A note on neutrino oscillations,
or
why MiniBooNE and MicroBooNE data are compatible

Andrea Gregori†

Abstract

The theoretical scenario introduced in Ref [1] implies a new approach to flavour mixing, and neutrino oscillations. It not only predicts non-vanishing neutrino masses, but also uniquely determines their values as functions of the age of the universe. We compare here the predicted neutrino mixing with the experimental data from atmospheric (Super-Kamiokande) and accelerator neutrinos (MiniBooNE, MicroBooNE), finding a substantial agreement with the experiments. In particular, a mismatch between MiniBooNE and MicroBooNE data is predicted that fits the measured data. Within this scenario, we can also make a prediction for the forthcoming SBDN and ICARUS neutrino experiments.

†e-mail: gregori.research@gmail.com

Contents

1 Introduction

Neutrino physics is one of the frontiers of research, where the current approach based on the standard model (SM) may fail. The SM has room for plenty of free parameters, among which possible neutrino mass terms, off-diagonal with respect to the interaction currents. Non-vanishing neutrino masses allow to implement neutrino oscillations, a phenomenon by now confirmed by several experimental observations. However, once the mass terms and the mixing angles are fixed by fitting some experiments, they are no more free parameters, and the predictions of the model can be tested on other, independent experiments. The SM fails on an important test, the MiniBooNE experiment on accelerator neutrinos [2], proving that either so-called new physics (new kind of particles and interaction terms) or, perhaps, a new theoretical approach, are needed. In order to justify the mismatch with the experimental results, that in the case of MiniBooNE show a higher degree of neutrino mixing/oscillation than expected, several corrections to the SM have been proposed, mainly retaining the basic approach, and introducing heavy, sterile neutrinos. Among the problems of this approach is the naturalness of mass terms, namely, justifying highly unbalanced strengths of Higgs coupling. To complicate things, MicroBooNE, a more recent experimental set aimed at refining the measurements made by MiniBooNE, does not find any excess of electron's events, finding instead data compatible with the range of values allowed by the SM, apparently in contradiction with MiniBooNE (see Refs. [3, 4, 5, 6]).

In this note we consider the problem from the point of view of the theoretical framework described in [1], a non-field theoretical framework with no free parameters, in which neutrinos are massive Dirac spinors, with masses determined outside of gauge theory (which is there only an approximation) and therefore without Higgs-mechanism. In this scenario, the differences between the masses of the three neutrinos are huge: the muon's neutrino mass is around two orders of magnitude larger than the electron's neutrino mass (this last of the order of the electronvolt), and the tau neutrino is in turn around more than one order of magnitude heavier than the muon's neutrino. As a consequence, for any particle family pair the oscillation period of the heavier neutrino is much shorter than the oscillation of the lighter one. From the point of view of the lighter family, the wavefunction of the heavier neutrino appears as averaged overall several periods to a constant value. The oscillation expressions must therefore be modified accordingly, leading to remarkable differences as compared to the usual oscillation pattern.

In order to investigate, and test, this approach, we consider the detection of atmospheric neutrinos at Super-Kamiokande and the accelerator neutrinos of MiniBooNE and MicroBooNE. These experiments provide solid data, not much dependent on specific models concerning the initial amount of neutrinos, to which the detected amount has to be compared: in the case of Super-Kamiokande, this dependence is virtually absent, because the experiment consists in comparing two detections made at two different distances from the source. In the case of MiniBooNE and MicroBooNE, producing neutrinos in laboratory gives anyway more control on the source than relying on solar or supernova models, that should be entirely reconsidered within our theoretical framework. Therefore, for these reasons we will not consider here solar or supernova neutrino experiments.

In the following, we start by shortly introducing the theoretical scenario in which the neutrino masses and their mixing coefficients (the entries of the PMNS matrix) are calculated. The reader who is not interested in these aspects can just accept the results and jump to section 3 at page 4.

