

ELECTRON-MUON MASS RATIO DETERMINES NEUTRINO MASS

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The neutrino mass eigen-state is assumed to be a superposition of the left- and right-handed components of the corresponding charged lepton with appropriate quantum numbers. This expansion enables the neutrino to have only the left-handed chiral component and also enables it to interact with the Higgs field to obtain mass. The exact computation of the neutrino masses is enabled through the electron-muon mass ratio. The tau-neutrino mass is also obtained through the same logic. The neutrino oscillations are analyzed theoretically.

1.INTRODUCTION

There are three neutrinos, the electron neutrino, the muon neutrino and the Tau- neutrino. All the neutrinos interact with other leptons through weak interactions. Their gravitational interaction depends on their mass. The neutrinos are chiral fields and hence the mass eigen states are presumed to be non-existent. But neutrino oscillations are detected. The neutrinos of one type oscillate into another type and this is possible only if they have mass. To make things clear let us first

concentrate on the electron- neutrino which is left handed and its right handed field finds no place in the Standard model. Hence it cannot acquire mass through its interaction with the Standard Higgs field. Particles like the electron acquire mass by interaction with the Higgs field. The electron has left handed & right handed chiral fields. The mass eigen state is a sum of the left & right handed chiral fields. Let Ψ be the wave -function of a field and Ψ_L and Ψ_R be its left-handed and right -handed fields and $P_L \Psi_L = \Psi_L$ and $P_R \Psi_R = \Psi_R$ where $P_L = \frac{1}{2} (1 - \gamma_5)$ and $P_R = \frac{1}{2} (1 + \gamma_5)$. Also, $P_L \Psi_R^C = \Psi_R^C$, $P_L \Psi_R = P_L \Psi_L^C = 0$, $P_R \Psi_L = P_R \Psi_R^C = 0$, $P_R \Psi_L^C = \Psi_L^C$. The superscript C indicates charge conjugate field (the anti-particle). Note that the charge conjugate of a right-handed field is left handed. Let the electron wave function be e where $e = e_L + e_R$ couples to the Higgs field to generate mass. In other words the total combination $e_L + e_R$ couples to the Higgs field to acquire mass. If $e_L + e_R$ interacts with the Higgs Field, then the field $e_L + \bar{e}_R$ can as well interact with the Higgs field. Here e_L has $Q = -1$, $Y^W = -2$, $I_3^W = 0$. The field \bar{e}_R has $Q = +1$, $Y^W = 2$, and $I_3^W = 0$.

In the standard model ν_L and e_L together are taken as a doublet and e_R is a singlet. Here we consider the neutrino wave function $\nu = e_L + \bar{e}_R$.

The left-handed field of the electron- neutrino is given by,

$$P_L \nu = P_L [e_L + \bar{e}_R] = e_L + \bar{e}_R . \quad (1)$$

The right handed field of the electron neutrino is given by ,

$$P_R \nu = P_R [e_L + \bar{e}_R] = 0 . \quad (2)$$

The above, are a consequence of the definitions of P_L and P_R . Also the neutrino wave function $e_L + \bar{e}_R$ has all the right properties of the

electron -neutrino. The electron-neutrino is superposition of basic entities e_L and \bar{e}_R . Similarly the muon neutrino $\nu_\mu = \mu_L + \bar{\mu}_R$. Like the electron neutrino it has left-handed field only. Still it can couple to the Higgs field to acquire mass because $\mu_L + \mu_R$ couples to the Higgs field to acquire mass. Moreover though both the electron-neutrino and muon neutrino are left-handed these are different as the charged muon is not identical to the charged electron. The Tau -neutrino wave function is given by $\nu_\tau = \tau_L + \bar{\tau}_R$. Though the neutrinos have left-handed fields, they can still interact with the Higgs field in view of our expansion. In the standard model the photon and the neutral Z bosons are given by, $A = W^0 \sin\vartheta_W + B \cos\vartheta_W$, and $Z^0 = W^0 \cos\vartheta_W - B \sin\vartheta_W$.

