$\frac{q}{m} L^i = \mu^i$$

which is the relation for $g = 2$ (for example see Singh and Raghuvanshi[1])

V. Conclusions

The center of rotation and center of mass are taken to be on the world line of the particle. That requirement together with the requirement that the mechanical momentum be zero in the rest frame leads to a constraint equation which, as the size of the particle is reduced and the self-fields due to rotation are neglected, reduces to the requirement that $g = 2$.

We have neglected some of the self-field terms in this derivation, and neglected nonlinear terms in the acceleration and some higher order derivatives. In spite of this it appears that in certain situations $g = 2$ is a requirement for a classical spinning charged particle when relativity is taken into account. This fact along with the fact that $g = 2$ for a spinning black hole appear to indicate that $g = 2$ is a classical effect due to relativity and not an effect due to quantum mechanics.

Appendix

This is a derivation of eq. (9). If we integrate eq. (2) over the 4-space within the 3-surface, eq. (2) takes the form

$$\int d^4x M^{\alpha\beta\gamma}{}_{,\gamma} = -u_\gamma(\tau+\delta\tau)\int_{top} M^{\alpha\beta\gamma}\, d^3\Sigma + u_\gamma(\tau)\int_{bottom} M^{\alpha\beta\gamma}\, d^3\Sigma$$

$$= \int d^4x\{\left(x^\alpha - x^\alpha{}_0(\tau)\right)F^\beta{}_\gamma j^\gamma - \left(x^\beta - x^\beta{}_0(\tau)\right)F^\alpha{}_\gamma j^\gamma\} \qquad \text{(A1)}$$

Multiply eq. (A1) by $u_\alpha(\tau)e^i{}_\beta(\tau)$ where $e^i{}_\beta(\tau)$ are the spatial vectors normal to $u_\alpha(\tau)$ at the bottom rest frame. Eq. (A1) then becomes

$$-u_\alpha(\tau)e^i{}_\beta(\tau)u_\gamma(\tau+\delta\tau)\int_{top} M^{\alpha\beta\gamma}\, d^3\Sigma + u_\alpha(\tau)e^i{}_\beta(\tau)u_\gamma(\tau)\int_{bottom} M^{\alpha\beta\gamma}\, d^3\Sigma$$

$$= u_\alpha(\tau)e^i{}_\beta(\tau)\int d^4x\{\left(x^\alpha - x^\alpha{}_0(\tau)\right)F^\beta{}_\gamma j^\gamma - \left(x^\beta - x^\beta{}_0(\tau)\right)F^\alpha{}_\gamma j^\gamma\} \qquad \text{(A2)}$$

Consider the two rest frames, the bottom frame the unprimed frame and the top frame the primed frame. Look at the first term in eq. (A2) in terms of the primed coordinates and the second and third terms in the unprimed coordinates so that eq. (A2) becomes

$$u_{\alpha'}(\tau)e^i{}_{\beta'}(\tau)\int_{top} M^{\alpha'\beta'0'}\, d^3\Sigma + e^i{}_\beta(\tau)\int_{bottom} M^{0\beta0}\, d^3\Sigma$$

$$= -e^i{}_\beta(\tau)\int d^4x\{\left(x^0 - x^0{}_0(\tau)\right)F^\beta{}_\gamma j^\gamma - \left(x^\beta - x^\beta{}_0(\tau)\right)F^0{}_\gamma j^\gamma\} \qquad \text{(A3)}$$

where a primed subscript indicates it is in the primed system.

Then using a Lorentz transformation we have to order $\delta\tau$

$$u_{0'}(\tau) = -1\ , \quad u_{i'}(\tau) = -a_i\delta\tau\ , \quad e^l{}_{0'}(\tau) = a^l\delta\tau\ , \quad e^l{}_{k'}(\tau) = \delta^l{}_k$$

along with $e^l{}_0(\tau) = 0$ and $e^l{}_k(\tau) = \delta^l{}_k$. Using these in eq. (A3) it becomes

$$-\int_{top} M^{0'i'0'}\, d^3\Sigma - a_j\delta\tau\int_{top} M^{j'i'0'}\, d^3\Sigma + \int_{bottom} M^{0i0}\, d^3\Sigma$$
$$= -\int d^4x\{\left(x^0 - x^0{}_0(\tau)\right)F^i{}_\gamma j^\gamma - \left(x^i - x^i{}_0(\tau)\right)F^0{}_\gamma j^\gamma\} \qquad \text{(A4)}$$

using $M^{00\gamma} = 0$ and ignoring second order $\delta\tau$ terms. The 4-space integral on the left takes the form

$$\int d^4x\left\{\left(x^0 - x^0{}_0(\tau)\right)F^i{}_\gamma j^\gamma - \left(x^i - x^i{}_0(\tau)\right)F^0{}_\gamma j^\gamma\right\}$$
$$= \delta\tau\int_{bottom}(1 + a_k r^k) r^i F^0{}_\gamma j^\gamma\, d^3\Sigma \qquad \text{(A5)}$$

Ignoring second order $\delta\tau$ terms, using $r^i = x^i - x^i{}_0(\tau)$, and the fact that to first order in $\delta\tau$

$$\int d^4x = \delta\tau\int_{bottom}(1 + a_k r^k)\, d^3\Sigma$$

Now on the bottom slice

$$M^{0i0} = \left(x^0 - x^0{}_0(\tau)\right)S^{i0} - \left(x^i - x^i{}_0(\tau)\right)S^{00} = -\left(x^i - x^i{}_0(\tau)\right)S^{00} = -r^i S^{00} \qquad \text{(A6)}$$

and on the top slice

$$M^{0'i'0'} = \left(x^{0'} - x^{0'}_0(\tau)\right) S^{i'0'} - \left(x^{i'} - x^{i'}_0(\tau)\right) S^{0'0'}$$

$$= u^{0'}(\tau)\delta\tau S^{i'0'} - \left(x^{i'} - x^{i'}_0(\tau+\delta\tau) + u^{i'}(\tau)\delta\tau\right) S^{0'0'}$$

$$= \delta\tau S^{i'0'} - (r^{i'} - a_i\delta\tau\delta\tau) S^{0'0'} \qquad \text{(A7)}$$

using $x^{\alpha'}_0(\tau+\delta\tau) = x^{\alpha'}_0(\tau) + u^{\alpha'}(\tau)\delta\tau$.

Using $L^{ij} = \int M^{ij0} d^3\Sigma$ and eqs. (A5-A7) eq. (A4) becomes

$$-\int_{top} \{\delta\tau S^{i'0'} - r^{i'} S^{0'0'}\} d^3\Sigma - a_j \delta\tau L^{ji} - \int_{bottom} r^i S^{00}\, d^3\Sigma$$

$$= \delta\tau \int_{bottom} (1 + a_k r^k) r^i F^0{}_\gamma j^\gamma\, d^3\Sigma \qquad \text{(A8)}$$

Ignoring second order $\delta\tau$ terms.

Dividing by $\delta\tau$ and taking the limit as $\delta\tau$ goes to zero, eq. (A8) becomes eq. (9) using the fact that $L^{ji} = -L^{ij}$ and that $\int_{top} S^{i'0'}\, d^3\Sigma = \int_{bottom} S^{i0}\, d^3\Sigma$ to order $\delta\tau$.

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