The effective theory of the theoretical scenario

The scenario we are going to consider is a non-field theoretical framework in which the space of the expanding universe is of finite extension, and there is a minimal distance, the Planck length. The appendix at page 10 is a brief introduction to the setup, in particular to concepts we will refer to, such as "entropy of geometries" and "entropy of string constructions" (for a more detailed description we refer the reader to Ref. [1]). Although the framework is non-field theoretical, the dynamics of local experiments, such as those relevant for this discussion, can nevertheless be approximately described by a superstring-derived field theoretical effective action, in which masses and couplings are treated as external inputs, and are not introduced by a Higgs mechanism [1]. They depend on the age of the universe. In the limit in which the space becomes infinitely extended, the spectrum becomes massless. Therefore, although the introduction of masses as external parameters without a Higgs mechanism explicitly breaks gauge symmetry, it makes sense to approximate the dynamics by an effective gauge field theory with massless fields, because the approximation gets better and better as the universe evolves. At any finite time, we need not care about cancellation of infinities because we know that the theory is finite, being just an approximate description of a theory defined on a space of finite extension and with a minimal distance. It can therefore be treated as if it were a gauge theory, with the following important modification: the results of Feynman diagram computations performed on gauge field theory with the same massless spectrum can be used, except for the dynamics of the Higgs field, which in our context does not exist. In practical computations, it is like working with a very heavy Higgs field. The latter can be considered decoupled to all effects, and the only term that matters is its coupling to the matter states. This just results in mass terms, because all what pertains the dynamics of the Higgs field is suppressed: the Higgs propagation is neglected here.

A key point of this scenario is that quantization is introduced as an implementation of the fact that the world at any instant is a collective effect in which any observable receives contribution from all possible geometries of space-time. As a consequence, differently from the field theory approach, the expansion in Feynman diagrams is here a representation of paths that contribute to the statistics, and therefore to the entropy of the physical process, with vertices weighted by coupling amplitudes, which in our framework measure the relative weights of interactions.

As long as one just considers a purely perturbative phase of the theory, the usual field theory computations through Feynman diagrams are valid for the determination of the amplitude also in our framework. As discussed in [7], when non-perturbative aspects play a relevant role, this scenario presents significant departures from the field theory approach.

Only one free parameter

As discussed in Ref. [8], in the most entropic string geometries, those that shape the physical world, S-duality is broken at the Planck energy scale. Below this scale, the spectrum admits an approximate description in terms of fields and couplings of an effective quantum field theory with gauge interactions. In this theory there is only one free parameter, the age of the universe. Couplings and masses of the elementary particles scale like powers of the inverse of the age of the universe. Leaving a detailed discussion of this rather non-trivial result to another place (see Ref. [1]), let us just give here a flavour of how this may be possible. In this theory the universe has basically the geometry of a 3-sphere, whose radius expands at the speed of light, hence the identification, up to appropriate conversion of units, of radius and age of the universe. Masses arise as ground momenta in a quantum space of finite extension. The existence of a more complex structure attached to

[1]The scattering resonances usually referred to as evidence of the production of a Higgs boson receive in this scenario a different interpretation (for a detailed discussion, see Ref. [1], chapter 4, section 5.2).

space-time, namely a superposition of higher dimensional string structures, plays a determinant role in giving rise to a spectrum with an entire hierarchy of masses, not simply just the inverse of the radius of the sphere, or age of the universe. Moreover, what makes possible to relate different masses to different powers of such a dimensional parameter like the age of the universe is the fact that the 3-sphere is just a representation in the continuum of a theory that fundamentally deals with combinatorics of abstract, dimensionless discrete "cells" of unit length that may be occupied or not, by a unit of an occupation number that rather in abstract we call "energy". In this way we can identify geometries through Einstein's equations, but fundamentally the radius of the universe, or its total energy, or its age, are just dimensionless numbers, of which we can take any power *before* giving them a physical meaning by introducing the units c, $\hbar$ and the Planck mass. Dimension is an attribute we give to numbers, fundamental are numbers and their combinatorics.

2 Flavour mixing from a different perspective

In our scenario physics is the result of a weighted sum over all geometries. There is no *pure* state, and flavour mixing is a "built-in" aspect of the theory. The amount of overlapping, or better the amount of projection of one family into the other, is basically determined by their relative weight in the phase space. From this perspective, the description in terms of propagating mass states rotated with respect to the interaction's current states is not the fundamental picture, but just an effective description, which doesn't always correctly account for the physical phenomenon. As discussed in [1], the spectrum of elementary particles is analyzed in a representation of the scenario in which the kinetic contribution to the phase space is separated from the treatment of couplings and masses, dealt with like external parameters. In this picture, once decoupled from the kinetic contribution the relative weights of particles families are given by mass ratios, and are related to the coupling strength of the symmetry group that rotates one family into the other. For the leptons, this turns out to be in first approximation an $SU(2)$ group (not to be confused with the chiral $SU(2)_w$ group of weak interactions). In the case of quarks the relation is a bit more complicated [1].