2. CHARGED LEPTON AND NEUTRINO MASSES.

The electron mass eigen state, e can couple to the Higgs field to acquire mass. The electron neutrino also couples to the Higgs field because its wave-function is given by $\nu = e_L + \bar{e}_R$. Both the fields e , and ν couple to the Higgs field with the same coupling constant, say h . But the electron has charge. So the electron can have another coupling with the Higgs field in addition. The mass generating Lagrangian is given by,

$$L = -h \bar{\nu} \nu \phi - h \bar{e} e \phi - i a_1 \bar{e} \gamma_5 e \phi, \quad (3)$$

Where, Higgs field is ϕ with the VEV V_0 , and h in Eq. (3), is very small as it determines the mass of the neutrino, after symmetry breaking the electron has the same mass as its neutrino at this stage. Also it interacts with the Higgs field through a γ_5 term. We recover Standard model scenario when h is set zero and $i \gamma_5$ is replaced by one. Given a Dirac field say, ψ , the Hermitian Scalar $\bar{\psi} \psi$ and $i \bar{\psi} \gamma_5 \psi$

have opposite CP and T transformation properties (In this respect they are unlike the vector and axial vector.)

After spontaneous symmetry breaking from Eq. (3) , we note that,

$$L = -hV_0\bar{\nu}\nu -h\bar{\nu}\nu\phi' -hV_0\bar{e}e -h\bar{e}e\phi' -ia_1\bar{e}\gamma_5eV_0 -ia_1\bar{e}\gamma_5e\phi' .$$

$$L = -m\bar{\nu}\nu -h\bar{\nu}\nu\phi' -m\bar{e}e -h\bar{e}e\phi' -ia_1\bar{e}\gamma_5eV_0 -ia_1\bar{e}\gamma_5e\phi' . \quad (4)$$

From Eq.(4) ,we note that the electron neutrino acquired mass m, where

$$m = hV_0 . \quad (5)$$

$$\text{Let, } e = \exp\left(-\frac{1}{2}ia_1\gamma_5\right)e' , \quad (6)$$

where α_1 is a real parameter. Vector and axial vector interactions are not affected by this transformation. We choose α_1 in such a way that the constant coefficient of $\bar{e}'\gamma_5e'$ is zero. This gives,

$$\begin{aligned} & [m\cos\alpha_1 + V_0a_1\sin\alpha_1]\bar{e}'e' - [-ims\sin\alpha_1 + ia_1V_0\cos\alpha_1]\bar{e}'\gamma_5e' \\ & -a_1\bar{e}'[\sin\alpha_1 + i\gamma_5\cos\alpha_1]e'\phi' . \end{aligned}$$

The mass of the electron is given by,

$$m_e^2 = m^2\sec^2\alpha_1 = m^2\left[1 + \frac{V_0^2a_1^2}{m^2}\right] = mV_0\left[\frac{V_0a_1^2}{m} + \frac{m}{V_0}\right]. \quad (7)$$

The very last term within the bracket is independent of the VEV. It is the

sum of $\left[\frac{a_1^2}{h} + h\right]$, which are the interaction constants of the Higgs field

with the electron and its neutrino. Let,

$$q_0 = \left[\frac{a_1^2}{h} + h\right]. \quad (8)$$

In the above expression a_1 and h are the coupling constants of the

Higgs field with the electron . The neutrino field interacts with the Higgs with the constant h through $m = hV_0$.

