George Bailey
Berlin, Germany
baileygeorgebln@gmail.com

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Uncovering the link between the proton-electron mass ratio and the fine structure constant through reflections on the proton radius

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Abstract

In the previous work, we established a connection between the age of the universe, or the resulting Hubble radius, and the proton radius. Thereby we already noticed that the dimensionless fine structure constant α (`1/137.036`) is a link of several numerical correlations of the atomic world and the universe.

And we also noticed that the delta value `1.00602` (we will call it δ_H hereafter) not only arises, if one considers the ratio from the mass square of the hydrogen atom by the product of electron mass and proton mass ($m_H^2/(m_p m_e)$ and divides this value by `10/3 * 4/alpha`, but also arises in other contexts, if one assumes that the ratio between electromagnetic force and gravitational force between proton and electron exactly corresponds to the value `20/3 * 2^128` and makes for this a minimal adjustment of the currently accepted value of the gravitational constant.

Based on the consideration that 3/10 of the classical electron radius r_e would be the physically more sensible electron radius and that this radius is only a touch away from the proton radius r_p - and that this small difference between $3/10 * r_e$ and r_p can be described by δ_H, we will derive two very similar equations for determining m_p/m_e, both only dependent on alpha and of the form:

$$\frac{m_p}{m_e} = \frac{10}{3} \cdot \frac{4}{\alpha} + 9 + x(\alpha)$$

- where x(α) in both cases is a value of the order of δ_H -1 = 0.00602

With ***1836.15267*446** and ***1836.15267*148** they deliver values that correspond to 9 digits with the current CODATA value for the ratio m_p/m_e and, with this accuracy, are even within the range of the current precision measurements for the fine structure constant alpha.

There can be only one - only one proton radius

In the previous work [1] we have concluded from considerations of a general energy uncertainty, which results from the finite age of the universe, on an equation for the determination of the proton radius:

$$r_p = \frac{4 \cdot \hbar \cdot m_p}{c \cdot m_H^2} \qquad (1)$$

ħ: reduced Planck constant = 1.0545718·10^-34 J·s

c: speed of light in vacuum = 299792458 m/s

m_p: mass of proton = 1,6726·10^-27 kg

$m_H = m_p + m_e$

with m_e: mass of electron = 9.109·10^-31 kg

This equation returns a value of r_p = **8.403· 10^{-16} m**. Already in [1] we had remarked that this equation can be simplified, if we assume that the mass of the hydrogen atom is equal to the mass of the proton:

$$r'_p = \frac{4 \cdot \hbar}{c \cdot m_p} \qquad (2)$$

This returns a value of r'_p = **8.412· 10^{-16} m**. Other authors have also come across this simplified equation, e.g. Gupta [2] and Rohrbaugh (via Rydberg Constant) [3].

The values from both equations differ only slightly, but it is necessary to understand this difference correctly if one tries to derive connections in fundamental physics from numerical matches.

The simplified equation actually seems physically more obvious:

- Only the mass of the proton determines its radius - and not its mass added with the mass of the electron.

- The simplified equation spits out a radius value that is closer to the values measured experimentally in 2010 and 2013 at the Swiss Paul Scherrer Institute, i.e. in those experiments that triggered the so-called "proton radius puzzle"[4][5]: 0.84184 fm in 2010 and 0.84087 fm in 2013. This does not mean much, since the error tolerance of these experiments would also accept the somewhat smaller value of 0.8403 fm - but when asked whether 0.8403 fm or 0.8412 fm is the true value, one would rather opt for the latter in view of these results.
- And this simplified radius has a third advantage: It connects in a simple way other well known quantities of the atomic world. In particular, the electron's (reduced) Compton wavelength, which is a measure for the rest energy of the electron, is related to the simplified proton radius like the mass ratio of electron and proton:

$$r'_p = 4 \cdot \lambda\!\!\!^{-}_c(e) \cdot \frac{m_e}{m_p} \tag{3}$$

$\lambda\!\!\!^{-}_c(e)$: reduced Compton wavelength of electron = 1,6726·10^-27 kg

