Gravitational Dynamics from the Kolmogorov Entropy

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Abstract

The Kolmogorov (K-) entropy quantifies the continuous transition from deterministic evolution to fully developed chaos. We argue here that, in the early Universe, multi-body Newtonian gravity emerges from the properties of the K-entropy. This finding also suggests that, far above the Fermi scale, gravitational physics and field theory are coexisting manifestations of Hamiltonian chaos in large systems of interacting components.

Key words: Classical gravity, Kolmogorov entropy, N-body systems, early Universe cosmology, Hamiltonian chaos, field unification.

The near-equilibrium regime of star clusters and galaxies can be modeled using the dynamics of Newtonian N-body systems, whose Lagrangian takes the form [1-4]

$$L(x,\mathrm{v}) = \frac{1}{2}\sum\nolimits_{1}^{N} m_a \left|\mathrm{v}_a\right|^2 - U(x) \tag{1}$$

where

$$U(x) = -G\sum\nolimits_{a<b} \frac{m_a\, m_b}{\left|x_a - x_b\right|} \tag{2}$$

As these systems settle down to thermodynamic equilibrium, several of their physical parameters (such as surface luminosity and velocity dispersion) approach stationary values within a relaxation time window (τ) [1-4].

To proceed with our derivation, we introduce a couple of assumptions:

A1) The relaxation time of star clusters and galaxies is given by the Gurzadyan-Savvidy (GS) theory, according to which [1-4].

$$\tau_{GS} \propto \frac{\mathrm{v}}{G M_0 n^{2/3}} \tag{3}$$

Here, v is the average stellar velocity, M_0 the mean stellar mass and n the mass density. The derivation of (3) is based upon treating the N-body problem of Newtonian gravity as geodesic flows on Riemannian manifolds.

A2) The non-relativistic model (1) is a reasonable approximation of the near-equilibrium regime describing star clusters and galaxies in the early Universe.

Unlike the approach behind A1), the basic premise of A2) is that the weak field limit of General Relativity reduces to the Poisson equation [5]

$$R_{00} = \Delta\varphi = 4\pi G n_0 \tag{4}$$

where R_{00} is the temporal component of the curvature tensor, φ the Newtonian potential,

$$g_{00} = 1 + 2\varphi \tag{5}$$

and n_0 the stellar spacetime density.

Straightforward dimensional analysis of densities entering (3) and (4) shows that, while n scales as the inverse of the spatial volume, n_0 scales as the inverse of the four-dimensional spacetime volume as in

$$\left[n\right] = M^3 \tag{6}$$

$$\left[n_0\right] = M^4 \tag{7}$$

Based on (6)-(7), to ensure a sensible comparison between (3) and (4) we choose the parameterization

$$n_0 = M_0 n \tag{8}$$

One obtains, on account of (3), (4) and (8)

$$n_0 = \frac{R_{00}}{4\pi G} \propto (\frac{\mathrm{v}}{\tau_{GS} G})^{3/2} M_0^{-1/2} \tag{9}$$

K-entropy relates to the inverse of the relaxation time τ, as a vanishing τ corresponds to maximal entropy at thermal equilibrium [1]. The magnitude of the K-entropy is computed from the sum of all positive Lyapunov exponents integrated over phase-space according to [6]

$$S_K = \int_{\Omega} \sum_i \lambda_i \, d\rho; \quad \lambda_i > 0 \tag{10}$$

in which Ω denotes the phase space volume, whose differential measure is $d\rho$. The GS relaxation time can be thus expressed as

$$\tau_{GS} = \frac{1}{(dS_K/d\rho)} = \frac{1}{\sum_i \lambda_i}; \quad \lambda_i > 0 \tag{11}$$

Combined use of (9), (10) and (11) yields

$$\boxed{\Delta g_{00} \propto \frac{8\pi}{\sqrt{M_0 G}} \, \mathrm{v}^{3/2} \left(\frac{dS_K}{d\rho}\right)^{3/2}} \tag{12}$$

Within the approximations made above, it is apparent from (3), (4), (11) and (12) that a diverging entropy rate $dS_K/d\rho \to \infty$ leads to an infinite temporal curvature R_{00}, which reflects a highly unstable regime of entropy fluctuations. By contrast, a vanishing entropy rate means a vanishing temporal curvature R_{00} and the onset of thermodynamic equilibrium.

However, this latter setting is unphysical, since – by (4) - it implies zero mass density and an absolute cosmological vacuum.

In closing, we bring up several points that are important for follow up work on the topic:

1) According to the equation (3) of [4], an intriguing relationship exists between τ_{GS} and the fractal dimension of large systems of gravitating bodies. Taking the space dimension to represent a continuous variable $d = 3 - \varepsilon$, in which $\varepsilon << 1$, bridges the gap between the K-entropy, gravitational dynamics and the concept of *minimal fractal manifold*, conjectured to come into play above the Fermi scale [7].

2) An alternative derivation is possible upon using the analogy between gravitational physics and the theory of geodesic flows on Riemannian manifolds. To this end, one can start from the relationship between τ_{GS} and the kinetic energy of gravitational motion W expressed by equation (60) of [1]

$$\tau_{GS} = \sqrt{\frac{W}{(\nabla W)^2}} \tag{13}$$

The proper time interval considered in [1] has the form

$$ds = \sqrt{2}W dt \tag{14}$$

and stands in a one-to-one correspondence with the expression of the proper time interval in the weak field non-relativistic limit of General Relativity [5]

$$ds = \sqrt{g_{00}}\, dt = \sqrt{(1+2\varphi)}\, dt \tag{15}$$

Combining (11) with (13)-(15) links the K-entropy to the Newtonian potential of the N-body problem.

3) It is known that Hamiltonian dynamical systems lie at the heart of classical physics and field theory. Aside from a handful of cases, these systems are *nonintegrable*, as the instability of their phase-space trajectories drives the dynamics into Hamiltonian chaos. It is also known that the chaotic behavior of N-body gravitational systems is on par with Hamiltonian chaos.

In this context and in line with the ideas of [8-12], relation (12) suggests that, far above the Fermi scale, gravitational physics and field theory are coexisting manifestations of Hamiltonian chaos in large systems of interacting components.

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