On the Gravitational Energy of Planet Mercury

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Abstract. The Sun is travelling around the Solar System barycentre so that *on average*, orbiting Mercury is attracted to a position between the Sun's centre and the barycentre. This toroidal-Sun-effect generates a small quadrupole moment in the gravitational energy of Mercury, which cannot be reduced to zero by using heliocentric coordinates.

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1. Introduction

In this article it is shown how the Sun, by orbiting the barycentre, simulates a toroidal Sun and adds a quadrupole moment to Mercury's potential energy. This addition cannot be precluded by using heliocentric coordinates because a toroidal Sun-effect or oblate Sun cannot be transformed away. To comprehend this, consider the analogy of an oscillating pendulum bob suspended from a slowly *moving* pivot. The pivot drags the bob and destroys its simple harmonic motion. Likewise, the Sun drags Mercury around the barycentre and modifies its orbit *in form and energy*. Investigators have always applied Newton's law to Mercury, as for a static Sun; see for example, Clemence (1947), Brouwer & Clemence (1961), Pireaux & Rozelot (2003). The inverse square law was applied to set up the instantaneous differential equations of motion, but subsequent integration process made no allowance for the Sun dragging Mercury, increasing its potential energy. The elemental quadrupole moment will be evaluated in two steps, as follows.

2. Potential of Mercury due to a theoretical speeding Sun

First of all, *imagine* the Sun orbiting the barycentre rapidly so that it appears like a *toroidal* Sun from Mercury; then Mercury would orbit the averaged Sun position at the barycentre. This would introduce a quadrupole term to the Newtonian potential for Mercury, as follows. In Figure 1, let Mercury (mass M_1) be regarded as *stationary* at distance (r_{1C} = 57.9x10^{6}km) from the barycentre C, while the Sun (mass M) travels *rapidly* around C at radius (r_{SC} = 7.43x10^{5}km). Then, the instantaneous distance r_1 to Mercury is given by:

$$r_1{}^2 = r_{1C}{}^2 + r_{SC}{}^2 - 2r_{1C}r_{SC}\cos\theta \quad . \qquad (1a)$$

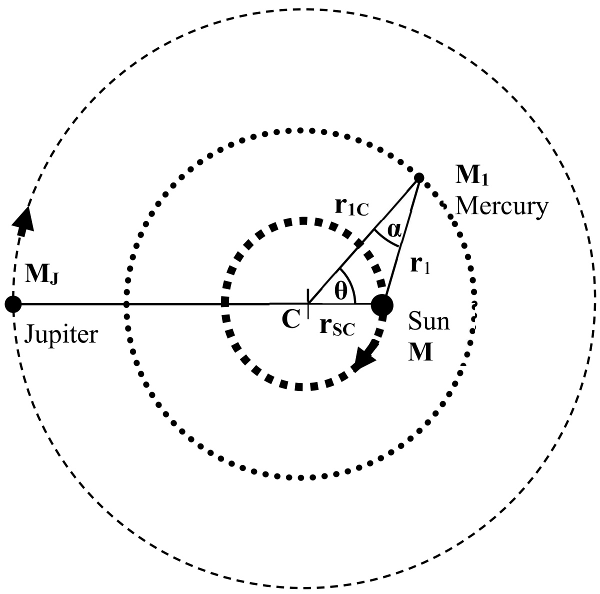

Fig. 1 Schematic diagram showing Jupiter and the Sun moving around their barycentre C. Mercury is here considered to be stationary during one orbit of the Sun.

