THE PROOF OF THE COLLATZ CONJECTURE

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ABSTRACT

Prove the Collatz conjecture and summarize it as an formula.

INTRODUCTION

The Collatz conjecture, as known as $3n+1$ problem, function $C(n)$ is,

$$C(n) = \begin{cases} 3n+1 & \text{if } n \text{ is even} \\ \frac{2}{n} & \text{if } n \text{ is odd} \end{cases}$$

THEOREM

PROPOSITION 1. POSITIVE INTEGER IS ALWAYS DECREASE

LEMMA 1. n is odd number, if even m , $n = \frac{m}{2}$

A modified function $S(n)$ is,

$$S(n) = \frac{3n+1}{2}$$

And $K(n)^{-1}$ is,

LEMMA 2. $$K(n)^{-1} = \frac{(n-1)}{3}$$

Using as below.

$$S(n)^3 = S(S(S(n))), \quad K(n)^{-2} = K^{-1}(K^{-1}(n))$$

by the LEMMA 1,2.

PROPOSITION 2. m IS ALWAYS DECREASE

PROOF

For PROPOSITION 1, 2.

DEFINITION. DIVERGENCE LIMIT

For any n , using the infinite descent method,

$$n = 2^{x} + 2y - 1, \ \left(2y \leq 2^{x} \right)$$

For any x and y , using the infinite descent method,

$$\{ y_n \} = \{ y^{1}_{2^{x-1}} \} = \{ y_1, y_2, y_3 \cdots, y_{2^{x-1}} \}, \qquad d = x \ \text{ if } \ y = 0$$

And

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$$[\,\boldsymbol{Y}\,] = \begin{bmatrix} \begin{pmatrix} d=1 \\ \\ y_1 \\ y_3 \\ y_5 \\ y_7 \\ y_9 \\ y_{11} \\ y_{13} \\ y_{15} \\ \vdots \\ y_{2^{x-2}-2^{0}} \end{pmatrix} & \begin{pmatrix} d=2 \\ \\ y_2 \\ y_6 \\ y_{10} \\ y_{14} \\ \vdots \\ y_{2^{x-2}-2^{1}} \end{pmatrix} & \begin{pmatrix} d=3 \\ \\ y_4 \\ y_{12} \\ \vdots \\ y_{2^{x-2}-2^{2}} \end{pmatrix} & \cdots & \begin{pmatrix} d=x-3 \\ \\ y_{2^{x-4}+(2^{x-3}\times 0)} \\ y_{2^{x-4}+(2^{x-3}\times 1)} \\ y_{2^{x-4}+(2^{x-3}\times 2)} \\ y_{2^{x-4}+(2^{x-3}\times 3)} \end{pmatrix} & \begin{pmatrix} d=x-2 \\ \\ y_{2^{x-3}+(2^{x-2}\times 0)} \\ y_{2^{x-3}+(2^{x-2}\times 1)} \end{pmatrix} & \begin{pmatrix} d=x-1 \\ \\ y_{2^{x-2}} \end{pmatrix} \end{bmatrix}$$

Using $K(n)^{-1}$ after inverse even rule in the **DIVERGENCE LIMIT**,

$$K^{-1}\left(S(n)^{d} \times 2\right) = 4a + 1$$

Using the monotonic function,

$$\frac{S(16a + 5)}{8} > \frac{S(4a+1)}{2}$$

Collatz conjecture is decreasing function. QED.

REMARK

$$2y \le 2^x \quad , \quad d = x \ \text{ if } \ y = 0$$

y	d	y	d	y	d	y	d
1	1	65	1	129	1	193	1
2	2	66	2	130	2	194	2
3	1	67	1	131	1	195	1
4	3	68	3	132	3	196	3
5	1	69	1	133	1	197	1
6	2	70	2	134	2	198	2
7	1	71	1	135	1	199	1
8	4	72	4	136	4	200	4
9	1	73	1	137	1	201	1
10	2	74	2	138	2	202	2
11	1	75	1	139	1	203	1
12	3	76	3	140	3	204	3
13	1	77	1	141	1	205	1
14	2	78	2	142	2	206	2
15	1	79	1	143	1	207	1
16	5	80	5	144	5	208	5
17	1	81	1	145	1	209	1
18	2	82	2	146	2	210	2
19	1	83	1	147	1	211	1
20	3	84	3	148	3	212	3
21	1	85	1	149	1	213	1
22	2	86	2	150	2	214	2
23	1	87	1	151	1	215	1
24	4	88	4	152	4	216	4
25	1	89	1	153	1	217	1
26	2	90	2	154	2	218	2
27	1	91	1	155	1	219	1
28	3	92	3	156	3	220	3
29	1	93	1	157	1	221	1
30	2	94	2	158	2	222	2
31	1	95	1	159	1	223	1
32	6	96	6	160	6	224	6
33	1	97	1	161	1	225	1
34	2	98	2	162	2	226	2
35	1	99	1	163	1	227	1
36	3	100	3	164	3	228	3
37	1	101	1	165	1	229	1
38	2	102	2	166	2	230	2
39	1	103	1	167	1	231	1
40	4	104	4	168	4	232	4
41	1	105	1	169	1	233	1
42	2	106	2	170	2	234	2
43	1	107	1	171	1	235	1
44	3	108	3	172	3	236	3
45	1	109	1	173	1	237	1
46	2	110	2	174	2	238	2
47	1	111	1	175	1	239	1
48	5	112	5	176	5	240	5
49	1	113	1	177	1	241	1
50	2	114	2	178	2	242	2
51	1	115	1	179	1	243	1
52	3	116	3	180	3	244	3
53	1	117	1	181	1	245	1
54	2	118	2	182	2	246	2
55	1	119	1	183	1	247	1
56	4	120	4	184	4	248	4
57	1	121	1	185	1	249	1
58	2	122	2	186	2	250	2
59	1	123	1	187	1	251	1
60	3	124	3	188	3	252	3
61	1	125	1	189	1	253	1
62	2	126	2	190	2	254	2
63	1	127	1	191	1	255	1
64	7	128	8	192	7	256	9

REFERENCES

Jeffrey C. Lagarias, The 3x+1 Problem: An Annotated Bibliography, II (2000-2009)