Let us first briefly consider the case of quarks. In order to make contact with the usual parametrization of the experimental data, we will convert our scenario in terms of the CKM mixing matrix. When translating the entropy-sum driven scenario into the parameters of an effective field theory, one has to consider *how* quantities are measured, namely what kind of experiment we want to effectively describe. Quark mixing is measured via meson decays into other mesons, and can be parametrized through corrections to the coupling strength of their interaction:

$$g \;\to\; g \times (m_i/m_j)\,, \tag{2.1}$$

where i and j indicate the mesons π, K and B. The CKM matrix elements are of type $V_{\text{up,down}}$ and can be viewed as corrections to the effective coupling of the vertices. This implies that the amplitudes are proportional to:

$$|V_{\text{up}_i,\text{down}_j}|^2 \;\approx\; (m_i/m_j)^2 \tag{2.2}$$

where (m_i/m_j) is the ratio of the up (or down) mass of family i to the the up (or down) mass of family j (see figure 1) [2]. The CKM matrix entries are generated by unitarity from V_{us}, V_{ub} and V_{cb}. The relation of amplitudes to the elementary degrees of freedom is here complicated by the fact that quarks enter into the amplitude expressions through multiplications and resummations. Strong coupling corrections and the presence of several intermediate decay channels that contribute through a non-direct flavour changing to the overall decay amplitudes correct the ratios we would infer by simply considering bare mass values. The volumes of the events are therefore not so simply related to ratios of bare masses. A better approximation is obtained by using meson masses instead of quark masses. From the meson mass ratios we obtain (see appendix B):

$$\text{CKM} \;=\; \begin{pmatrix} 0.97549796 & 0.219999569 & 0.00198 \\ -0.219855965 & 0.974447209 & 0.04599991 \\ 0.008190555 & -0.045308133 & 0.998939482 \end{pmatrix}, \tag{2.3}$$

which parametrizes the quark mixing at the present age of the universe.

[2] Notice that, while the matrix elements relate the "up" of one family to the "bottom" of the other one, the $SU(2)$ symmetry relates bottom to bottom or up to up states.

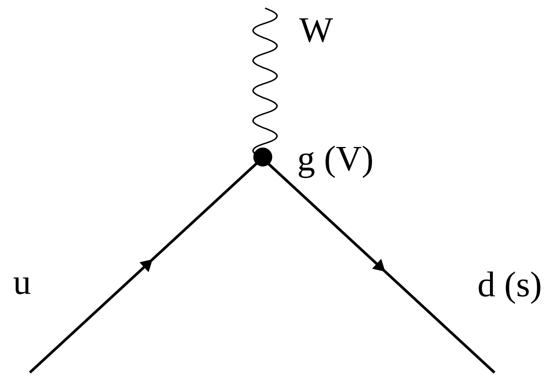

Figure 1: Quark mixing corrects interaction vertices: $g \to gV$.

3 Neutrino oscillations

In this theoretical framework neutrinos are massive Dirac fermions. This means that they have both a left and a right handed part. As we anticipated, the mass does not arise from a Higgs mechanism, but is kind of a Casimir effect in a universe whose dominant space geometry has finite extension. Since space expands, masses depend on the age of the universe. The relation between masses and age of the universe can only be investigated outside of field theory. Neutrino masses are therefore not related to Yukawa couplings. Despite the existence of the right-handed part, there is no need of thinking of a see-saw mechanism to balance heavy and light part: neutrino masses turn out to be small in a natural way. On the other hand, being neutrinos Dirac spinors, in this scenario we do not expect a neutrinoless double beta-decay to occur. The neutrino masses at the present age of the universe were computed in Ref. [1] [3]:

$$m_{\nu_e} = 1.40\,\mathrm{eV}\,, \tag{3.1}$$
$$m_{\nu_\mu} = 205.25\,\mathrm{eV} \tag{3.2}$$
$$m_{\nu_\tau} = 5.72\,\mathrm{KeV}\,. \tag{3.3}$$

Let us now consider the mixing, that should produce the phenomenon known as neutrino oscillations. In this case, the detection of the mixing does not occur like in the case of quarks: the rotation among neutrino families does not reflect in corrections to the vertices of a one-loop interaction, the phenomenon we consider involves free propagating neutrinos. Therefore, the probabilities are related in simple way to mass ratios of bare neutrinos. In our scenario, what we can determine is the overall amplitude for the mixing of the muon's to the electron's neutrino (or to the tau neutrino), given by the ratio of volume of the first to the volume of the second neutrino family, namely the mass ratio of the respective neutrinos (or the ratio of the second to the third family respectively). Let us concentrate on the first two families. In the SM approach neutrino oscillation probabilities go typically as:

$$P \sim \left| \sum U U^{\star} e^{-i\frac{m^2 L}{2E}} \right|^2 , \tag{3.4}$$

[3] These values must be considered as a first approximation of a computation that cannot compete with the degree of precision we are used to see in fine tests of the Standard Model. It is however sufficient for the purpose of our discussion.