Other interesting connections:

$$r'_p = 4\alpha \cdot \frac{m_e}{m_p} \cdot r_{Bohr} \tag{4}$$

α: fine structure constant = 1/137.035999

r_{Bohr}: Bohr radius = 5.29177·10^−11 m

$$r'_p = \frac{4}{m_p} \cdot l_{Planck} \cdot m_{Planck} \tag{5}$$

l_{Planck}: Planck length = 1.616255·10−35m

m_{Planck}: Planck mass = 2.1764·10−8 kg

On the other hand, there is the somewhat smaller value, which we continue to think results conclusively from the finite age of the universe and the thus general energy uncertainty of the universe:

$$r_p = \frac{2 \cdot G m_e m_p \cdot T_u}{\hbar} \tag{6}$$

G: gravitational constant = 6.6743·10−11 m³ / kg·s²

*T_U: a*ssumed age of the universe = 13.807 billion years

The whole could be solved, if we would assume that the age of the universe would not be connected with the mass of the hydrogen atom but only of the proton:

$$T_u = \frac{2 \cdot \hbar^2}{m_e c \cdot G \cdot m_H^2} = 13.807 Gyr$$

(7)

$$T_u ? = \frac{2 \cdot \hbar^2}{m_e c \cdot G \cdot m_p^2} = 13.822 Gyr ?$$

But this would contradict the intuitive approach which we have presented in our previous work. Furthermore, it would contradict the rather accurate final results of the Planck Space Telescope published in 2018 [6], which rather point to a universe age slightly below 13.8 billion years. 13.807 billion years would still be in the tolerance range, 13.822 Gyr no more.

The more realistic electron radius points to two delta values

To better understand this small mismatch, let's now see how these two slightly different proton radius values can be related to the classical electron radius r_e. What is noticeable is that they are somewhat smaller than 3/10 of r_e:

$$r_e = \frac{10}{3} \cdot 1.006024 \cdot r_p$$

$$r_e = \frac{10}{3} \cdot 1.004929 \cdot r_p' \qquad (8)$$

r_e: classical electron radius = 2,8179·10^-15 m

This factor 3/10 (or its reciprocal 10/3) we already met in the previous work and it can be justified physically well: The classical electron radius describes a sphere which is so large that the rest energy of the electron corresponds to the electric field energy of this sphere, but the charge is distributed only on the outer edge of the sphere. If one assumes that the charge is distributed evenly throughout the sphere, then the radius decreases by a factor of 3/5. And if one then considers that an electron (or positron) can only be brought to the size of such a radius if it is moving at nearly the speed of light, then the virial theorem comes into play, which states for Coulomb fields that the potential energy must be twice the total energy so that it can compensate for the kinetic energy of the same amount, but with the opposite sign. To do this, the radius of the charge source must be halved.

With these two additional considerations to the classical electron radius, we compress this by the factor 10/3 and land already very close to our two proton radius values.

How close this „compressed“ electron radius comes can be defined by two delta values, both of which are quite close to 1. We want to call the delta value for the proton radius originally determined by us δ_H, because the equation derived for it contains the mass of the hydrogen atom:

$$\delta_H = \frac{3}{10} \cdot \frac{r_e}{r_p} = \frac{3}{10} \cdot \frac{\alpha}{4} \cdot \frac{(m_p + m_e)^2}{m_p \cdot m_e} = 1.006024 \qquad (9)$$

The delta value for the simplified proton radius, on the other hand, we call δ_p, because in the equation simplified for it then only the proton mass occurs:

$$\delta_p = \frac{3}{10} \cdot \frac{r_e}{r'_p} = \frac{3}{10} \cdot \frac{\alpha}{4} \cdot \frac{m_p}{m_e} = 1.004929 \tag{10}$$

Almost hit the bull's eye from the left side...

Let's now take a close look at these two delta values δ_p and δ_H.

As already mentioned, an electron which should be compressed to the classical electron radius or even smaller, e.g. to the proton radius, would have to move at nearly the speed of light with a considerable time dilation factor gamma.
For masses m compressed to radii r with c << ħ/(2mr) the following applies:

$$\gamma \cdot c = \frac{\hbar}{2 \cdot m \cdot r} \tag{11}$$

At 3/10 of the classical electron radius, which we have identified as the more "realistic" electron radius, according to (11) the gamma factor would be exactly 5/3 * 1/α. For the simplified proton radius r'_p the gamma factor would be 5/3 * 1/α * δ_p.
And the eightfold value of this gamma factor is the mass ratio of proton and electron:

$$8 \cdot \frac{5}{3 \cdot \alpha} \cdot \delta_p = \frac{m_p}{m_e} \tag{12}$$

This is already an interesting insight: Eight electrons (or positrons), compressed to the size of the simplified proton radius, bring the mass of a proton on the balance!

And it gets even more interesting. Because now let's see if there is another way of bridging this gap between the gamma factors 5/3*1/α and 5/3*1/ α*δ_p, ideally even with our other delta value δ_H?