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1{}^2$), and the component of this force directed towards barycentre C is $F_1 \cos\alpha$; where $\cos\alpha$ is given by:

$$r_{SC}{}^2 = r_{1C}{}^2 + r_1{}^2 - 2r_{1C}r_1\cos\alpha \, . \qquad (1b)$$

Upon eliminating cos α, this force towards C is:

$$F = F_1 \cos\alpha = -\left(\frac{GMM_1}{r_1{}^2}\right)\left(\frac{r_{1C} - r_{SC}\cos\theta}{r_1}\right). \qquad (2a)$$

Now eliminate variable r_1 and get all the cosθ terms in the numerator:

$$F \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left(1 - \frac{3}{2}\frac{r_{SC}{}^2}{r_{1C}{}^2} + \frac{2r_{SC}}{r_{1C}}\cos\theta + \frac{9}{2}\frac{r_{SC}{}^2}{r_{1C}{}^2}\cos^2\theta\right). \qquad (2b)$$

By varying θ from zero to 2π over a complete orbit of the Sun, the *average* force towards C becomes:

$$\tilde{F} \approx -\left(\frac{GMM_1}{r_{1C}{}^2}\right)\left[1 + \frac{3}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right]. \qquad (2c)$$

By integrating from r_{1C} to infinity, the absolute potential energy of Mercury in this simulated system would be:

$$PE_a \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1 + \frac{1}{4}\left(\frac{r_{SC}{}^2}{r_{1C}{}^2}\right)\right]. \qquad (3)$$

These expressions would apply to the equation of motion for Mercury, as it orbits around the Sun which is orbiting C rapidly.

3. Potential of Mercury due to the real moving Sun

The Sun is actually moving slowly around the barycentre, so the quadrupole moment derived above will be much reduced, but not to zero. For example, the Sun orbits the barycentre C at radius ($r_{SC} = 7.43 \times 10^5$km) over 11.86 years due to Jupiter. Its slow movement allows Mercury to respond to the Sun's position more directly, such that the averaged centre of attraction for Mercury lies near to the Sun rather than at C. That is, r_{SC} in Eq.(2c) takes a smaller value r'_{SC} so the average acceleration of Mercury around this position is:

$$\tilde{a} \approx -\frac{GM}{r_{1c}{}^2}\left[1 + \frac{3}{4}\left(\frac{r'_{SC}}{r_{1c}}\right)^2\right]. \qquad (4)$$

After substituting ($u = 1/r_{1C}$), plus Mercury's specific angular momentum [$h \approx (GMr_{1C})^{1/2}$], then orbit theory yields a differential equation for the trajectory:

$$\frac{d^2u}{d\varphi^2} + u = \frac{-\tilde{a}}{h^2u^2} = \frac{GM}{h^2} + \left(\frac{GM}{h^2}\frac{3}{4}(r'_{SC})^2\right)u^2. \qquad (5)$$

This type of equation has previously been solved because General Relativity theory gives a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\varphi^2} + u = \frac{GM}{h^2} + \left(\frac{3GM}{c^2}\right)u^2 \quad . \tag{6}$$

The final term evaluates to 43arcsec/cy precession of Mercury's orbit, hence by direct comparison we can calculate the precession to expect from the quadrupole moment in Eq.(5):

$$\delta\omega \approx \left(\frac{c}{2h} r'_{SC}\right)^2 \times 43\text{arc sec/cy} . \tag{7}$$

The value of r'_{SC} can be derived by considering Figure 2, wherein Mercury is now depicted orbiting the Sun in period (T_1 = 88days), while the Sun is *slowly* orbiting barycentre C with its period due to Jupiter (T_{SC} = 4332.6days = 49.25T_1). Mercury is no longer attracted to C, but is on average attracted to an effective moving centre of gravity P at radius r_{PC} from C towards the Sun. During one orbit of Mercury over period T_1, the Sun moves distance ($2\pi r_{SC}/49.25$) and drags Mercury with it.

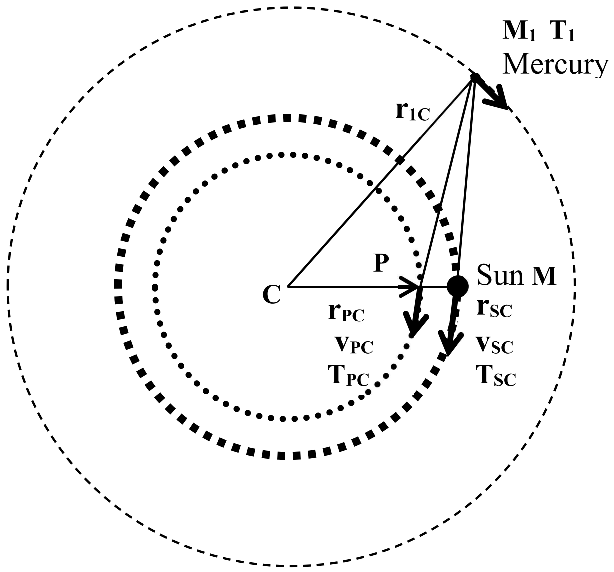

Fig. 2 Schematic diagram showing the Sun moving slowly around the barycentre C. Mercury now orbits the effective centre of gravity at P near the Sun.