The charged lepton, muon has a neutrino and it is different from the electron- neutrino The muon is a lepton and is different from electron . The muon-neutrino wave function is given by,

$$\nu_\mu = \mu_L + \overline{\mu_R} \quad . \quad (9)$$

A Lagrangian like(3) can be chosen to obtain the effective mass for the muon and its neutrino through their interaction with the same Higgs field

$$L = -h_1 \bar{\nu}_\mu \nu_\mu \phi - h_1 \bar{\mu} \mu \phi - ia_2 \bar{\mu} \gamma_5 \mu \phi \quad . \quad (10)$$

In Eq. (10), h_1 and a_2 are real positive numbers and after symmetry breaking, the muon neutrino obtains mass $m_1 = h_1 V_0$. (11)

Following the same steps as in Eq. (4, 6, and 7) for the Lagrangian (10) we readily observe that,

$$m_\mu^2 = m_1^2 \sec^2 \alpha_2 = m_1^2 \left[1 + \frac{V_0^2 a_2^2}{m_1^2} \right] = m_1 V_0 \left[\frac{V_0 a_2^2}{m_1} + \frac{m_1}{V_0} \right]. \quad (12)$$

Again, we note that in Eq. (12) the parameter at the end in the brackets is independent of the VEV and is equal to $\left[\frac{a_2^2}{h_1} + h_1 \right]$. Let,

$$q_1 = \left[\frac{a_2^2}{h_1} + h_1 \right]. \quad (13)$$

There is another massive charged lepton, the τ lepton. Its mass is,

$$m_\tau = 1.777 \text{ GeV} \quad . \quad (14)$$

This lepton also has a neutrino and its wavefunction is given by,

$$\nu_\tau = \tau_L + \bar{\tau}_R \quad . \quad (15)$$

Like the other leptons, they start out with no intrinsic mass and obtain effective mass with their interaction to the same Higgs field with the VEV $V_0 = 246.22 \text{ GeV}$. The Lagrangian in this case is,

$$L = -h_2 \bar{\nu}_\tau \nu_\tau \phi - h_2 \bar{\tau} \tau \phi - i a_3 \bar{\tau} \gamma_5 \tau \phi \quad . \quad (16)$$

Following by now the familiar steps, after spontaneous symmetry breaking, we have,

$$m_{\nu_\tau} = m_2 = h_2 V_0 \quad . \quad (17)$$

where m_2 is the mass of the τ -neutrino and the mass of the charged τ lepton is now given by,

$$m_\tau^2 = m_2^2 \sec^2 \alpha_3 = m_2^2 \left[1 + \frac{V_0^2 a_3^2}{m_2^2} \right] = m_2 V_0 \left[\frac{V_0 a_3^2}{m_2} + \frac{m_2}{V_0} \right] \quad . \quad (18)$$

Again, the very last factor in the brackets is independent of the VEV.

And it is given by, q_2 where

$$q_2 = \left[\frac{a_3^2}{h_2} + h_2 \right] \text{ and } m_\tau^2 = m_2 V_0 q_2 \quad . \quad (19)$$

The three neutrinos are left handed, neutral but they are different. All three acquire mass through their coupling to the same Higgs boson as

their charged partners. If $h = h_1 = h_2$, all the neutrinos will have the same mass. If $h = h_1$, the electron-neutrino and the muon neutrino will have equal mass. In order to predict their masses we should have some more information about the masses of the charged leptons.

3. ELECTRON- MUON MASS RATIO AND NEUTRINO MASS ESTIMATION

The electron muon mass ratio is still an unsolved problem of the electroweak gauge model. The mass of the electron is given by

$$m_e^2 = mV_0 \left[\frac{a_1^2}{h} + h \right], \quad (20)$$

Where a_1^2 and h are coupling constants with which the Higgs field interacts with the electron mass eigenstate. The mass of the muon is given by,

$$m_\mu^2 = m_1V_0 \left[\frac{a_2^2}{h_1} + h_1 \right]. \quad (21)$$

As in the case of the electron a_2^2 and h_1 are coupling constants with which the Higgs field interacts with the muon mass eigen state. All together there are four unknowns. Let us rewrite Eqs. (20) and Eqs.(21) in the following way with three constants.