And lo and behold:

$$\frac{5}{3 \cdot \alpha} + 1 + \frac{\delta_H}{8} \approx \frac{5}{3 \cdot \alpha} \cdot \delta_p \qquad (13)$$

or rearranged:

$$1 + \frac{3 \cdot \alpha}{5} + \frac{\delta_H \cdot 3\alpha}{40} = 1{,}0049290099 \approx \delta_p \qquad (14)$$

The exact value of δ_p is **1.00492900718** with the current CODATA-2018 values of α (1/137.035 999 084) and m_p/m_e (1836.15267343). That's damn close to hitting the bull's eye. So close that an exact calculation of this approach is worthwhile.

So we have two related equations. In following equation (II) we consider the approximation found in (13) as an equation for the sake of simplification:

$$\begin{array}{ll} \text{(I)} & 8 \cdot \frac{5}{3 \cdot \alpha} \cdot \delta_H = \frac{(m_p + m_e)^2}{m_p \cdot m_e} = \frac{m_p}{m_e} + 2 + \frac{m_e}{m_p} \\ \text{(II)} & \frac{5}{3 \cdot \alpha} + 1 + \frac{\delta_H}{8} = \frac{1}{8} \cdot \frac{m_p}{m_e} \end{array} \qquad (15)$$

(I) rearranged to δ_H and inserted in (II):

$$\frac{5}{3 \cdot \alpha} + 1 + \frac{1}{8} \cdot \frac{3 \cdot \alpha \cdot (\frac{m_p}{m_e} + 2 + \frac{m_e}{m_p})}{40} = \frac{1}{8} \cdot \frac{m_p}{m_e} \qquad (16)$$

For easier further rearrangements we substitute: x := 1/α and y := m_p/m_e :

$$\frac{5x}{3} + 1 + \frac{1}{8} \cdot \frac{3 \cdot (y + 2 + \frac{1}{y})}{40x} = \frac{y}{8} \qquad (17.1)$$

Rearranged:

$$(-40x + 3)y^2 + (\frac{1}{3} \cdot 1600x^2 + 320x + 6) \cdot y + 3 = 0 \qquad (17.2)$$

Further conversion using polynomial division:

$$y^2 - (\frac{40x}{3} - 9 + \frac{33}{-40x + 3}) \cdot y - \frac{3}{40x + 3} = 0 \qquad (17.3)$$

Solving quadratic equation:

$$y = \frac{1}{2} \cdot (\frac{40x}{3} + 9 + \frac{33}{40x + 3}) + \sqrt{\frac{1}{4} \cdot (\frac{40x}{3} + 9 + \frac{33}{40x + 3})^2 + \frac{3}{40x + 3}} \qquad (17.4)$$

Since

$$\frac{1}{4} \cdot (\frac{40x}{3} + 9 + \frac{33}{40x + 3})^2 \gg \frac{3}{40x + 3}$$

we can simplify:

$$y \approx \frac{40x}{3} + 9 + \frac{33}{40x + 3} \qquad (17.5)$$

and finally back-substituted:

$$\boxed{\frac{m_p}{m_e} \approx \frac{40}{3 \cdot \alpha} + 9 + \frac{33}{40 \cdot \alpha^{-1} + 3}} \qquad (18)$$

With the current CODATA value for α (1/137.035 999 084), this gives us a value for m_p/m_e of **1836.15267*148***. Since we saw early on that we do not have 100% agreement with our approximation in (13), we are not surprised that we do not within the tolerance range of the current CODATA value for m_p/m_e =**1836.15267*343(11)***. But we're so close that we're now certain that there must be an equation in exactly this form that expresses m_p/m_e only as a function of alpha.

...and now from the right side

Let's take a closer look at the direct relation of δ_p and δ_H to α. What first strikes you is that the two delta values subtracted by 1 are of the same order of magnitude as alpha. And especially with $\alpha/2$ an interesting numerical constellation results:

$$\frac{2 \cdot (1-\delta_p)}{\alpha} = 1.350902848 \qquad (19)$$

$$\frac{2 \cdot (1-\delta_H)}{\alpha} = 1.65098454 \qquad (20)$$

In both relations, a value close to a multiple of 1/20 is noticeable and that the difference between the two relation values corresponds to about 0.3 = 3/10. So 1.35 is 27/20 and 1.65 is 33/20. *This means that δ_p is roughly 27/40 times smaller than alpha and δ_H is about 33/40 smaller than alpha by the same consideration.* And yes - this relation value 33/40 * α corresponds almost to the last term of our approximation equation in the last section: ... + 33/(40*α^{-1}+3)

