The radiation of RLC circuit with the longitudinal capacitor

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Abstract

The RLC circuit is generalized in such a way that the capacitor has longitudinal form and the components are all in series with the voltage source $(R - L - C - v)$. The medium inside the capacitor is dielectric with the index of refraction n. The change of the amount of charges on the left and right side of the capacitor generate in dielectric medium special radiation which is not the Čerenkov radiation, no the Ginzburg transition radiation but the original radiation which must be confirmed in laboratories.

We have calculated the spectral form of the radiation of RLC circuit with the longitudinal capacitor. It depends on the dielectric constant n of the capacitor medium. The defect in medium is involved in the spectral form and can be compared with the original medium. Such comparison is the analog of the Heyrovský-Ilkovič procedure in the electro-chemistry (Heyrovský et al., 1965).

The article is the preamble for the future investigation of electronic physics and can be integral part of such institutions as Bell Laboratories, NASA, CERN and so on.

1 Introduction

An RLC circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series, or, in parallel. The circuit forms a harmonic oscillator for current.

The three circuit elements, R, L and C can be combined in a number of different topologies. All three elements in series or all three elements in parallel are the simplest in concept and the most straightforward to analysis. There are, however, other arrangements, some with practical importance in real circuits. One issue often encountered is the need to take into account inductor resistance. Inductors are typically constructed from coils of wire, the resistance of which is not usually desirable, but it often has a significant effect on the circuit.

2 Series RLC circuit

In the situation where we consider series RLC circuit, the three components are all in series with the voltage source $(R - L - C - v)$. The governing differential equation can be found by substituting into the Kirchhoff voltage law (KVL) the constitutive equation for each of the three elements. From KVL it follows

$$v_R + v_L + v_C = v(t), \tag{1}$$

where v_R, v_L, v_C are the voltages across R, L, C respectively and $v(t)$ is the time varying voltage from the source. Substituting the corresponding physical term, in eq. (1), we get the following integral differential equation (Nilsson et al., 2008):

$$Ri(t) + L\frac{di}{dt} + \frac{1}{C}\int_{-\infty}^{\tau=t} i(\tau)d\tau = v(t). \tag{2}$$

To our goal it is sufficient to consider the more simple situation with $v = 0$. Then, instead of eq. (2) we write

$$L\ddot{Q} + R\dot{Q} + Q/C = 0 \tag{3}$$

with stationary solution

$$Q = Ae^{-\frac{R}{2L}t}\sin(\omega t + \alpha), \tag{4}$$

where

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}. \tag{5}$$

The Thomson formula for the period of oscillations is when $R = 0$:

$$T = 2\pi/\omega = 2\pi\sqrt{LC}. \tag{6}$$

3 Two $R - L - C-$ circuits with the mutual induction

In this case the components R_1, L_1, C_1 are inductive boned with R_2, L_2, C_2 configuration. The problem was published by Landau et al. (1989) and the frequency of the inductive system was calculated in the form (for $R_1 = R_2 = 0$).

$$\omega_{1,2}^2 = \frac{L_1C_1 + L_2L_2 \pm [(L_1C_1 - L_2C_2)^2 + 4C_1C_2L_{12}^2]^{1/2}}{2C_1C_2(L_1L_2 - L_{12}^2)}. \tag{7}$$

4 Remarks on the Čerenkov and Ginzburg radiation

While the Čerenkov electromagnetic radiation is generated by a fast moving charged particle in a medium when its speed is faster than the speed of light in this medium, our bunches of charges are not in linear motion. Nevertheless, the oscillation of the magnitude of charges cause electromagnetic processes in the dielectric with the emergence of the electromagnetic radiation of this medium. The radiation is in no case the Čerenkov radiation or, the Ginzburg transition radiation.

The Ginzburg transition radiation was demonstrated theoretically by Ginzburg and Frank many decades ago (Ginzburg, 1940; Frank, 1942). They showed the existence of transition radiation when a charged particle perpendicularly passed through a boundary between two different homogeneous media. The frequency of radiation emitted in the backwards direction relative to the particle was mainly in the range of visible light. The application of the optical transition radiation for the detection and identification of individual particles is limited due to the low intensity of the radiation.

We derive in this paper[1] the spectral formula of the radiation of the longitudinal capacitor in the framework of the source theory. We suppose that the bunches of charges are at the ends of the capacitor and the quantity of the total charges is in the accord with the *RLC* oscillation, which manifests in the spectral formula.

5 The source theory formulation of the problem

Source theory (Schwinger et al., 1976; 1970; Dittrich, 1978) is the theoretical construction that uses quantum-mechanical particle language. It was found that the original formulation simplifies the calculations in the electrodynamics and gravity where the interactions are mediated by the photon, or, graviton, respectively.

The basic formula in the source theory is the vacuum to vacuum amplitude (Schwinger et al., 1976):

$$\langle 0_+ | 0_- \rangle = e^{\frac{i}{\hbar} W(S)}, \tag{8}$$

where the minus and plus signs on the vacuum symbol are causal labels, referring to any time before and after the space-time region where sources are manipulated. The exponential form is introduced with regard to the existence of the physically independent experimental arrangements, which has a simple consequence that the associated probability amplitudes multiply and corresponding W expressions add (Schwinger et al., 1976; 1970; Dittrich, 1978).