$$m_e^2 = mV_0 K(A - B)^2 \quad (22)$$

$$m_\mu^2 = m_1 V_0 K (A + B)^2 \quad . \quad (23)$$

Now there are three constants ,A, B, and K .In the Lagrangian (3) and (10)electron and muon are coupled to ϕ , as $h \bar{e} e \phi$ and $h_1 \bar{\mu} \mu \phi$.The neutral Z- boson couples $\bar{e} \gamma_\mu e$ and $\bar{\mu} \gamma_\mu \mu$ with the coupling g_V . There is then a possibility that h and h_1 may be proportional to g_V^2 . Moreover in the Lagrangian (3) and (10) for similar reasons a^2 and a_1^2 appear to be proportional to g_A^2 where g_A is the axial vector coupling constant with Z. These propositions lead to the conclusions that A, and B and K are just functions of g_V^2 and g_A^2 . With this presumption we found that [1,2] ,

$$m_e^2 = m V_0 \frac{1}{2} \frac{g_A^2}{g_V^4} \left[(g_A^2 + g_V^2)^{1/2} - (g_A^2 - g_V^2)^{1/2} \right]^2 \quad . \quad (24)$$

$$m_\mu^2 = m_1 V_0 \frac{1}{2} \frac{g_A^2}{g_V^4} \left[(g_A^2 + g_V^2)^{1/2} + (g_A^2 - g_V^2)^{1/2} \right]^2 \quad . \quad (25)$$

From the above relations it is clear that a_1^2 , h and a_2^2 and h_1 are all Functions of g_V^2 and g_A^2 . In the above relations the factor $\frac{1}{2}$ appears artificial. But a rearrangement of the above relations clears this doubt.

$$m_e^2 = m V_0 \frac{g_A^4}{g_V^4} \left[1 - \left(1 - \frac{g_V^4}{g_A^4} \right)^{1/2} \right] = m V_0 q_0 \quad . \quad (26)$$

$$m_\mu^2 = m_1 V_0 \frac{g_A^4}{g_V^4} \left[1 + \left(1 - \frac{g_V^4}{g_A^4} \right)^{1/2} \right] = m_1 V_0 q_1 . \quad (27)$$

In all the above expressions g_V and g_A refer to $(g_V)_{e\mu}$ and $(g_A)_{e\mu}$ which are the vector and axial vector coupling constants of e/μ with

The Z -boson. From Eqs. (26) and (27) it just follows that,

$$\frac{2m_e m_\mu}{m_e^2 + m_\mu^2} = (g_V/g_A)_{e\mu}^2 = (-1 + 4\sin^2\vartheta_W)^2 , \quad (28)$$

as and when the electron-neutrino and the muon-neutrino mass are nearly equal, $(m_e \approx m_\mu)$. If we treat $\sin^2\vartheta_W = x$, as an unknown, in the above equation, we will obtain the following values for the Weinberg mixing parameter,

$$\sin^2\vartheta_W = 0.2254 , \text{ and } 0.2746 . \quad (29)$$

The Weinberg mixing parameter is obtained experimentally from other experiments and found to be about 0.23. From Eq.(29) we observe that the sum of the two roots is just 0.5. If the electroweak model is based on $SU(2)_L \times SU(2)_R \times U(1)$, then the two mixing parameters are 0.2254 and 0.2746. The electroweak model is based only on $SU(2)_L \times U(1)$ because the electron -neutrino mass is not equal to the muon neutrino mass exactly. The observed neutrino oscillations also indicate that they

have different mass. However we can find a closed expression for the electron-neutrino and muon neutrino mass when $m = m_1$.

$$m = \frac{m_e m_\mu}{V_0} (g_V/g_A)_{e\mu}^2 = \frac{m_e m_\mu}{V_0} (-1 + 4\sin^2\vartheta_W)^2 . \quad (30)$$

$$\text{For } \sin^2\vartheta_W = 0.2254, m = m_1 = 2.123259 \text{ eV} . \quad (31)$$

$$\text{If } \sin^2\vartheta_W = 0.23 \quad m = m_1 = 1.4033 \text{ eV} . \quad (32)$$