where U are the entries of the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix), m the neutrino mass, L the travelled distance and E the average neutrino energy. This expression can be re-written as:

$$P \;\sim\; |UU^{\star}|^2 \sin^2\left(\frac{\Delta m^2 c^3 L}{4\hbar E}\right). \tag{3.5}$$

The argument of the $\sin^2$ function can be rewritten as:

$$1,27 \times \frac{\Delta m^2 L}{E} \tag{3.6}$$

where L is in Km, E is in GeV, and m^2 is in eV^2. Indeed, the $\sin^2$ behaviour is a quite general fact which is a direct consequence of the wave-like propagation, and of being probabilities defined as squares of amplitudes. Therefore, this behaviour does not depend on the specific model through which we implement the mixing, and can be assumed to hold also in our scenario. However, owing to the huge mass difference between muon's and electron's neutrino, in our scenario during one period of the electron neutrino wave the muon neutrino undergoes many oscillation periods. In practice, it just contributes for an averaged effect: the electron neutrino wave projects onto a constant muon neutrino state. The $\sin^2$ argument of expression 3.5 reduces therefore to:

$$\approx\; 1,27 \times \frac{m^2_{\nu_e} L}{E}. \tag{3.7}$$

What matters in our case is therefore the electron neutrino wave. From the values of the neutrino masses at present time, expressions 3.1–3.3, we can see that for $E \sim \mathcal{O}(1)$ GeV the typical period T is:

$$\frac{m^2_{\nu_e}\mathrm{T}}{\mathrm{E}} \sim 2\pi \;\Longrightarrow\; \mathrm{T} \sim \frac{\pi}{1.27}\,\mathrm{Km}. \tag{3.8}$$

For what we said in section 2, the overall value of the coefficient $|UU^{\star}|^2$ for the muon to electron transition, summed over all the internal states, is:

$$|UU^{\star}|^2 \;\approx\; \frac{\alpha_{SU(2)}}{1+\alpha_{SU(2)}} \;\approx\; \frac{m_{\nu_e}/m_{\nu_\mu}}{1+m_{\nu_e}/m_{\nu_\mu}}. \tag{3.9}$$

Averaging over the period of the $\sin^2$ part produces a normalization factor 1/2:

$$\frac{1}{2\pi}\int_{2\pi} \sin^2(x)dx \;=\; \frac{1}{2}. \tag{3.10}$$

Inserting the values of the $SU(2)$ coupling (or equivalently the neutrino mass ratio), and taking into account the integration over the period, we obtain the average value of the $\nu_\mu \to \nu_e$ mixing, that we indicate as M_{12}:

$$M_{12} \;\equiv\; \langle P_{12}\rangle \;=\; \frac{1}{2}\times\frac{m_{\nu_e}/m_{\nu_\mu}}{1+m_{\nu_e}/m_{\nu_\mu}} \;=\; 0.00342. \tag{3.11}$$

This allows us to *test* the theory on the experiments. The experimental data we are going to consider are those provided by the Super-Kamiokande [9], MiniBooNE [2] and MicroBooNE [3, 4, 5, 6]. These sources of experimental data are particularly important, because they are less dependent on specific hypotheses and models, like for instance the solar model in the case of solar neutrinos. Moreover, fitting both the Super-Kamiokande and the MiniBooNE data is a challenge for a theory of oscillations, not to speak of the puzzling "disagreement" of MiniBooNE and MicroBooNE data: in the most optimistic scenario the SM prediction lies 5-6 standard deviations away from the MiniBooNE data, a result that the MicroBooNE data seem to put into question, although in a seemingly non-explicable way.

3.1 Atmospheric neutrinos

Let us start by considering the detection of atmospheric neutrinos at Super-Kamiokande. This experiment compares the same muon's neutrino beam before and after the travel through earth, thereby getting rid of model-dependent systematic errors on the estimation of the absolute amount of neutrinos. Differently from the usual approach, that assumes the interaction of neutrinos during their travel through the earth to be negligible, in our scenario, owing to the shortness of the oscillation's wavelength (of the order of the kilometer) during their travel muon neutrinos can be assumed to be *in the average* electron's neutrinos by a constant mixing fraction M. This reflects in an increased interaction probability: since stable matter is made of particles of the first family, the interaction with matter of the muon's neutrino is in fact of second order in the weak coupling α_w as compared to the interaction of the electron's neutrino. Therefore, when travelling through matter electron neutrinos have a higher scattering amplitude than pure muon neutrinos. As a consequence, owing to the frequent oscillation, during the travel through matter the muon's neutrino beam decreases through its partial mutation to a more interacting state [4].

