The actuality of acousto-mechanical resonances for noise control

Roman VINOKUR

ResMed Motor Technologies, USA

ABSTRACT

A clarifying analytical study was done for the 2-DOF acousto-mechanical system modeled as the Helmholtz resonator with elastic vibrating bottom. Generally, such a system has two resonances that can generate and radiate an annoying tonal noise. As shown, the two resonance frequencies may get very close with mutual amplification and even merge together into one powerful resonance (a close-form equation describing this transition was derived). Such an effect can happen because the mass of air in the neck and the rigid mass may differ in several orders of magnitude. This is less possible in 2-DOF mechanical systems where the masses of both vibrating rigid bodies are commonly within one order of magnitude. The results may be helpful in particular for noise identification and reduction in blowers and other air moving devices. Here, the role of a vibrating bottom is played by the impeller in combination with the rotor and suspension system, and the air inlet (or outlet) serves as the Helmholtz resonator neck. Low-frequency acousto-mechanical resonances can also occur in rooms with open windows or doors.

Keywords: Noise and Vibration Control, Resonance, Loss factor, Air Moving Devices, Rooms.

I-INCE Classification of Subjects Number(s): 11.4, 11.6, 21.4, 21.6, 75.9.

1. INTRODUCTION

At their resonance frequencies, mechanical systems may develop harsh vibration and noise with a risk of structural failure. The elastic structures incorporating gas- or liquid-filled cavities (especially, blowers, pumps, and other air- or water-moving devices) can experience rather prominent acousto-mechanical resonances. The goal of this paper is to identify such resonance conditions in the simplest case: for 2-DOF in-series systems and in particular for the Helmholtz resonator with an elastic bottom).

2. ANALYSIS

2.1 Analysis of 2-DOF in-series mechanical model with constant loss factors

Consider a 2-DOF in-series mechanical system consisting of two inertial elements and two springs with hysteretic damping (Figure 1a). Such systems are commonly analyzed with dashpots simulating viscous friction (1-3, etc.). Here, the complex spring constants are defined as $K_1 = k_1(1 - i\eta)$ and $K_2 = k_2(1 - i\eta)$ where k_1 and k_2 are the factual spring constants, and $\eta = \text{const}$ is the loss factor; the masses of two inertial elements are m_1 and m_2. Hence, the partial natural angular frequencies are $\omega_{p1} = \sqrt{k_1/m_1}$ and $\omega_{p2} = \sqrt{k_2/m_2}$. The first spring is fixed to a solid base vibrating harmonically with displacement $Y_0 = A\exp(-i\omega t)$ where A is a constant value.

The equations of motion are

$$\begin{cases} m_1\ddot{Y}_1 + K_1Y_1 + K_2(Y_1 - Y_2) = K_1Y_0, \\ m_2\ddot{Y}_2 + K_2(Y_2 - Y_1) = 0. \end{cases} \quad (1)$$

