Towards the explanation of flatness of galaxies rotation curves

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Abstract

We suggest a new explanation of flatness of galaxies rotation curves without invoking dark matter. For this purpose a new gravitational tensor field is introduced in addition to the metric tensor.

1 Introduction

Velocities of stars and gas rotation around galaxies centers become independent of rotation radii at large enough radii, see e.g. [1] and references therein. This asymptotical flatness of galaxies rotation curves is in obvious contradiction with Newtonian dynamics which demands that velocities should decrease with radii as $1/\sqrt{r}$.

The contradiction is explained within the dark matter paradigm [2]. Within the paradigm a galaxy is placed in a spherical halo of dark matter [3] with mass density of dark matter decreasing as $1/r^2$. Hence the Newtonian gravitational potential becomes logarithmically dependent on r providing a flat rotation curve. But hypothetical constituents of dark matter are still not discovered in direct experiments inspite of numerous searches.

In this situation it is worthwhile to develope explanations of flatness of galaxies rotation curves which do not invoke dark matter. Probably the most known attempt of this type is modified Newtonian dynamics [4] where it is assumed that gravitational forces depend on accelerations of objects participating in interactions. Modified Newtonian dynamics has essential phenomenological successes. In particular it described the known Tully-Fisher relation [5] which establishes correlation between a galaxy luminosity and a corresponding flat rotation speed. But modified Newtonian dynamics does not describe e.g. gravitational lensing by galaxies and galaxies clusters. Modified Newtonian dynamics is not a completely formalized theory although there was an attempt to construct its complete Lagrangian version [6] which was not supported experimentally.

It should be mentioned that the Tully-Fisher relation is more precise in its barionic form which states that a flat rotation speed in a galaxy correlates with its barionic mass i.e. a sum of stars and gas masses, see [7] and references therein. The barionic Tully-Fisher relation states that $M_{bar} \propto V_{flat}^4$. This tigh correlation between visible matter of a galaxy and a corresponding flat rotation speed is also a rather strong motivation to find the explanation of flatness of galaxies rotation curves without dark matter.

In the present paper we suggest a new explanation of flatness of galaxies rotation curves without invoking dark matter. For this purpose a new tensor gravitational field is introduced in addition to the metric tensor.

2 Main part

We consider the General Relativity action plus tems with a new gravitational tensor field $f_{\mu\nu}$:

$$S = \int d^4x\sqrt{-g}(-M_{Pl}^2 R + g_{\mu\nu}T^{\mu\nu} + M_{Pl}^2\Lambda \tag{1}$$

$$+f_{\mu\nu}(D^\lambda D_\lambda)^{3/2} f^{\mu\nu} + G^* \sum_i \frac{1}{\sqrt{1+m_i/m^*}} f T_i^{\mu\nu}),$$

here the R-term is the Einstein-Hilbert Lagrangian of General Relativity. $M_{Pl}^2 = 1/(16\pi G)$ is the Planck mass squared.

D_λ is the covariant derivative, for example:

$$D_\lambda f^{\mu\nu} = \frac{\partial}{\partial x^\lambda} f^{\mu\nu} + \Gamma^\mu_{\lambda\sigma} f^{\sigma\nu} + \Gamma^\nu_{\lambda\sigma} f^{\mu\sigma}, \tag{2}$$

where the Christoffell symbols are

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta} \left(\partial_\nu g_{\mu\beta} + \partial_\mu g_{\nu\beta} - \partial_\beta g_{\mu\nu}\right). \tag{3}$$

$f_{\mu\nu}$ is the new tensor field.

G^* is the new coupling constant of interaction of the tensor field $f_{\mu\nu}$ with matter fields.

m^* is a new mass parameter.

The sum $\sum_i$ goes over matter objects drscribed by energy-momentum tensors $T_i^{\mu\nu}$ and having masses m_i. Couplings of them with the field $f_{\mu\nu}$ depend on m_i via $\frac{1}{\sqrt{1+m_i/m^*}}$; this is used below to reproduce the barionic Tully-Fisher relation.

Numerical values of new constants G^* and m^* should be fixed from fitting experiments, but this is a subject for a separate publication.

The Λ-term is not essential in perturbation theory which we will consider.

We use the system of units $\hbar = c = 1$.