Let us call I_{ν_μ} the amount of muon neutrinos which can be *measured* at any time of the neutrino flight. This is proportional to the total amount of neutrinos by a factor that we don't know, and we don't actually need to know. We can write:

$$\frac{\partial I_{\nu_\mu}}{\partial t} \approx -IMA_{\nu_e}\,, \tag{3.12}$$

where M is the average amount of mixing over the oscillation period, and A_{ν_e} is the scattering amplitude of the electron's neutrino. Since we are interested in deriving the fraction of remaining neutrinos as compared to the initial amount by comparing the amount of decays before and after travelling through the earth (in other words, since we are not interested in absolute quantities but in relative ones), let us normalize I_{ν_μ} by dividing it by its initial value. I_{ν_μ} will therefore always be lower than one. In order to determine A_{ν_e} we consider that between muon's and electron's neutrino there is an $SU(2)$ rotation among first and second family. We can therefore write:

$$A_{\nu_\mu} = \alpha^2_{SU(2)} \times A_{\nu_e}\,. \tag{3.13}$$

where $\alpha_{SU(2)}$ is the strength of the families-rotating $SU(2)$ coupling, the group that determines the mass ratios. Its value must be run to the appropriate energy scale. In our case, we evaluate it at the mean energy scale of the beam we want to consider. In turn, at every time A_{ν_μ} is given by the measured amount of muon neutrinos, namely, I_{ν_μ} itself. We obtain therefore:

$$\frac{\partial I_{\nu_\mu}}{\partial t} \approx -I^2 M\alpha^{-2}_{SU(2)}\,. \tag{3.14}$$

Assuming that the cross sections of the electron's and muon's neutrino scattering remain constant during the path through the earth [5] and are the same as at the point of measurement, we can integrate 3.14 to:

$$\frac{1}{I^{out}_{\nu_\mu}} = 1 + \alpha^{-2}_{SU(2)} M\Delta t\,, \tag{3.15}$$

where Δt is the duration of the travel through earth. The neutrino mass is so small that we can consider it to practically travel at almost the speed of light. Therefore,

$$\Delta t \approx 0.0425\,s\,. \tag{3.16}$$

From 3.11 we calculate then:

$$M = 0.00342\,. \tag{3.17}$$

[4] Oscillations to the tau neutrino can be ignored here, because they do not significantly contribute to the interaction with matter. We can therefore assume their contribution to the detected events to be basically the same before and after the travel through the hearth, so that it gets systematically subtracted from the experimental data.

[5] This approximation is justified by the fact that the interaction of the electron's neutrino with matter mostly concerns valence electrons, so that the higher density of earth, five times that of the water, does not play any role.

Inserting in 3.15 these values, and the value of the $SU(2)$ coupling run to the appropriate scale (see appendix 3.2), we obtain:

$$\left.\frac{1}{I^{out}_{\nu_\mu}}\right|_{\langle E\rangle=\mathcal{O}(0.1\,\mathrm{GeV})} \approx 2.09\,, \tag{3.18}$$

$$\left.\frac{1}{I^{out}_{\nu_\mu}}\right|_{\langle E\rangle=\mathcal{O}(10\,\mathrm{GeV})} \approx 1.86\,. \tag{3.19}$$

Both the values 3.18 and 3.19 are in agreement with the Super-Kamiokande results [9], that also report a higher oscillation rate of neutrino events below, but close to, the GeV energy scale.

3.2 The MiniBooNE and MicroBooNE results

Let us now consider the case of neutrinos produced in laboratory. In the usual interpretation, both these data and those of atmospheric neutrinos (as well as those of solar and supernova neutrinos) correspond to measurements made at a different phase of the oscillation. Once parameters such as the mass difference and the PMNS mixing angles are fixed by the other experiments, in order to obtain the prediction for the MiniBooNE experiment it remains only to plug a different energy E and distance L in the same expression 3.4. Indeed, the experimental data do not fix the parameters in a unique way, but impose constraints on their values. As a consequence, one speaks rather of a range of predictions. Nevertheless, in the most optimistic case the SM fails to account for the experimental result by several standard deviations (> 4) ([2]): the experimental data show a higher degree of mixing than expected. Several solutions have been proposed to this puzzle, typically sticking on the idea of oscillation between neutrinos of comparable masses, therefore with very long oscillation period. This is a natural assumption if one i) tries to justify neutrino masses within a field-theoretical framework, necessarily based on Higgs mechanism and naturalness of Yukawa couplings, ii) as a consequence explains also the Super-Kamiokande results in terms of single period oscillation. In this case one can try to improve the model by introducing see-saw mechanisms involving non-interacting (sterile) highly massive neutrinos, that would contribute to the oscillation without nevertheless being detected.