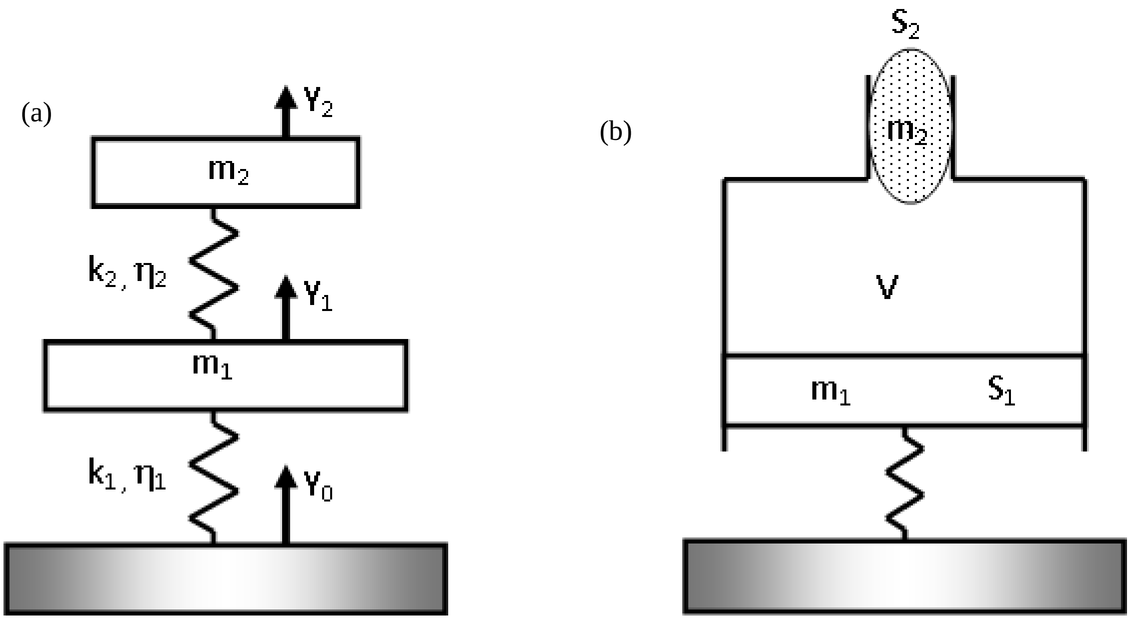

Figure 1 – Simplified engineering systems: (a) 2-DOF in-series mechanical model, (b) acousto-mechanical model (Helmholtz resonator with the vibrating elastic bottom).

The solution to the system of differential equations (1) is $Y_{1,2} = y_{1,2}(\omega)\exp(-i\omega t)$ where

$$\frac{y_1}{A} = \frac{[1-(\omega/\omega_{p2})^2 - i\,\eta]\;(1-i\,\eta)}{\Psi(\omega)}, \tag{2}$$

$$\frac{y_2}{A} = \frac{(1-i\,\eta)^2}{\Psi(\omega)}. \tag{3}$$

The complex function in the denominator of Eqs (2) and (3) is

$$\Psi(\omega) = [1-(\omega/\omega_1)^2 - i\,\eta]\;[1-(\omega/\omega_2)^2 - i\,\eta] \tag{4}$$

where the natural angular frequencies of the 2-DOF system

$$\omega_{1,2} = \omega_{p2}\sqrt{D_{1,2}}\,. \tag{5}$$

Here, $\omega_1 < \omega_2$. The dimensionless values

$$D_{1,2} = \frac{1+b}{2}\left(1 \mp \sqrt{1-\frac{1}{1+\mu}\,\frac{4\,b}{(1+b)^2}}\right) \tag{6}$$

are functions of two dimensionless parameters:

$$\begin{cases} b = (1+\mu)\,(\omega_{p1}/\omega_{p2})^2, \\ \mu = m_2/m_1\,. \end{cases} \tag{7}$$

2.2 Effect of the ratio squared of two natural frequencies of 2-DOF system on its vibration

The system transmissibility (the transmissibility related to the second mass)

$$T_2(\omega) = \left|\frac{y_2}{A}\right| = \frac{\left(1+\eta^2\right)}{|\Psi(\omega)|}. \tag{8}$$

is of most importance at the resonance frequencies of the system. Using Eqs (4) and (8), calculate the system transmissibility at the natural angular frequencies ω_1 and ω_2 :

$$\begin{cases} T_2(\omega_1) = \sqrt{\dfrac{1+\eta^2}{(1-p)^2+\eta^2}}, \\ T_2(\omega_2) = p\sqrt{\dfrac{1+\eta^2}{(1-p)^2+p^2\eta^2}} \end{cases} \tag{9}$$

where

$$p = \left(\frac{\omega_1}{\omega_2}\right)^2 < 1 \tag{10}$$

is the ratio squared of the two natural frequencies defined by Eq. (5).
As seen from Eqs (9), the closer the parameter p to 1, the higher the system transmissibility at both natural frequencies.