Equal mass for the electron-neutrino and muon-neutrino is not supported by neutrino oscillations . The electron-neutrino mass is given

approximately by, $m = \frac{2m_e^2}{V_0} = 2.12095\text{eV}$. The exact mass of the

electron -neutrino is given by,(from Eq.26),

$$m = 2.120904 \text{ eV} , \text{ If, } \sin^2\vartheta_w = 0.2254 \quad \text{and} \quad (33)$$

$$m = 2.123011 \text{ eV} , \text{ If, } \sin^2\vartheta_w = 0.23 . \quad (34)$$

The exact mass of the muon -neutrino is given by from Eq.(27)

$$m_1 = 2.125604 \text{ eV} , \text{ when } \sin^2\vartheta_w = 0.2254 , \quad \text{and} \quad (35)$$

$$m_1 = 0.929486 \text{ eV} , \text{ when } \sin^2\vartheta_w = 0.23 . \quad (36)$$

4.TAU-NEUTRINO MASS .

An expression for the Tau-neutrino mass is to be obtained through analogy with the expressions for the e/mu neutrino masses.

If $m_e m_\mu$ is replaced by m_τ^2 , such that equal neutrino mass scenario works for the tau neutrino mass in Eq.(30), we have,

$$m_2 = \frac{m_\tau^2}{V_0} (g_V/g_A)_\tau^2 \quad . \quad (37)$$

If in the above for the charged Tau lepton, $(g_V/g_A)_\tau^2 = 1$,

$$m_2 = 12.825 \text{ MeV} \quad . \quad (38)$$

If the measured tau- neutrino mass turns to be 12.825 MeV, e-mu-tau Universality is violated.² As of now there are no indications about it.

Eq.(37) be treated as obtained from Eq.(30). Assuming universality the factor $(g_V/g_A)^2$ of tau is $(g_V/g_A)_{e\mu}^2 = (-1 + 4\sin^2\vartheta_W)^2$, in Eq.(37).

With this,

$$m_2 = 0.124183 \text{ MeV} \quad , \text{if, } \sin^2\vartheta_W = 0.2254 \quad (39)$$

$$m_2 = 0.082079 \text{ MeV} \quad , \text{if, } \sin^2\vartheta_W = 0.23 \quad . \quad (40)$$

While arriving at Eq.(39 or 40) we have to have, $q_2 = (g_A/g_V)^2$ and

hence Eq.(37) gives the mass of the Tau-neutrino.(see Eq.18 & 19).On

the other hand, let,

$$m_\tau^2 = m_2 V_0 \frac{g_A^4}{g_V^4} \left[1 - \left(1 - \frac{g_V^4}{g_A^4} \right)^{1/2} \right] \quad . \quad (41)$$

From the above, whenever e-mu-tau universality holds ,

$$m_2 \approx \frac{2m_\tau^2}{V_0} = 25.65 \text{ MeV}. \quad (42)$$

$$\text{The above amounts to, } q_2 = q_0 . \quad (43)$$

$$\text{Similarly if, } m_\tau^2 = m_2 V_0 \frac{g_A^4}{g_V^4} \left[1 + \left(1 - \frac{g_V^4}{g_A^4} \right)^{1/2} \right] ,$$

$$\text{So that } q_2 = q_1, \& m_2 \approx \frac{m_\tau^2}{2V_0} \frac{g_V^4}{g_A^4} = 601.19 eV . \quad (44)$$

Yet another possibility exists for the tau mass. It is given by,

$$m_\tau^2 = m_2 V_0 \frac{q_0 + q_1}{2} = m_2 V_0 \frac{g_A^4}{g_V^4} . \quad (45)$$

The above expression leads to the following value for the Tau-neutrino.

$$m_2 = 1202.38 eV . \quad (46)$$

5. NEUTRINO OSCILLATIONS

As in the case of quarks the lepton mass matrix may not be diagonal.