Let us now see how things look like in our theoretical scenario. Since we are interested in catching the core of the phenomenon, for simplicity we just consider what could be the overall effect collectively accounting for all the channels, at a reference energy of 1 GeV. While in the case of Super-Kamiokande the period of oscillation is short in comparison to the travelled distance, in the MiniBooNE case the detector is placed at around 1/5th of oscillation's wavelength away from the source [6]. This turns out therefore to correspond to just after the point of maximal rate of increase of the mixing probability, when the $\sin^2$ function attains the value $\sin^2 \sim 0.91$. The MicroBooNE experiment is placed some 70 meters upstream (see for instance [3]), where $\sin^2 \sim 0.75$. Therefore, the MiniBooNE data should in first approximation correspond to a mixing:

$$P(\nu_\mu \to \nu_e) \;=\; 0.91 \times M_{12} \;\approx\; 0.0031\,. \tag{3.20}$$

For energies below 1 GeV we obtain a slightly higher value. For instance, at 900 MeV the period is 20% shorter, and we obtain:

$$P(\nu_\mu \to \nu_e) \;=\; 0.95 \times M_{12} \;\approx\; 0.00326\,. \tag{3.21}$$

This has to be compared with the experimental observation for the neutrino channel, here extrapolated from the data of [2]:

$$\sim\; 0.00323 \pm 0.00014\,. \tag{3.22}$$

According to [2], the best fit of the SM expectation is instead ~ 0.0026, more than 4σ away from the experimental result. Let us now come to the result of MicroBooNE. In this case, for energies of 1 GeV we obtain:

$$P(\nu_\mu \to \nu_e) \;=\; 0.72 \times M_{12} \;\approx\; 0.0025\,, \tag{3.23}$$

[6] For an estimation of the travelled distance, we do not consider the whole distance of the detector from the accelerator's target [10], but the length between the absorber and the neutrino detector, ~ 450m, plus half the detector's length/diameter.

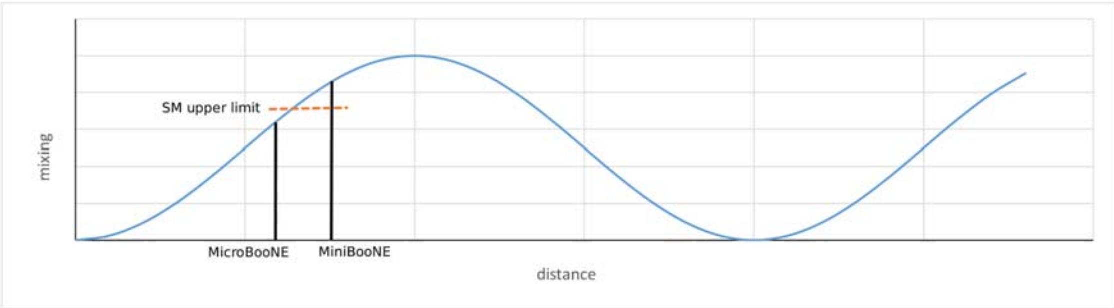

Figure 2: The apparent contradiction of the MiniBooNE and MicroBooNE results depends on the architecture of the two detectors, by coincidence placed precisely just before and after the threshold of compatibility with the SM allowed values.

and for 900 MeV:

$$P(\nu_\mu \to \nu_e) \;=\; 0.82 \times M_{12} \;\approx\; 0.0028\,. \tag{3.24}$$

These calculations are very sensitive to the exact position of the experiment. If the actual center of the experiment is some ten meters more upstream [7], we obtain:

$$P(\nu_\mu \to \nu_e) \;\approx\; 0.0024 \;\;(1\,\mathrm{GeV})\,, \tag{3.25}$$

and:

$$P(\nu_\mu \to \nu_e) \;\approx\; 0.0027 \;\;(900\,\mathrm{MeV})\,. \tag{3.26}$$