2.3 Maximum ratio squared of the natural frequencies of 2-DOF in-series system

Using Eq. (5)-(7) and (10), obtain

$$p = \frac{D_1}{D_2} = \left(\frac{1-\sqrt{1-1/B}}{1+\sqrt{1-1/B}}\right)^2 = \frac{1}{\left(\sqrt{B}+\sqrt{B-1}\right)^2} \tag{11}$$

which is a monotonically decreasing function of the dimensionless parameter

$$B = \left(1+\mu\right)\frac{(1+b)^2}{4b} > 1$$

For any constant value μ, this parameter attains its minimum

$$\min_{\mu=\text{const}}\{B\} = 1+\mu \tag{12}$$

at the condition $b=1$ or, as follows from Eq. (7), for

$$\frac{\omega_{p1}}{\omega_{p2}} = \frac{1}{\sqrt{1+\mu}}. \tag{13}$$

Using Eq. (11) and (13), introduce the function

$$U(\mu) = \max_{\mu=\text{const}}\{p\} = \frac{1}{\left(\sqrt{1+\mu}+\sqrt{\mu}\right)^2} \tag{14}$$

decreasing with the parameter μ.

It should be noted that extremely small values μ are common for acousto-mechanical systems because the mass of air "plugs" is much lower than the mass of the solid elements.

2.4 Numeric illustration of the analytical results

For more clear illustration of the analytical results, the system transmissibility described by Eq. (8) was plotted in Figure 2 for loss factor $\eta = 0.1$ and three different values of the ratio squared of two natural frequencies: $p = 0$, $p = 0.5$ and $p = 0.75$. As seen, the transmissibility increases notably with the parameter p: (1) at $p = 0.75$ and $\eta = 0.1$, the second peak even merges into the first peak; (2) at $p = 0$, the second natural frequency is infinite, so, the 2-DOF system turns into a 1-DOF system with a relatively low transmissibility.

2.5 Important case: Helmholtz resonator with elastic vibrating bottom

A special type of 2-DOF in-series mechanical system is the Helmholtz resonator with elastic vibrating bottom (Figure 1b). Here, the role of the second mass is played by the air "plug" in resonator's neck and the inside air volume serves as the second spring. Using the existing theory (1-3), those parameters can be expressed in the form

$$\begin{cases} m_2 = \rho_0 \, S_2 \, L_{eff} \, , \\ k_2 = \dfrac{\rho_0 \, c_0^2 \, S_2^2}{V} \end{cases} \tag{15}$$

where ρ_0 and c_0 are respectively the air density and speed of sound in air, S_2 and L_{eff} are the area and effective length of the air "plug", and V is the air volume.
It can be shown that the results obtained in chapters 2.1 -2.4 are applicable for this special model too if

$$\begin{cases} \mu = \dfrac{m_2}{m_1} \left(\dfrac{S_1}{S_2} \right)^2 , \\ \omega_2 = c_0 \sqrt{\dfrac{S_2}{V \, L_{eff}}} \end{cases} \tag{16}$$

where S_1 is the area of the resonator bottom.

3. Shaker experiment

To check the analytical results for the acousto-mechanical model, the author made a miniature cubic "room" incorporating the ceramic floor and side walls, 1 mm thick, with horizontal rectangular slots (4 mm long and 1 mm wide each) in two opposite walls. The room roof incorporated the silicon disc (3 mm in diameter and 0.5 mm thick) and circular elastomer membrane (Figure 3). Each edge of the room was 10 mm long, so, the "room" can operate like a Helmholtz resonator up to ≈8000 Hz.

The "room" was fixed on a 1-DOF shaker for sweep sine test, and the disk vibration was measured with a single-point laser-vibrometer. The transmissibility (here, the ratio of the amplitudes of the disk and shaker velocities) demonstrated two peaks (Figure 4) caused by the disk-membrane resonance at ≈ 2000 Hz and the Helmholtz resonance at ≈3500 Hz.

