Introduction to the n-SuperHyperGraph - the most general form of graph today

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Abstract: We recall and improve our 2019 and 2020 concepts of *n-SuperHyperGraph, Plithogenic n-SuperHyperGraph, n-Power Set of a Set*, and we present some application from the real world. The n-SuperHyperGraph is the most general form of graph today and it is able to describe the complex reality we live in, by using n-SuperVertices (groups of groups of groups etc.) and n-SuperHyperEdges (edges connecting groups of groups of groups etc.).

Keywords: n-SuperHyperGraph (n-SHG), n-SHG-vertex, n-SHG-edge, Plithogenic (Crisp, Fuzzy, Intuitionistic Fuzzy, Neutrosophic, etc.) n-SuperHyperGraph, n-Power Set of a Set, MultiEdge, Loop, Indeterminate Vertex, Null Vertex, Indeterminate Edge, Null Edge, Neutrosophic Directed Graph

1. Definition of the n-SuperHyperGraph

Let $V = \{v_1, v_2, \dots, v_m\}$, for $1 \le m \le \infty$, be a set of vertices, that contains Single Vertices (the classical ones), Indeterminate Vertices (unclear, vague, partially known), and Null Vertices (totally unknown, empty).

Let $P(V)$ pe the power of set V, that includes the empty set ϕ too.

Then $P^n(V)$ be the n-power set of the set V, defined in a recurent way, i.e.:

P(V), $P^2(V) = P(P(V))$, $P^3(V) = P(P^2(V)) = P\left(P(P(V))\right)$, …,

$P^n(V) = P(P^{n-1}(V))$, for $1 \le n \le \infty$, where by definition $P^0(V) \overset{def}{=} V$.

Then, the **n-SuperHyperGraph (n-SHG)** is an ordered pair:

$$\text{n-SHG} = (G_n, E_n),$$

where $G_n \subseteq P^n(V)$, and $E_n \subseteq P^n(V)$, for $1 \le n \le \infty$.

G_n is the set of vertices, and E_n is the set of edges.

The set of vertices G_n contains the following types of vertices:

- Singles Vertices (the classical ones);
- Indeterminate Vertices (unclear, vagues, partially unkwnown);
- Null Vertices (totally unknown, empty);

and:

- SuperVertex (or SubsetVertex), i.e. two ore more (single, indeterminate, or null) vertices put together as a group (organization).
- n-SuperVertex that is a collection of many vertices such that at least one is a $(n-1)$-SuperVertex and all other r-SuperVertices into the collection, if any, have the order $r \le n-1$.
- The set of edges E_n contains the following types of edges:
- Singles Edges (the classical ones);

- Indeterminate Edges (unclear, vagues, partially unkwnown);
- Null Edges (totally unknown, empty);

and:

- HyperEdge (connecting three or more single vertices);
- SuperEdge (connecting two vertices, at least one of them being a SuperVertex);
- n-SuperEdge (connecting two vertices, at least one being a n-SuperVertex, and the other of order r-SuperVertex, with $r \leq n$);
- SuperHyperEdge (connecting three or more vertices, at least one being a SuperVertex);
- n-SuperHyperEdge (connecting three or more vertices, at least one being a n-SuperVertex, and the other r-SuperVertices with $r \leq n$;
- MultiEdges (two or more edges connecting the same two vertices);
- Loop (and edge that connects an element with itself).

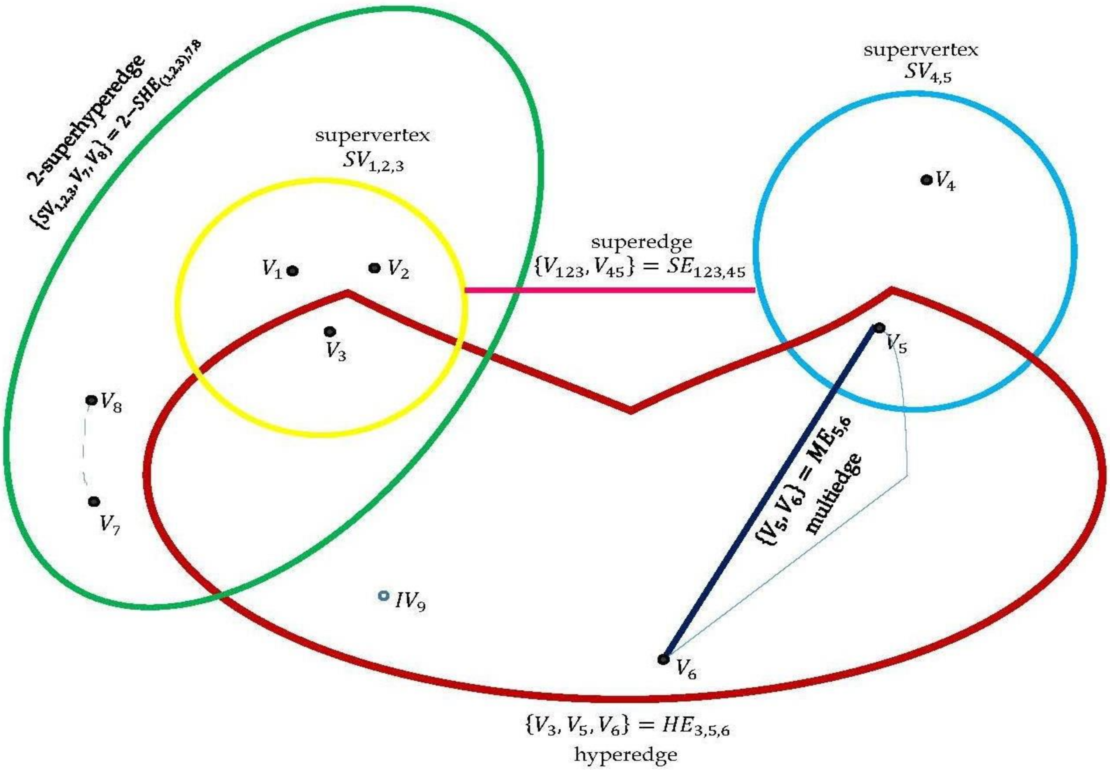
and:

- Directed Graph (classical one);
- Undirected Grpah (classical one);
- Neutrosophic Directed Graph (partially directed, partially undirected, partially indeterminate direction).

2. SuperHyperGraph

When $n = 1$ we call the *1-SuperHyperGraph* simply **SuperHyperGraph**, because only the first power set of V is used, P(V).

3. Examples of 2-SuperHyperGraph, SuperVertex, IndeterminateVertex, SingleEdge, Indeterminate Edge, HyperEdge, SuperEdge, MultiEdge, 2-SuperHyperEdge [2]



$IE_{7,8}$ is an Indeterminate Edge between single vertices V_7 and V_8, since the connecting curve is dotted;

IV_9 is an Indeterminate Vertex (since the dot is not filled in);

while $ME_{5,6}$ is a MultiEdge (double edge in this case) between single vertices V_5 and V_6.

4. Types of n-SuperHyperGraphs

The attributes values degrees of appurtenance of a vertex or an edge to the graph may be: crisp / fuzzy / intuitionistic fuzzy / picture fuzzy / spherical fuzzy / etc. / neutrosophic / refined neutrosophic / degrees with respect to each *n-SHG-vertex* and to each *n-SHG-edge* respectively.

For example, one has:

5. Plithogenic n-SuperHyperGraph

We recall the Plithogenic n-SuperHyperGraph.

A *Plithogenic n-SuperHyperGraph (n-PSHG)* is a n-SuperHyperGraph whose each *n-SHG-vertex* and each *n-SHG-edge* are characterized by many distinct attributes values $(a_1, a_2, \dots, a_p)$, $p \geq 1$.

Therefore one gets n-SHG-vertex$(a_1, a_2, \dots, a_p)$ and n-SHG-edge$(a_1, a_2, \dots, a_p)$.

6. Plithogenic Fuzzy-n-SHG-vertex $(a_1(t_1), a_2(t_2), \dots, a_p(t_p))$

and Fuzzy-n-SHG-edge$(a_1(t_1), a_2(t_2), \dots, a_p(t_p))$;

7. Plithogenic Intuitionistic Fuzzy-n-SHG-vertex $(a_1(t_1, f_1), a_2(t_2, f_2), \dots, a_p(t_p, f_p))$

and Intuitionistic Fuzzy-n-SHG-edge$(a_1(t_1, f_1), a_2(t_2, f_2), \dots, ap(t_p, f_p))$;

8. Plithogenic Neutrosophic-n-SHG-vertex $(a_1(t_1, i_1, f_1), a_2(t_2, i_2, f_2), \dots, a_p(t_p, i_p, f_p))$

and Neutrosophic-n-SHG-edge $(a_1(t_1, i_1, f_1), a_2(t_2, i_2, f_2), \dots, ap(t_p, i_p, f_p))$;

etc.

Whence in general we get:

9. The Plithogenic (Crisp / Fuzzy / Intuitionistic Fuzzy / Picture Fuzzy / Spherical Fuzzy / etc. / Neutrosophic / Refined Neutrosophic) n-SuperHyperGraph

10. Conclusions

The n-SuperHyperGraph is the most general for of graph today, designed in order to catch our complex real world.

First, the SuperVertex was introduced in 2019, then the SuperHyperGraph constructed on the power set P(V), and further on this was extended to the n-SuperHyperGraph built on the n-power set of the power set, $P^n(V)$, in order to overcome the complex groups of individuals and the sophisticated connections between them.

References

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