The slope of the typical Standard Model oscillation is not uniquely determined by the experimental data. In any case, roughly speaking the MiniBooNE results can be regressed to the MicroBooNE point by considering that the typical wavelength of SM oscillation models is much larger, at least one order of magnitude larger, than the one of this scenario. This implies that, in the space of the small distance between the two experiments, the MiniBooNE most optimistic mixing, 0.26% according to [2], can be regressed almost like a constant (it would at most decrease by some $2\% - 3\%$ if $\Delta m^2 \approx 0.2\mathrm{eV}^2$). As a consequence, in the average our prediction for the MicroBooNE experiment, 3.23–3.26, lies below the upper limit of the values allowed by the Standard Model, whereas the prediction for MicroBooNE lies above. The situation is illustrated in figure 2, in which we represent the SM upper limit around the two experiments as a dashed line. Our results are therefore in line with the data reported in Refs. [3, 4, 5, 6], and justify the absence of electron's events excess. According to our analysis, the excess is larger for lower energies, because the wavelength of the oscillation is larger, and the MiniBooNE detector is effectively placed more upstream in the period. At the time of writing this paper, the analysis of the MicroBooNE data is not yet complete, and two new experiments, SBDN and ICARUS, located respectively upstream and downstream of these two, will start in the future. It will be interesting to test the shape illustrated in figure 3 also against the data produced by these experiments.

[7] This correction is perhaps necessary in order to account for the different sizes of the MiniBooNE and MicroBooNE detectors: considering also the different size of the respective detectors, it could be that 85 meters is a better estimate of the distance between the centers of the two experiments.

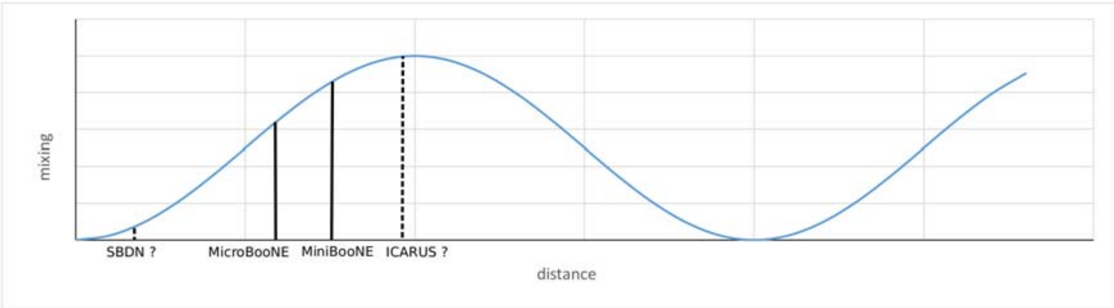

Figure 3: The SBDN and ICARUS detectors are placed along the same streamline of Mini- and MicroBooNE. Here we illustrate the wavelength for energies around 1 GeV. According to our model, SBDN should measure in the average a lower mixing than MicroBooNE. Depending on the energy range, ICARUS could fall around the maximum of oscillation (for energies around 1 GeV), or down the slope after the maximum (lower energies).

Conclusion

In this note we investigated flavour mixing, and in particular neutrino oscillations, in the light of a theoretical framework that unifies quantum mechanics and general relativity at a non-field-theoretical level. In this framework, all masses and couplings are determined by construction as functions of the only free parameter, the age of the universe. Matching the experiments is therefore not a matter of data-fitting, but pure checking of the predictive power of the theory, a theory that in part implies a re-thinking of the oscillation scenario. Despite the rough approximation in the computation of the neutrino mixing, our results show a remarkable agreement with the experimental data of atmospheric neutrinos as measured at Super-Kamiokande and of accelerator neutrinos (both the MiniBooNE *and* the MicroBooNE experiments). Besides constituting a test of the neutrino mass values predicted some years ago within this scenario (see Ref. [8], then updated in [1]), this result provides further support for this theoretical approach, which goes to add to the successful computation of the muon's anomalous magnetic moment (see [7]) and several other quantities, not only concerning elementary particle's physics but including also the critical temperatures of hight-temperature superconductors ([1], [11]). All this suggests that perhaps it is time for the field-theoretical approach being overcome by a more fundamental approach. The one presented in [1] could constitute a first step in the right direction. In the future, it will be possible to test this hypothesis also on the data produced by SBDN and ICARUS, for which we can already make a quantitative prediction.

Proceeding to more refined tests of the predictions on the neutrino physics within this scenario will on the other hand require developing computational tools specific for this theoretical framework. In particular, for consistency a test involving also solar and supernova neutrinos would require reconsidering also the solar and supernova models within the same theoretical framework.