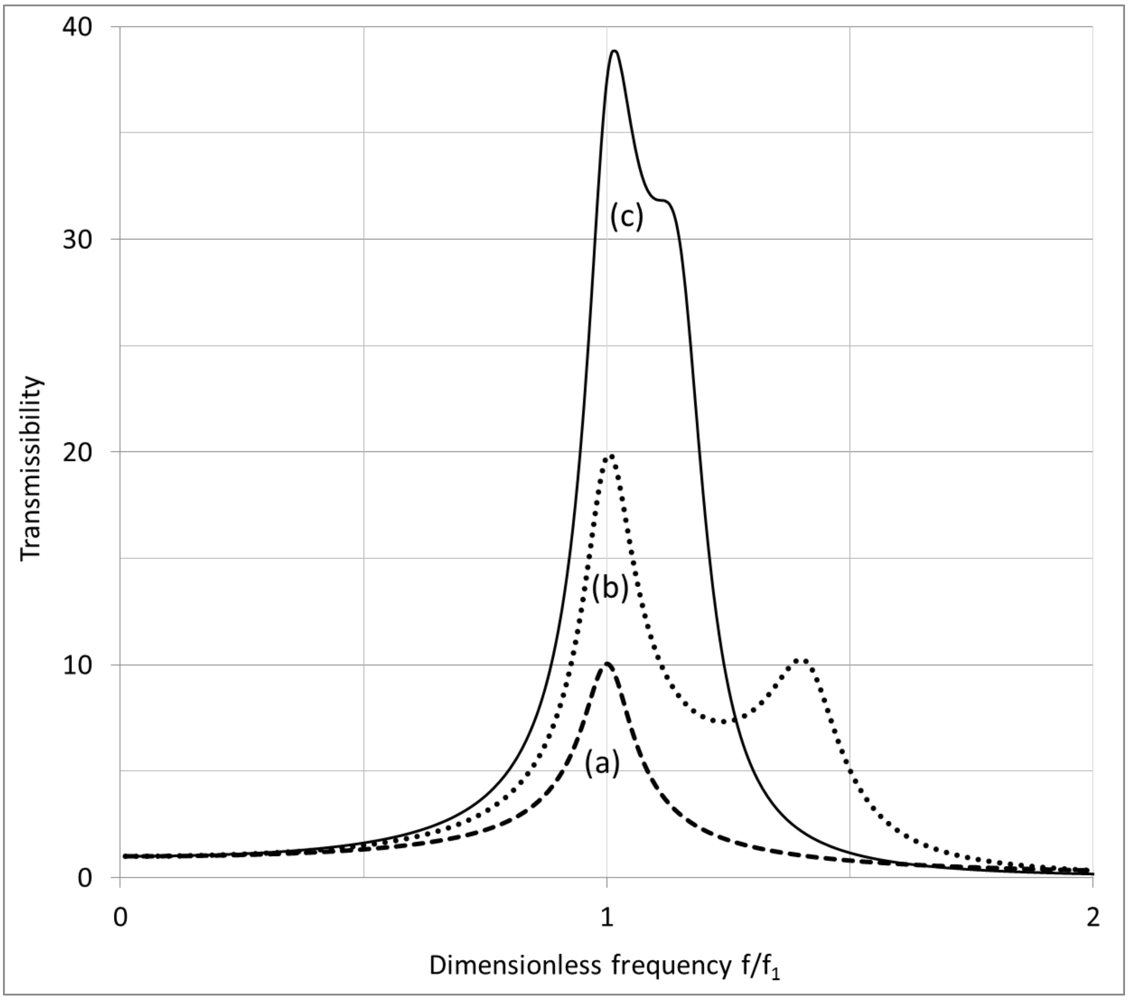

Figure 2 – Transmissibility of 2-DOF mechanical system vs. dimensionless frequency for loss factor η = 0.1 and three various values of parameter p: (a) p = 0, (b) p = 0.5, (c) p = 0.75. Here, f is frequency and f_1 is the first natural frequency of the system.

Here, the test setup shown in Figure (3) is defined as (b). The other test (a) was done for comparison: the two opposite walls with the slots were completely removed, so, the Helmholtz resonance frequency became infinite. As a result, the second peak disappeared (Figure 4).
Such an experiment was first described by the author in his paper (4) but just to simulate Helmholtz's resonance in real rooms at low-frequency environmental vibration caused in particular by earthquakes.
One more test setup (c) was investigated too but the results were not published and were lost later: the right slot was boarded up to reduce the Helmholtz resonance frequency from ≈3500 to ≈2500 Hz.
In this case, the ratio squared of two natural frequencies increased from 0.33 to 0.65, the peak transmissibility increased up to ≈60 (about twice as much), and both peaks all but merged together, so, the measured spectrum looked much similar to the calculated spectrum (c) in Figure 2.

