Bose-Einstein condensate with repulsive interatomic interaction in an anharmonic trap X^4

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Abstract

An ansatz for the Bose-Einstein condensate in an external potential of the form x^4 is proposed. The ansatz was chosen by numerical comparison of the stationary wave packet with different ansatz.
Keywords: Bose-Einstein condensate, anharmonic potential

1. Introduction

The study of the low energy of collective excitations is important for understanding the dynamics of an atomic quantum liquid [1]. Most of these theoretical and experimental studies have been carried out for a condensate in a harmonic trap. The description of the dynamics of the wave function of the condensate in such a trap has many simplified properties for both repulsive and attractive interactions between atoms . The theory is based on the Gross- Pitaevskii equation , which is a non-linear Schrödinger equation with a linear oscillator potential.

An analysis of the repulsive condensate shows that in such a potential the motion of the center of mass of the condensate does not depend on fluctuations in the width of the condensate, and vice versa. The same properties are also valid for the case with possessive BEC, where waves of matter soliton can exist in quasi-1 D geometry. This property can be shown as at the level of symmetries of the 1-dimensional Gross - Pitaevskii equation . and using the method of moments. Individual resonances in the oscillations of the width and position of the soliton were studied in [2].

In the case of an anharmonic trap, the properties of the dynamics of translational motion relative to the trap (movement of the center of mass) and the internal regime (width fluctuation) become coupled. This results in the ability to control internal oscillations by manipulating the position of the trap. This possibility can also be useful in the creation of new technical devices, including quantum computers, and in ultra-sensitive interferometers.

For a soliton wave in a parabolic potential, it is well known that the oscillations of the center of mass are completely separated from the internal excitation, and this is an analogue of Kohn's theorem for a soliton wave packet . the centers of mass of the condensate are independent of each other. This property makes it impossible to control the width of the condensate by changing the position of the condensate relative to the parabolic trap.

In [1], the case of an anharmonic trap $V(x) = x^2 + m x^4$, with a small parameter m , and it is shown that in an anharmonic trap, the fluctuations in the width and coordinates of the center of mass of the condensate are mutually related, namely, the fluctuations of the center of mass lead to fluctuations in the width of the condensate. In asymmetric potentials, it is also possible to control the dynamics of the center of mass by manipulating the values of the condensate width. In this paper, we study the dynamics of a condensate with a focusing nonlinearity in an external potential of the form $V(x) = x^4$. An ansatz for the derivation of ordinary differential equations in the variational approximation is proposed.

2. Model

The dynamics of the quasi-one-dimensional Bose-Einstein condensate is described by the one-dimensional Gross- Pitaevskii equation ,

$$i\hbar\phi_t = -\frac{\hbar^2}{2m}\Delta\phi + sx^4\phi + 2\hbar a_s w|\phi|^2\phi . \quad (1)$$

where m is the mass of the condensate atom, a_s is the scattering length, and w is the frequency of the oscillator in the perpendicular direction. This equation is derived from the 3D Gross- Pitaevskii equation with a strong anisotropic external potential, where the frequencies of the harmonic potential are many times greater in the perpendicular directions than in the longitudinal direction. Dynamics in perpendicular directions is averaged and fluctuations in the longitudinal direction of the condensate are described by equation (1). For the convenience of calculations, and also for the universality of the results, we reduce equation (1) to the following dimensionless form (where *t* and *x* are dimensionless quantities):

$$i\phi_t = -\frac{1}{2}\phi_{xx} + x^4\phi + g|\phi|^2\phi , \quad (2)$$

Here, time *t* and distance *x are* normalized to the frequency and length of the gramonic oscillator, respectively. The subscripts t and x mean differentiations with respect to time and coordinate, respectively, g is the coefficient of two-particle interaction, which we consider to be negative.

Equation (1) can be obtained from the variational equations

$$\frac{\partial L}{\partial \phi *} - \frac{\partial}{\partial x}\frac{\partial L}{\partial \phi_x *} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \phi_t *} = 0, \quad (3)$$

where the superscript * denotes complex conjugation, the Lagrangian density for equation (1) is given by the equation

$$L = \frac{i}{2}(\phi_t \phi * - \phi_t * \phi) - \frac{1}{2}|\phi_x|^2 - \frac{x^2}{2}|\phi|^2 - \frac{g_1}{2}|\phi|^4$$

3. Ansats

For the wave function of the condensate u (x,t) we examined the following ansatzes :

$$\phi = A(t)\exp(-\frac{(x-x_0(t))^2}{2a^2(t)} + ik(t)(x-x_0(t)) + \frac{ib(t)(x-x_0(t))^2}{2}), \quad (4)$$

$$\phi = A(t)\exp(-\frac{|x-x_0(t)|^3}{3a^3(t)} + ik(t)(x-x_0(t)) + \frac{ib(t)(x-x_0(t))^2}{2}), \quad (5)$$

$$\phi = A(t)\exp(-\frac{(x-x_0(t))^4}{4a^4(t)} + ik(t)(x-x_0(t)) + \frac{ib(t)(x-x_0(t))^2}{2}), \quad (6)$$

$$\phi = A(t)\sec h(\frac{x-x_0(t)}{w(t)})\exp(ik(t)(x-x_0(t)) + ib(t)(x-x_0(t))^2) \quad (7)$$

$$\phi = A(t)\sec h(\frac{(x-x_0(t))^2}{a^2(t)})\exp(ik(t)(x-x_0(t)) + ib(t)(x-x_0(t))^2) \quad (8)$$

$$\phi = A(t)\sec h(\frac{(x-x_0(t))^3}{a^3(t)})\exp(ik(t)(x-x_0(t)) + ib(t)(x-x_0(t))^2) \quad (9)$$

where the parameters A , a , b , x 0 k are amplitude, width, chirp , center of mass and speed , respectively. The first of these ansatz , the Gaussian ansatz , showed a good approximation, in the case of a harmonic external potential.

For our case, of the considered ansatz, according to our calculations , ansatz (3) is the most suitable. Ansatz (7-8) have a problem in getting the averaged Lagrangian.

4. Links

1. Abdullaev , F., Galimzyanov , R.M., Ismatullaev , Kh ., // J. Phys. B. 2008. 41. P. _ 015301

2. Abdullaev , F., Garnier, J., Phys. Rev. A // 200. 70, P. 053604