It will lead to a mixed state of the two neutrinos much like Cabibbo mixing of quarks. For obtaining the electron-neutrino & muon-neutrino mixing we proceed in the following [3,4] way.

Let the electron and its neutrino mass matrix be given by, M_e , where,

$$M_e = \begin{pmatrix} 0 & \sqrt{m_e m} \\ \sqrt{m_e m} & m_e - m \end{pmatrix}. \quad (47)$$

This mass matrix is diagonalized by an orthogonal matrix , O_e , where,

$$O_e = \begin{pmatrix} \cos\phi_1 & -\sin\phi_1 \\ \sin\phi_1 & \cos\phi_1 \end{pmatrix}, \quad (48)$$

$$\text{Where } , \tan\phi_1 = \sqrt{\frac{m}{m_e}} = \sqrt{\frac{2.120904}{0.51099 \times 10^6}} = 0.002037, \quad (49)$$

$$\text{From the above we note that, } \phi_1 = 0.116711 \text{ degree.} \quad (50)$$

Let the mass matrix for the muon and its neutrino be given by the matrix, M_μ where,

$$M_\mu = \begin{pmatrix} 0 & \sqrt{m_1 m_\mu} \\ \sqrt{m_1 m_\mu} & m_\mu - m_1 \end{pmatrix}. \quad (51)$$

The above mass matrix is diagonalized by an orthogonal matrix $O_\mu(\phi_2)$,

$$\text{Where } , \tan\phi_2 = \sqrt{\frac{m_1}{m_\mu}} = \sqrt{\frac{2.125604}{105.65839 \times 10^6}} = 0.000142. \quad (52)$$

$$\text{The angle } \phi_2 = 0.008136 \text{ degree.} \quad (53)$$

The absolute mass eigen-states ν_e of the electron- neutrino and the absolute mass eigen state ν_μ of the muon-neutrino are not the eigen States that take part in the electroweak model. Instead they mix and appear as a mixed states much like the Cabibbo-mixed states.

$$\begin{aligned} \nu'_e &= \nu_e \cos\vartheta_1 - \nu_\mu \sin\vartheta_1 \\ \nu'_\mu &= \nu_e \sin\vartheta_1 + \nu_\mu \cos\vartheta_1, \end{aligned} \quad (54)$$

$$\text{Where } \vartheta_1 = 0.008575 \text{degree} = \phi_1 - \phi_2 \quad (55)$$

This is much like the Cabibbo mixing in the quark sector. To obtain this Mixing angle we used the non-diagonal mass matrices, M_e and M_μ . In view of the mixing of ν_e and ν_μ with the mixing angle ϑ_1 the relative Phase of ν_e and ν_μ changes because of the mass difference so that a neutrino originating as ν'_e has a non-zero probability of being detected as ν'_μ . If an electron-type of neutrino is propagating with momentum P_e at time $t=0$, it will have a probability of oscillation $P_1 = P_{\nu_e \rightarrow \nu_\mu}$, where,

$$P_1 = \sin^2 2\vartheta_1 \sin^2 \left[\frac{1.27 \Delta m^2 L}{E_e} \right]. \quad (56)$$

Here, ϑ_1 is given by Eq. (55), and,

$$\Delta m^2 = m_1^2 - m^2 = (2.125604 \text{eV})^2 - (2.120904 \text{eV})^2. \quad (57)$$

Moreover E_e is the initial energy of the electron- neutrino in GeV and L is in km [5].