It is noteworthy that the loss factors for the disk-membrane and Helmholtz resonances are different: applying the half-bandwidth method (5) to the spectrum (b) in Figure 4, obtain 0.20 and 0.03, respectively. Besides, the measurements done as shown in Figure 3 describe the transmissibility related to the first mass

$$T_1(\omega) = \left|\frac{y_1}{A}\right| = \sqrt{\frac{\left[1-\left(\omega/\omega_{p2}\right)^2\right]^2+\eta^2}{1+\eta^2}}\, T_2(\omega).$$

However, such differences insignificantly affect the trend illustrated in Figure 2.

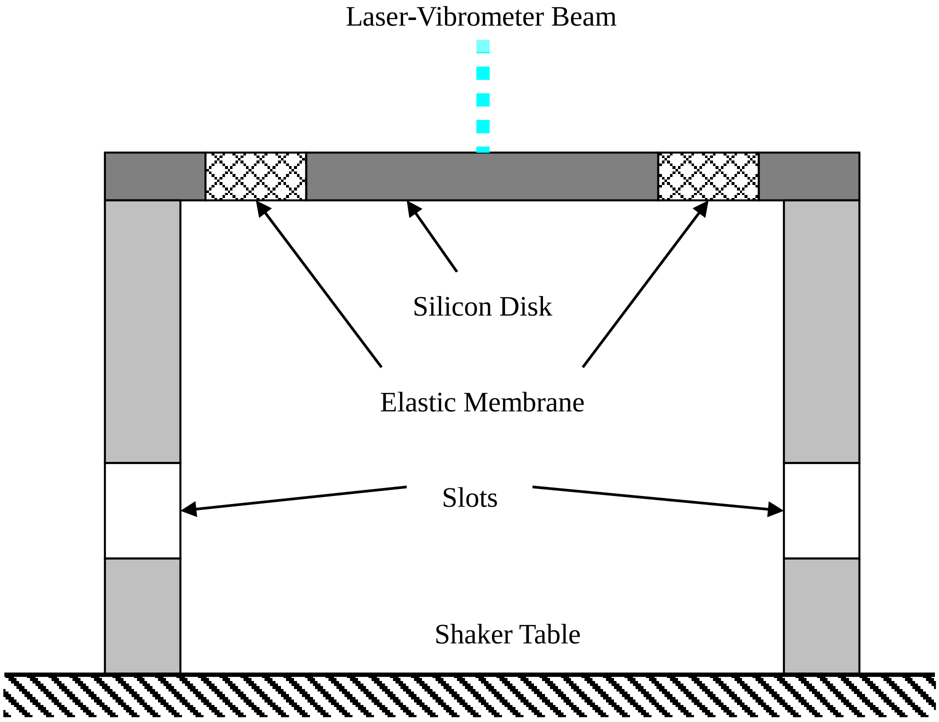

Figure 3 – Test setups: (a) two opposite “walls” are removed (not shown), (b) both slots were open (as shown), (c) the right slot was boarded up (not shown).