The electromagnetic field is described by the amplitude (4) with the action

$$W(J) = \frac{1}{2c^2} \int (dx)(dx') J^\mu(x) D_{+\mu\nu}(x - x') J^\nu(x'), \tag{9}$$

[1]This contribution is the analogue version of the article: M. Pardy, Phys. Rev. A **55**, No. 3, 1647 (1997).

where the dimensionality of $W(J)$ is the same as the dimensionality of the Planck constant $\hbar$. J_μ is the charge and current densities. The symbol $D_{+\mu\nu}(x - x')$ is the photon propagator and its explicit form will be determined later.

It may be easy to show that the probability of the persistence of vacuum is given by the following formula (Schwinger et al., 1976):

$$|\langle 0_+|0_-\rangle|^2 = \exp\{-\frac{2}{\hbar}\mathrm{Im}\, W\} \stackrel{d}{=} \exp\{-\int dt d\omega \frac{P(\omega, t)}{\hbar\omega}\}, \tag{10}$$

where we have introduced the so-called power spectral function (Schwinger et al., 1976) $P(\omega, t)$. In order to extract this spectral function from $\mathrm{Im}\, W$, it is necessary to know the explicit form of the photon propagator $D_{+\mu\nu}(x - x')$.

The electromagnetic field is described by the four-potentials $A^\mu(\phi, \mathbf{A})$ and it is generated by the four-current $J^\mu(c\varrho, \mathbf{J})$ according to the differential equation (Schwinger et al., 1976)

$$\left(\Delta - \frac{\mu\varepsilon}{c^2}\frac{\partial^2}{\partial t^2}\right) A^\mu = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2 - 1}{n^2}\eta^\mu\eta^\nu\right) J_\nu \tag{11}$$

with the corresponding Green function $D_{+\mu\nu}$:

$$D_+^{\mu\nu} = \frac{\mu}{c}\left(g^{\mu\nu} + \frac{n^2 - 1}{n^2}\eta^\mu\eta^\nu\right) D_+(x - x'), \tag{12}$$

where $\eta^\mu \equiv (1, \mathbf{0})$, μ is the magnetic permeability of the dielectric medium with the dielectric constant ε, c is the velocity of light in vacuum, n is the index of refraction of this medium, and $D_+(x - x')$ was derived by Schwinger, Tsai and Erber (Schwinger et al., 1976) in the following form:

$$D_+(x - x') = \frac{i}{4\pi^2 c}\int_0^\infty d\omega \frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|} e^{-i\omega|t - t'|}. \tag{13}$$

Using formulas (9), (10), (12), and (3), we get for the power spectral formula the following expression (Schwinger et al., 1976):

$$P(\omega, t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'dt' \frac{\sin\frac{n\omega}{c}|\mathbf{x} - \mathbf{x}'|}{|\mathbf{x} - \mathbf{x}'|}\cos[\omega(t - t')] \times$$
$$\times \left\{\varrho(\mathbf{x}, t)\varrho(\mathbf{x}', t') - \frac{n^2}{c^2}\mathbf{J}(\mathbf{x}, t)\cdot\mathbf{J}(\mathbf{x}', t')\right\}. \tag{14}$$

Now, we are prepared to apply the last formula to the situations of the two equal charges moving in the dielectric medium.

6 Radiation of the longitudinal capacitor

While the Čerenkov radiation in electrodynamics is produced by uniformly moving charge with constant velocity, author (Pardy, 1997) considered the system of two equal charges e with the constant mutual distance $a = |\mathbf{a}|$ moving with velocity $\mathbf{v}$ in the dielectric medium. In this situation the charge and the current densities for this system are given by following equations:

$$\varrho = e[\delta(\mathbf{x}-\mathbf{v}t)+\delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t)], \tag{15}$$

$$\mathbf{J} = e\mathbf{v}[\delta(\mathbf{x}-\mathbf{v}t)+\delta(\mathbf{x}-\mathbf{a}-\mathbf{v}t)], \tag{16}$$

where $\mathbf{a}$ is the vector going from the left charge to the right charge with the length of $a = |\mathbf{a}|$ in the system S. This system was used by author to determine the Lorentz contraction from the Čerenkov spectral formula (Pardy, 1997).