To quantize the theory (1) selfconsistently one should add to the Lagrangian all possible terms quadratic in the Riemann tensor $R_{\mu\nu\rho\sigma}$, see [8], [9] where pertutbatively reormalizable and unitary model of quantum gravity was for the first time formulated. But these terms are not essential for the present considerations.

We work within perturbation theory, hence a linearized theory around the flat metric $\eta_{\mu\nu}$ is considered:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{4}$$

here the convention in four dimensions is $\eta_{\mu\nu} = diag(+1, -1, -1, -1)$. Indexes are raised and lowered by means of the tensor $\eta_{\mu\nu}$.

Within perturbation theory one makes the shift of the field

$$h_{\mu\nu} \to M_{Pl} h_{\mu\nu}. \tag{5}$$

Perturbative expansion goes in inverse powers of M_{Pl} or in powers of the Newton coupling constant $G = \frac{1}{16\pi M_{Pl}^2}$.

Let us get the propagator of the tensor field $f_{\mu\nu}$. We take the quadratic in $f_{\mu\nu}$ part of the Lagrangian and perform the Fourier transform:

$$Q = i(2\pi)^4 \int d^4k \ f^{\mu\nu}(-k) \left[(k^2)^{3/2} \eta_{\mu\rho} \eta_{\nu\sigma} \right] f^{\rho\sigma}(k), \tag{6}$$

To obtain the propagator $D_{\mu\nu\rho\sigma}$ of the field $f_{\mu\nu}$ we invert the matrix in brackets of (6):

$$[Q]_{\mu\nu\kappa\lambda} D^{\kappa\lambda\rho\sigma} = \frac{1}{2}(\delta^\rho_\mu \delta^\sigma_\nu + \delta^\sigma_\mu \delta^\rho_\nu). \tag{7}$$

Then the propagator has the form

$$D_{\mu\nu\rho\sigma} = \frac{1}{2i(2\pi)^4} \frac{\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho}}{(k^2)^{3/2}}. \tag{8}$$

The propagator (8) generates the logarithmic gravitational potential. At large disrances r (i.e. at galactic and extragalactic scales) this $\log r$- potential starts to dominate over the Newtonian $1/r$- potential which is generated by the field $h_{\mu\nu}$ of General Relativity for small potentials.

Thus the gravitational field $f_{\mu\nu}$ generates a $1/r$- force between a point object with a mass M_{bar} having the energy-momentum tensor $T_{\mu\nu} = \delta^0_\mu \delta^0_\nu M_{bar} \delta^3(x)$ (describing a galaxy with the baarionic mass M_{bar} of stars plus gas) and an analogous object with a mass M_{star} (describing a star with the mass M_{star}):

$$F = (G^*)^2 \frac{M_{bar}}{\sqrt{1 + M_{bar}/m^*}} \frac{M_{star}}{\sqrt{1 + M_{star}/m^*}} \frac{1}{r}. \tag{9}$$

At M_{bar} large compared to m^* the above relation (9) become

$$F \approx (G^*)^2 \sqrt{M_{bar}} \frac{M_{star}}{\sqrt{1 + M_{star}/m^*}} \frac{1}{r}. \tag{10}$$

From the other side according to the Newton law

$$F = M_{star} \frac{V_{flat}^2}{r}. \tag{11}$$

Equating (10) and (11) we get the relation

$$M_{bar} \approx \frac{V_{flat}^4}{(G^*)^2} \sqrt{1 + M_{star}/m^*}. \tag{12}$$

The expression (12) reproduces the barionic Tully-Fisher relation (which states that $M_{bar} \propto V_{flat}^4$). It is interesting to note that r.h.s. of (12) has the M_{star}-dependence.

Thus flat rotation speeds of stars satisfy the barionic Tully-Fisher relation in our model.

We should also mention that the introduced tensor field $f_{\mu\nu}$ interacts with light and that is why adds additional, as compared to the metric field $h_{\mu\nu}$, gravitational lensing due to barionic matter.

3 Conclusions

We suggested a new explanation of flatness of galaxies rotation curves without invoking dark matter. For this purpose a new tensor gravitational field is introduced in addition to the metric tensor. Flat rotation speeds of stars in our model satisfy the known barionic Tully-Fisher relation.

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