At this point we have to say that we are not enthusiastic supporters of the Standard Model with its quarks. After all, it is a conceptual model in which the elementary charge is further chopped up and the carriers of this chopped up charge do not even reveal themselves as free particles. But (unfortunately) factors like 27/20=1.35 would probably explain better with a chopped elementary charge than without. For example, according to this model, the so-called up quarks form the positively charged components in a proton. And the electromagnetic force between two up quarks is smaller by a factor of 4/9 (2/3 * 2/3) or 12/27 than between two electrons/positrons - which would result in the reciprocal value 20/27 when considering the pre-factor 5/3 explained above: 4/9 * 5/3 = 20/27. However, we continue to hope that these numerical matches can also be explained with a more economical model than the quark model.

But now let's take a closer look at the small numerical difference that lies in equation (19) between 1.35 and the precisely calculated value 1.350902848. And then it is noticed that:

$$\frac{0.35}{0.000902848} \approx \sqrt{8} \cdot \alpha^{-1} \qquad (21)$$

with a deviation of less than 0.017% with the current CODATA value for alpha.
Together with (19) and rearranged for δ_p, this results in:

$$\delta_p - 1 \approx ((\frac{\alpha}{\sqrt{8}} + 1) \cdot 0.35 + 1) \cdot \frac{\alpha}{2} \qquad (22)$$

With 0.35 = 7/20, we can this rearrange to:

$$\delta_p \approx 1 + \frac{\alpha}{2} + \frac{7\alpha}{40} + \frac{7\alpha^2}{40 \cdot \sqrt{8}} \qquad (23)$$

With the definition of δ_p in (10) we get:

$$\frac{m_p}{m_e} \approx \frac{40}{3\alpha} \cdot (1 + \frac{\alpha}{2} + \frac{7\alpha}{40} + \frac{7\alpha^2}{40 \cdot \sqrt{8}}) \qquad (24)$$

and finally rearranged:

$$\frac{m_p}{m_e} \approx \frac{40}{3\alpha} + 9 + \frac{7\alpha}{3 \cdot \sqrt{8}} \qquad (25)$$

With the current CODATA value for α (1/137.035 999 084), this gives us a value for m_p/m_e of **1836.15267*446***. Just like with the approximation approach from the previous section - see equation (18) - we determine a value that corresponds to the current CODA TA value of m_p/m_e - **1836.15267*343(11)*** - to within 9 digits, but is again outside the tolerance range specified by CODATA. In contrast to (18), however, the approximate value is now above this tolerance range – a near hit from the right side.

Close look at the two dart arrows

Using equations (18) and (25), we have two approximations, both of which yield a value for m_p/m_e that matches the current CODATA value to within 9 digits. While (18) gives a value slightly below this reference value, (25) gives a value slightly above.

Both equations are of the form: 40/(3 α) + 9 + x(α). The difference is only in the third term x(α). In the first approach, this third term is $33/(40\alpha^{-1}+3)$, in the second approach it is 7·α/(3sqrt(8)). It is interesting that the values of both terms are within sight of δ_H-1:

$$\frac{7\alpha}{3 \cdot \sqrt{8}} = 0.006020009$$

$$\frac{33}{40\alpha^{-1} + 3} = 0.00601702$$

$$\delta_H - 1 = 0.0060239$$

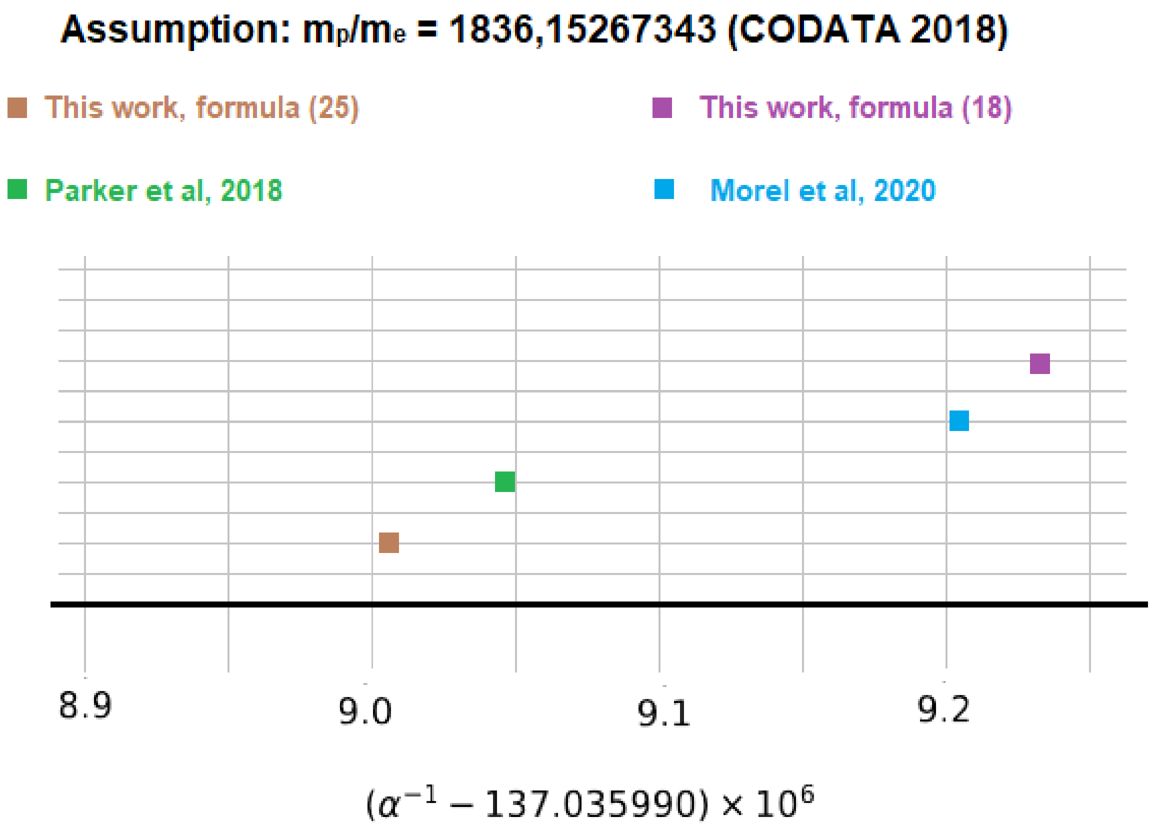
With the current CODATA values for alpha, the second approximation of (25) is slightly closer to the CODATA specified value for m_p/m_e than the first in equation (18). But that could change with the next CODATA release. Both approximations are in the field of tension of the last two precision measurement experiments of the fine structure constant alpha: First, there is the experiment published in 2018 by Parker et al from the University of California [7]. It was published with the key message: "the most accurate measurement of the fine-structure constant to date: $\alpha = 1/137.035999046(27)$ at 2.0×10^{-10} accuracy." This experiment appears to have been instrumental in CODATA setting the current value for alpha below 1/137.0359991.

And then there's the experiment published in 2020 by Morel et al. of the Kastler Brossel Laboratory in Paris [8][9], which headlined "determine the fine-structure constant $\alpha^{-1}=137.035999206(11)$ with a relative accuracy of 81 parts per trillion." This experiment could result in the next CODATA value for α^{-1} being well over 137.0359991.

The reported accuracy of both experiments is impressive. Unfortunately, it means that the results of the two high-precision measurements are initially incompatible. And one can get the impression that the two teams approached the accuracy with a little too much self-confidence.

Now, if we make the assumption that the CODATA value for $m_p/m_e=1836.15267344$ is a safe bet up to the last digit (which is by no means the case, just look at the development of the m_p/m_e-CODATA values over the last 20 years), then the first experiment by Parker et al. is more consistent with our second approximation from (25). With the fixed m_p/m_e assumption, we arrive at a value for α^{-1} of 137.0359990065.

On the other hand, with the first approximation of (18) we are closer to the results of Morel et al. With the same assumption, the α^{-1} -value is 137.03599923054. The graphic below illustrates this situation. So we look forward to the upcoming high-precision experiments to determine alpha.



Outlook

Whether r_p = 0.8403 fm or r'_p = 0.8412 fm is the true proton radius - we cannot yet clarify - but we have now become more certain that these two radii are even more closely interwoven than originally thought. And if r'_p is the actual radius of the proton, then this should be directly derivable from the minimally smaller "fundamental radius" r_p, which in turn can be directly derived from the minimal energy uncertainty of the universe. We hope to find out more about this in future work.

Above all, we are more convinced than ever with our two approximations that the proton-electron mass ratio can be expressed solely with the fine structure constant - and thus also shown in this way that if the fine structure constant is a time-invariant quantity, then that is also the case for the mass ratio of proton to electron.

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