Prime Numbers Estimation in A Special Palindromic Number

Zhi Li and Hua Li

(lizhi100678@sina.com, lihua2057@yeah.net)

Abstract: The repunit number refers to the natural number composed of all the numbers 1, which is a special palindromic number. Whether the number of prime numbers in the repunit number is infinite, and how to calculate the number of prime numbers less than a certain integer have not been determined. This paper proves that the number of prime numbers of this kind is infinite by using the method of probability and statistics, and derives the calculation formula of it. On this basis, two conjectures are further proposed, that is, the ratio of the number of prime numbers in this palindromic number to the number of prime numbers in Mersenne number is 16ln (2)/(15ln (10)), and the number of prime numbers less than 24862048 digits is close to 15.

Keywords: palindromic number, repunit number, prime number, probability statistics, repunit prime number conjectures

The repunit number is a special palindromic number, which refers to

$$\frac{10^n - 1}{9} = \{1,11,111,\dots\}, n \geq 1$$

The number of prime numbers in it has not been determined [1].

Till now, it is known that the repunit numbers are all prime when n is 2, 19, 23, 317, 1031, 49081. And it has not been proved that they are all prime numbers when n is 86453, 109297, 270343, 5794777, 8177207. The exact number of repunit prime numbers less than 24862048 digits has not been reported [1].

This paper proves that the repunit prime numbers are infinite by using the method of probability statistics, derives the calculation formula of the number of repunit prime numbers, and gives two conjectures.

1. The number of repunit prime numbers

Let n be a natural number, and the algebraic form of repunit number is

$$\frac{10^n - 1}{9}$$

According to the distribution law of prime number [2], the probability that the repunit number (n ≥ 2) is a prime number is

$$\frac{1}{ln(\frac{10^n-1}{9})}$$

By using the probability statistics method [3] [4] [5] [6], when n ≥ 2, the cumulative probability that the repunit numbers are prime numbers is

$$\sum_{\mathrm{x=2}}^{\mathrm{n}} \frac{1}{ln(\frac{10^x-1}{9})}$$

Because as x increases, 10^x-1 approaches 10^x, x ln (10) - ln (9) approaches x ln (10), and the above expression can be simplified as

$$\frac{1}{\ln(10)}\sum_{\mathrm{x=2}}^{\mathrm{n}}\frac{1}{x}$$

For the harmonic series is not convergent, when n tends to infinity, the number of prime numbers in the repunit number is infinite.

For a finite number n, the conservative estimate of the number of prime numbers is about

$$\frac{1}{\ln(10)}\sum_{\mathrm{x=2}}^{\mathrm{n}}\frac{1}{x}$$

And because the mantissa of all repunit numbers is the number 1, according to the prime number theorem, the more accurate probability that such repunit number is a prime number is

$$\frac{2}{ln(\frac{10^n-1}{9})}$$

Therefore, the more accurate estimate of the repunit prime numbers is

$$\frac{2}{\ln(10)}\sum_{\mathrm{x=2}}^{\mathrm{n}}\frac{1}{x}$$

2. Conclusion

The number of repunit prime numbers is infinite, and for a fixed n, the number of prime numbers is about

$$\frac{2}{\ln(10)}\sum_{\mathrm{x=2}}^{\mathrm{n}}\frac{1}{x}$$

3. Conjectures

According to [6], it has been proved that the number of prime numbers in Mersenne number is

$$\frac{15}{8\ln(2)}\sum_{x=2}^{n}\frac{1}{x}$$

Therefore, the ratio of the number of prime numbers in the repunit number to the number of prime numbers in the Mason number is 16ln (2)/(15ln (10)).

According to the current results, the largest known Mersonne prime number is the 51st, and the number of digits is 24862048 [6]. According to the above formula, the calculated value of the number of prime numbers in the repunit number less than 24862048 digits is 14.42~16.38, and it is estimated that the number of the repunit prime numbers is close to 15.

3. Main References

[1] https://primes.utm.edu/glossary/page.php?sort=Repunit
[2] https://encyclopediaofmath.org/wiki/Distribution_of_prime_numbers
[3] Zhi Li and Hua Li.A Proof of Goldbach Conjecture. https://vixra.org/abs/2111.0093
[4] Zhi Li and Hua Li.Proofs of Twin Prime Number Conjecture and First Hardy-Littlewood Conjecture. https://vixra.org/abs/2111.0098?ref=13171890
[5] Zhi Li and Hua Li.Proof of N^2+1 Conjecture. https://vixra.org/abs/2209.0059
[6] Zhi Li and Hua Li.A Proof of Mersenne Prime Conjecture. https://vixra.org/abs/2209.0018