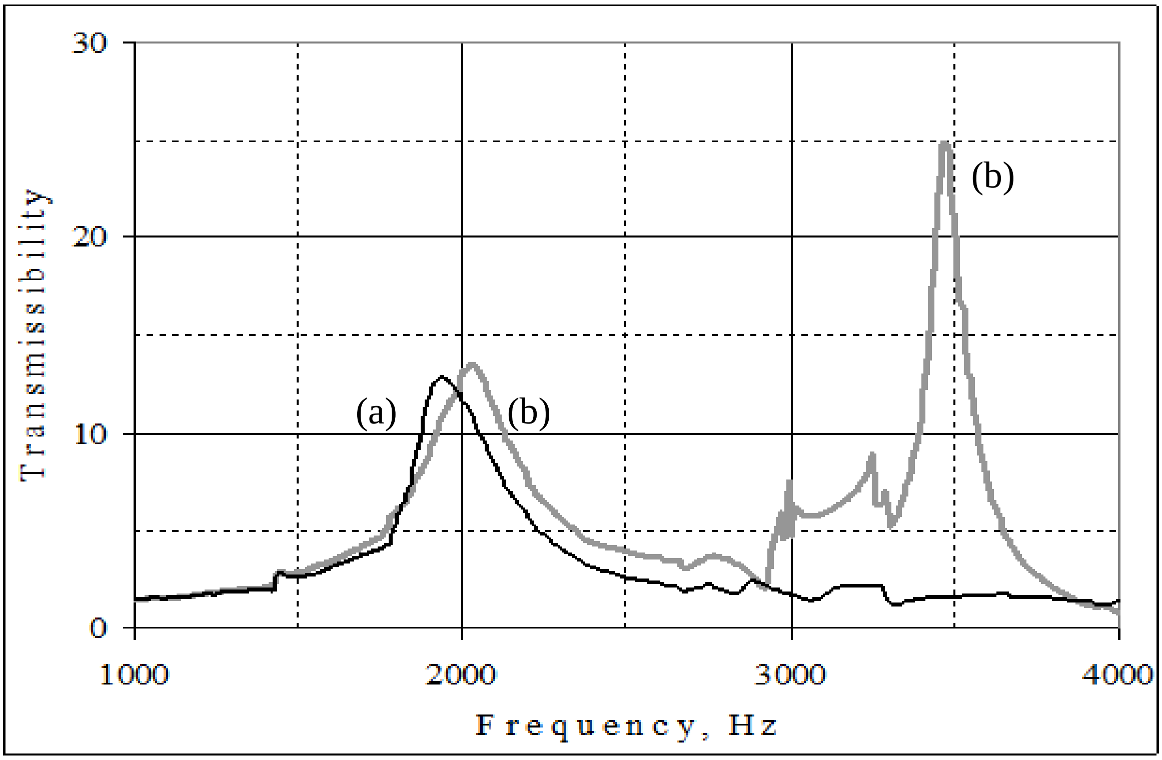

Figure 4 – Experimental vibration spectra measured for the test setups (a) and (b).

4. APPLICATIONS

4.1 Rooms at very low frequencies

Notable acousto-mechanical resonances can occur in a room with open window (or door) and vibrating wall (or floor) at very low frequencies, in particular in the infrasound range.

4.2 Blowers and air moving devices with blowers

The role of a vibrating bottom can be played by the impeller in combination with the rotor and suspension system, and the air inlet (or outlet) serves as the Helmholtz resonator neck.

The function of Helmholtz's resonator may be also performed by an air moving device with blower. Here, the air inlet (or outlet) of the whole system serves as the Helmholtz resonator neck. It is noteworthy that even if the blower itself may be relatively quiet, it may generate significant noise and vibration in the air moving device because of the resonance interaction described above.

5. CONCLUSIONS

The resonance peaks in 2-DOF in-series mechanical system may approach closely (with mutual amplification) and even merge together into one powerful resonance (a close-form equation describing this transition was derived). In the particular case of acousto-mechanical system modeled as the Helmholtz resonator with elastic vibrating bottom, such an effect can be very notable because the mass of air in the neck and the rigid mass are commonly within one order of magnitude. The results may be helpful for noise reduction in blowers and other air moving devices where the role of a vibrating bottom is played by the impeller itself or in combination with the rotor and suspension system, and the air inlet (or outlet) serves as the Helmholtz resonator neck. Low-frequency acousto-mechanical resonances can also occur in rooms with open windows or doors.

REFERENCES

1. Morse PM. Vibration and sound. Acoustical Society of America, NY, USA; 1981.

2. Beranek LL. Acoustics. Acoustical Society of America, NY, USA; 1996.

3. P. Den Hartog JP. Mechanical vibrations. Dover Publications, Inc., NY, USA; 1985.

4.Vinokur R. Infrasonic sound pressure in dwellings at the Helmholtz resonance actuated by environmental noise and vibration. Applied Acoustics, 2004; 65(2), p 143-151.

5. Lindeburg MR. Mechanical engineering reference manual. Ninth edition. Professional Publications, Inc., Belmont, USA; 1994.