Critical Behavior in Continuous Dimensions and Early-Universe Cosmology

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Abstract

It is known that large-scale dynamical systems can sustain a rich variety of collective phenomena. This brief note argues that the cosmology of the early Universe can be viewed as critical behavior in continuous dimensions. We find that the self-similar properties of the metric near the Big Bang singularity are comparable to the effects produced by minimal fractality of spacetime far above the electroweak scale.

Key words: critical phenomena, metric oscillations, early Universe cosmology, gravitational singularity, minimal fractal spacetime.

According to [1], the behavior of the spatial metric $\gamma_{\alpha\beta}$ near the time singularity $t=0$ can be studied starting from

$$\gamma_{\alpha\beta} = a^2 l_\alpha l_\beta + b^2 m_\alpha m_\beta + c^2 n_\alpha n_\beta \tag{1}$$

where a^2, b^2, c^2 represent the diagonal elements of the matrix $\gamma_{ab}(t)$ and $\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}$ are unit vectors. Introducing the time-like variable $\eta(t)$ divides the evolution of (1) into a couple of distinct regimes:

1) at large times $\eta >> 1$, the metric coefficients a and b oscillate, while the coefficient c varies exponentially according to

$$a = a_0 \sqrt{\frac{\eta}{\eta_0}} [1 + \frac{A}{\sqrt{\eta}} \sin(\eta - \eta_0)] \tag{2a}$$

$$b = a_0 \sqrt{\frac{\eta}{\eta_0}} [1 - \frac{A}{\sqrt{\eta}} \sin(\eta - \eta_0)] \tag{2b}$$

$$c = c_0 \exp[-A^2(\eta_0 - \eta)] \tag{2c}$$

in which A is a constant. As η falls off from ∞ to about $\eta \approx 1$, the oscillations (2a) and (2b) occur with a slow reduction of their average values ($O(\sqrt{\eta})$) and the functions a and b stay close in magnitude. On the other hand, the function (2c) is monotonically decreasing during all this time. Relations (2a)

– (2c) no longer apply as the parameter η drops below 1 and shifts towards $\eta << 1$.

2) at ultrashort times ($\eta << 1$), metric coefficients and the original time variable t evolve as power law functions, namely,

$$a \propto \eta^{\frac{1+k}{2}} = \eta^{\beta_a(k)} \tag{3a}$$

$$b \propto \eta^{\frac{1-k}{2}} = \eta^{\beta_b(k)} \tag{3b}$$

$$c \propto \eta^{-\frac{1-k^2}{4}} = \eta^{\beta_c(k)} \tag{3c}$$

$$t \propto \eta^{\frac{3+k^2}{4}} = \eta^{\beta_t(k)} \tag{3d}$$

where the arbitrary parameter k lies in the interval $-1 < k < +1$. Using the notation

$$h_i = (a, b, c); \;\; i = 1, 2, 3$$

renders (3a) - (3c) in the condensed form

$$h_i \propto \eta^{\beta_i(k)} \tag{4}$$

Unlike the regime of $\eta >> 1$ determined by (2a) - (2c), the coefficients a and b start to fall off while the magnitude of c ramps up.

These considerations suggest that, passing from early times near the singularity ($t = 0$) to far later times ($t >> 0$), generates a transition from a Universe having a *single space dimension* to a Universe with *two space dimensions*. This behavior is consistent with the *dimensional reduction* conjecture [2-3], according to which spacetime near the Big Bang singularity is effectively two dimensional, having one space and one time dimension only.

The power law relationships (3) and (4) bear a striking resemblance to the scaling of parameters in classical critical phenomena [4 – 5]. A textbook example of such phenomena is provided by spin systems approaching criticality in four spacetime dimensions ($d = 4$), where the correlation length ξ diverges with the reduced temperature τ as in

$$\xi \propto \tau^{-\nu} \tag{5}$$

Here, ν is a positive critical exponent and

$$\tau = (\frac{T}{T_c} - 1) \tag{6}$$

The overall magnetization M of the system assumes the role of the order parameter and scales with τ according to

$$M \propto \tau^{\beta} \tag{7}$$

Here, the critical exponent β also depends on the number of spacetime dimensions and on the critical exponent of the correlation function η^*, i.e.

$$\beta(d) = \frac{1}{2}\nu[d - 2 + \eta^*] \tag{8}$$

The perturbative treatment of the system is based on the dimensionless spin coupling constant

$$\bar{g}(d) = g\xi^{4-d} \tag{9}$$

It is seen from (9) that, near the phase transition at $T = T_c$, the correlation length diverges in less than four spacetime dimensions ($d < 4$) and the perturbative treatment breaks down. On the other hand, the perturbation analysis is enabled again when $d > 4$, as (9) is bounded to stay finite. Solving the tension between $d < 4$ and $d > 4$ stems from the so-called *epsilon expansion method*, whereby spacetime dimension flows in a continuous range of non-integer (fractal) values defined by [2]

$$d = 4 - \varepsilon \tag{10}$$

These remarks indicate that there is a natural analogy between critical behavior of spin systems described by (5) - (10) and the scaling of metric coefficients described by (3) and (4). Replacing (10) in (8) yields

$$\beta(\varepsilon_i) = \frac{1}{2}\nu[2 - \varepsilon_i + \eta^*] \tag{11}$$

where the dimensional deviation is taken to be coordinate dependent, that is, $\varepsilon_i = 4 - d_i$, with $i = 1, 2, 3$.

In this context, a reasonable assumption is that the metric coefficients $h_i = (a, b, c)$ are analogs of the magnetization parameter (7), the time variable η an analog of the reduced temperature (6), and the exponents entering (3) and (4) are analogs of (11). The side-by-side comparison is captured below,

$$h_i \Leftrightarrow M \tag{12a}$$

$$\eta \Leftrightarrow \tau \tag{12b}$$

$$\beta_i(k) \Leftrightarrow \beta(\varepsilon_i) \tag{12c}$$

Furthermore, to make (11) compatible with both the $+$ and $-$ signs of (3a) - (3d), forces one to assume that, in the crossover region $\eta \to 1$, the metric oscillation regime (2a) – (2b) induces large variations of the correlation length (5) and its exponent ν. A possible form of this expected behavior for ν is supplied by

$$\nu(\varepsilon_i)=\begin{cases}>0, & \varepsilon_i>0\\ <0, & \varepsilon_i<0\end{cases} \tag{13}$$

It follows from (13) that (8) turns into

$$\beta(\varepsilon_i)=\frac{1}{2}\nu(\varepsilon_i)[2-\varepsilon_i+\eta] \tag{14}$$

Piecing everything together, relations (3) – (14) link the metric coefficients h_i to the dimensional deviations ε_i, as summarized below

$$\boxed{h_i \propto \eta^{\beta_i(k)} \Leftrightarrow M_i \propto \tau^{\beta(\varepsilon_i)}} \tag{15}$$

This is our main result. In closing, we note that the approach developed here is in alignment with the content of [6 – 10].

References

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