6. Electron And Tau Neutrino Mixing.

We follow an exact mass mixing procedure to obtain the electron and Tau-neutrino mixing, again we assume two flavor mixing. In order to find the mixing angle of the Tau-neutrino with the electron-neutrino, we consider the following mass matrix for the Tau and its neutrino again assuming as though there are two generations.

$$M_\tau = \begin{pmatrix} 0 & \sqrt{m_2 m_\tau} \\ \sqrt{m_2 m_\tau} & m_\tau - m_2 \end{pmatrix}. \quad (58)$$

This matrix is diagonalized by the orthogonal matrix $O_\tau(\phi_3)$, where

$$\tan\phi_3 = \sqrt{\frac{m_2}{m_\tau}} = \sqrt{\frac{0.124183}{1.777 \times 10^3}} = 0.00835964 \quad (59)$$

From the above ,the angle ϕ_3 is given by,

$$\phi_3 = 0.478961 \text{ degrees.} \quad (60)$$

The electron-neutrino and the Tau-neutrino mix where the mixing angle

$$\text{is , } \vartheta_2 = \phi_3 - \phi_1 = 0.36225 \text{ degrees ,} \quad (61)$$

The mixed e-tau neutrino states are given by,

$$\begin{aligned} \nu'_e &= \nu_e \cos\vartheta_2 - \nu_\tau \sin\vartheta_2 , \\ \nu'_\tau &= \nu_e \sin\vartheta_2 + \nu_\tau \cos\vartheta_2 . \end{aligned} \quad (62)$$

Because of this mixing ,a neutrino ν'_e which starts with energy E_e will

Oscillate into a neutrino ν'_τ with a probability , $P_2 = P_{\nu_e \rightarrow \nu_\tau}$ given by

$$P_2 = \sin^2 2\vartheta_2 \sin^2 \left[1.27 \frac{\Delta m^2 L}{E_e} \right] , \quad (63)$$

Where ϑ_2 is given by Eq.(61),(E_e in GeV, and L in Km),and

$$\Delta m^2 = m_2^2 - m^2 = (0.124183 \times 10^6 \text{ eV})^2 - (2.120904 \text{ eV})^2. \quad (64)$$

7. Muon-Tau Neutrino Mixing.

The muon neutrino can also mix with the tau neutrino exactly like

the electron neutrino. The Cabibbo type of mixing angle in this case

is given by , ϑ_3 , where,

$$\vartheta_3 = \phi_3 - \phi_2 = 0.470825 \text{ degrees .} \quad (65)$$

The mixed neutrino states are given by,

$$\begin{aligned} \nu'_\mu &= \nu_\mu \cos\vartheta_3 - \nu_\tau \sin\vartheta_3 \\ \nu'_\tau &= \nu_\mu \sin\vartheta_3 + \nu_\tau \cos\vartheta_3 . \end{aligned} \quad (66)$$

Because of the above mixing a ν'_μ neutrino with initial Energy E_μ oscillates into a ν'_τ with a probability ,

$P_3 = P_{\nu_\mu \rightarrow \nu_\tau}$, where ,

$$P_3 = \sin^2 2\vartheta_3 \sin^2 \left[\frac{1.27 \Delta m^2 L}{E_\mu} \right]. \quad (67)$$

$$\text{Here, } \Delta m^2 = (0.124183 \times 10^6 eV)^2 - (2.125604 eV)^2. \quad (68)$$

where E_μ is the initial energy of the muon neutrino in GeV and L is in Km .

8. CONCLUSIONS.

The left-handed and the right-handed chiral states of the charged lepton mix together to form the corresponding neutrino with all the appropriate quantum numbers. The left-handed and the right-handed fields of the charged lepton mix together to form a mass eigen state which interacts with the Higgs field to obtain mass. The corresponding neutrino which is also such a mixed state also obtains mass through the same Higgs boson. The electron -neutrino mass is given very approximately by, $m = 2 m_e^2 / V_0$, where V_0 is the VEV of the Standard model. The TAU neutrino mass can be determined easily to confirm our estimations here. Finally all the data is available here to verify NEUTRINO OSCILLATIONS.

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