Radius r_{PC} will be derived by using the conservation of action principle, where *action* is (Kinetic Energy x Time). Thus, let the real action for the Sun at velocity (v_{SC}) during one period of Mercury (T_1) be theoretically equivalent to the action for P at velocity (v_{PC}) during the total time for Mercury to regain rectilinearity with C, P, and the Sun:

$$\frac{1}{2}Mv_{SC}^2 \times T_1 = \frac{1}{2}Mv_{PC}^2 \times T_1\left(\frac{49.25}{48.25}\right) . \qquad (8a)$$

[This is analogous to the way clock hands align after 1hr 5.45min = (12/11) hours]. Here, the last term describes how much Mercury's orbit extends beyond $2\pi r_{1C}$ in order for Mercury to align with the Sun and C again. After 49.25 orbits of Mercury ($49.25T_1 = T_{SC}$), this expression becomes:

$$\frac{1}{2}Mv_{SC}^2 \times T_{SC} = \frac{1}{2}Mv_{PC}^2 \times [T_{PC} + T_1(\frac{49.25}{48.25})] . \qquad (8b)$$

Given ($T_{PC} = T_{SC}$), then ($r_{SC}/v_{SC} = r_{PC}/v_{PC}$), and this simplifies to:

$$r_{PC} = r_{SC}\left(\frac{T_{SC}-T_1}{T_{SC}}\right)^{1/2} . \qquad (9)$$

Consequently, the position of P is determined by Mercury's period T_1 relative to the Sun's period T_{SC} around the barycentre C. From Mercury's viewpoint, the Sun moves around P in a circle of radius r'_{SC} :

$$r'_{SC} = r_{SC} - r_{PC} = r_{SC}\left[1 - \left(\frac{T_{SC}-T_1}{T_{SC}}\right)^{1/2}\right] . \qquad (10)$$

It is this motion *around* P which determines the quadrupole moment of force operating on Mercury by introducing r'_{SC} into Eq.(4). Centre of gravity P lies between the barycentre and the Sun's centre. Given ($T_1 = 88.0$days), ($T_{SC} = 4331$days) and ($r_{SC} = 7.43\text{x}10^5$km), then ($r'_{SC} = 7433$km). Substitution in Eq.(7), with ($h = 2.76\text{x}10^{15}m^2s^{-1}$), yields the precession due to the quadrupole moment:

$$\delta\omega \approx 0.162 \times 43 \approx 7.0\text{arc sec/ cy} . \qquad (11)$$

Therefore, the well-known 43arcsec/cy residual precession attributed to GR must be reduced by 7arcsec/cy to 36arcsec/cy for GR.

The absolute potential energy of Mercury, in this realistic Solar System, has the same form as Eq.(3):

$$PE_{abs} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1+\frac{1}{4}\left(\frac{r'^{2}_{SC}}{r_{1C}^{2}}\right)\right]. \qquad (12)$$

The other planets will cause some fluctuation of (r'_{SC}) (Landscheidt, 2007).

Precessions currently attributed to GR for Venus, Earth and Icarus, will also be affected by the Sun's acceleration around the barycentre; see Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992).

4. Conclusion

The acceleration of the Sun around the barycentre produces a small quadrupole moment in the gravitational energy of Mercury, just as a toroidal Sun would. Application of heliocentric coordinates cannot reduce this toroidal Sun effect to zero. Thus, averaged over its period, Mercury is attracted to a position inside the Sun's arc, towards the barycentre. The Sun increases the binding energy of Mercury.

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