The charge of moving particle was constant. We consider here the situation where a charge in the experiment is dependent on time, or $e \to Q(t)$ and it is at rest. So, instead of eqs. (15) and (16) we have:

$$\varrho = Q(t)[\delta(\mathbf{x})+\delta(\mathbf{x}-\mathbf{a}], \tag{17}$$

$$\mathbf{J} = 0 \tag{18}$$

However, the problem can be simplified if we consider only the left side of the capacitor in calculation of the spectral density of radiation. Or,

$$\varrho = Q(t)\delta(\mathbf{x}), \tag{19}$$

$$\mathbf{J} = 0. \tag{20}$$

Then, after insertion of eq. (19) and (20) into eq. (14), putting $\tau = t' - t$, we get instead of the formula (14) the following relation

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\int d\mathbf{x}d\mathbf{x}'d\tau\frac{\sin\frac{n\omega}{c}|\mathbf{x}-\mathbf{x}'|}{|\mathbf{x}-\mathbf{x}'|}\cos[\omega(\tau)]$$

$$Q(\mathbf{x},t)Q(\mathbf{x}',\tau+t)\delta(\mathbf{x})\delta(\mathbf{x}'). \tag{21}$$

The density of charges on the left and right side of capacitor are evidently periodic, so we can write

$$Q(t) = A\sin\Omega t, \quad Q(t') = A\sin\Omega(\tau+t), \tag{22}$$

where A is some experimental constant.

After some modification of the formula (21) with regard to the formula (22), we get:

$$P(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\left(\frac{n\omega}{c}\right)A^2\sin\Omega t\int_{-\infty}^{\infty}d\tau\cos(\omega\tau)\sin\Omega(\tau+t) \tag{23}$$

where

$$\cos(\omega\tau)\sin\Omega(\tau-t) = \cos(\omega\tau)\left[\sin(\Omega\tau)\cos(\Omega t)+\cos(\Omega\tau)\sin(\Omega t)\right] \tag{24}$$

So we can write formula (23) as a sum:

$$P(\omega,t) = P_1(\omega,t)+P_2(\omega,t), \tag{25}$$

where

$$P_1(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\left(\frac{n\omega}{c}\right) A^2 \sin\Omega t \cos\Omega t \int_{-\infty}^{\infty} d\tau \cos(\omega\tau)\sin(\Omega\tau) \tag{26}$$

$$P_2(\omega,t) = -\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\left(\frac{n\omega}{c}\right) A^2 \sin(\Omega t)\sin(\omega t) \int_{-\infty}^{\infty} d\tau \cos(\omega\tau)\cos(\Omega\tau) \tag{27}$$

We have for τ-integrals

$$\int_{-\infty}^{\infty} d\tau \cos(\omega\tau)\sin(\Omega\tau) = \left[\frac{\sin(\Omega-\omega)\tau}{2(\Omega-\omega)} + \frac{\sin(\Omega+\omega)\tau}{2(\Omega+\omega)}\right]_{-\infty}^{\infty} = I_1 \tag{28}$$

$$\int_{-\infty}^{\infty} d\tau \cos(\omega\tau)\cos(\Omega\tau) = -\left[\frac{\sin(\Omega+\omega)\tau}{2(\Omega+\omega)} + \frac{\sin(\Omega-\omega)\tau}{2(\Omega-\omega)}\right]_{-\infty}^{\infty} = I_2 \tag{29}$$

So, with regard to eqs. (26)–(29) we have

$$P(\omega,t) = P_1(\omega,t) + P_2(\omega,t) =$$

$$\left[-\frac{\omega}{4\pi^2}\frac{\mu}{n^2}\right]\left(\frac{n\omega}{c}\right) A^2 \sin(\Omega t)\left[\sin(\Omega t) I_1 + \cos(\Omega t) I_2\right] \tag{30}$$

Now, the problem is only to evaluate integrals I_1, I_2 using the corresponding regularization technique. Nevertheless, the fundamental information of the original longitudinal capacitor effect is involved in eq. (30).

7 Discussion

We have calculated the spectral form of the radiation of *RLC* circuit with the longitudinal capacitor. It depends on the dielectric constant n of the capacitor medium. The defect in medium is involved in the spectral form and can be compared with the original medium. Such comparison is the analog of the Heyrovský-Ilkovič procedure in the quantum electrochemistry (Heyrovský et al., 1965).

We have considered only the *RLC* circuit where the components of it are all in series with the voltage source, or in the form $(R - L - C - v)$. The generalization to the more complex situation was not considered.

We have demonstrated that in the case of the system of two equal bunches of charges, the new original effect was realize which was not still considered in theory and experiment. After performing the experiment with the *RLC* circuit the unique effect will be definitely confirmed.

We have not considered for the sake of simplicity the radiative corrections to the photon propagator and in order to get the modified power spectral formula of the emitted radiation. However, the radiative corrections have meaning for the gamma photons rather than the optical ones. Nevertheless, the possibility of the existence of the gamma radiation can be considered as the relevant physical effect.

The experiments suggested by us are feasible in the sense that the bunches of charges are generate in *RLC* circuit and therefore it is not necessary to prepare substantially the new arrangement of the equipment for the verification of the new effects. Our result represents the synergism of the invention and the discovery of the new effect. We hope that experiments with the *RLC* circuit with the longitudinal capacitor will be sooner or later performed by all electronic laboratories over the world.

The article forms the preamble of the future investigation of electronic systems (Nillson et al. 2008) and it will be, no doubt, the integral part of such institutions as Bell Laboratories, NASA, CERN and all laboratories over the world.

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