⋆ ⋆ ⋆

Appendix A: The universe of codes in summary

Let us briefly recapitulate the key aspects of the theoretical framework used in this paper. As discussed in Ref. [12], the basic formulation is given in terms of mappings from the set of natural numbers to a product of discrete spaces. The mapping from a discrete set to a product of discrete spaces can be interpreted as an assignment of units of what we may call "energy" to a set of "positions". We build up in this way a discrete geometry: in the limit to the continuum one can introduce an energy density, and a curvature related to the energy density according to Einstein's equation. The total number of energy units which are going to be distributed by such a mapping can be interpreted as the total energy of this geometry. Let us call E this total amount. It is clear that, if we consider the set of all the possible mappings distributing E energy units, this contains as sub-geometries the set of all the possible mappings distributing $E' < E$ energy units. Therefore, in the space of all such mappings for any value of E we can introduce an order by grouping the geometries according to their total energy content. As surprising as it may seem, the ordering of these sets by increasing E has all the characteristics of a temporal order [1]. Any set $\{\Psi\}(E)$ containing all the geometries $\{\Psi\}(E)$ corresponding to a distribution of E energy units is a "universe" at total energy E, and the E-ordered sequence of sets $\{\Psi\}(E)$ a history of the universe. We are used to view the physical universe as corresponding, at any time in its history, to just *one* geometry; how can a collection of geometries correspond then to the universe? In Ref. [1] we introduced the concept of superposition of geometries. There is no room here for going into details, but let us at least take a glimpse of this point, which is a fundamental concept of the entire construction. According to this approach, what an observer sees is not one geometry, but an "ensemble of geometries leading to weighted averages of observables". For instance, the amount of energy measured around a certain point in the space is the result of observing a set of geometries, each one with its own energy distribution. The average observed effect is a weighted sum, in which each geometry contributes to the detected energy according to its statistical weight in the space of all the geometries: geometries that occur more often contribute more. What determines the weight of a geometry is the number of equivalent times it can be realized, i.e. the number of times in which distributing a certain amount of energy units produces the same geometry. This is the order of the symmetry group of the geometry. The basic expression in this scenario is the sum over all the possible energy distributions:

$$\mathcal{Z}(E) = \sum_{\Psi(E)} \mathrm{e}^{S(\Psi(E))} . \tag{A.1}$$

where $\Psi(E)$ is a geometry that corresponds to the distribution along the space of a total amount of energy E. $S(\Psi(E))$ is the entropy of the geometry $\Psi(E)$ in the phase space $\{\Psi\}(E)$ of all the geometries of energy E. It is related to the volume of occupation in the phase space, $W(\Psi)$, in the usual way: $S = \log W$ [8]. Contact with the real world is established by identifying the size of the unit cell in the discrete space with the Planck length, the energy unit with the Planck mass times c^2, and taking the limit to the continuum. Indeed, the sum A.1 is not restricted to three-dimensional spaces. Nevertheless, three dimensional space arises as the most entropic, i.e. the most frequently realized, dimension within the class of the most symmetric geometries. Owing to the interpretation of the path through sets of geometries ordered according to their energy content E as a history of the universe, with an appropriate conversion of units [9], up to a multiplicative constant we can also identify the total energy E with a time coordinate $\mathcal{T}$, that we call "age of the universe". As astonishing as it may be, this leads, in the continuum limit, to a universe with the physical and geometrical properties of the universe we live in, with a three-dimensional space governed by relativistic quantum mechanics. Indeed, this setup leads in a natural way to the concept of mean value of an observable, and to an uncertainty principle: if we consider as "classical space" the most entropic geometry at any age $\mathcal{T}$, we have that, as seen from the classical space, the contribution to the mean energy around a certain point is not just the value assigned by the most entropic geometry, the "classical" value, but is corrected by an amount Δ, that collects the contribution of all the other geometries. Since any physical observation, any experiment, can only occur during a certain non vanishing interval of time, we cannot observe an "instant value" of energy.

[8] This volume is related to the volume of the symmetry group of the energy distribution. Since the sum A.1 is always regularized to discrete spaces of finite volume, the volume of the symmetry group turns out to correspond to the order of a finite group.

[9] Based as usual on the constants c, $\hbar$, and the Planck length.

It only makes sense to speak of a variation of energy during a certain interval of time. It can be shown that ΔE and $\Delta \mathcal{T}$ are inversely proportional, leading to a relation that from a formal point of view coincides with the Heisenberg's uncertainty relation. Indeed, as discussed in Ref. [1], this gives us an idea about why the physical world is necessarily quantum mechanical, even before introducing the notion of light and measurements made through wavelike probes: in this scenario quantum mechanics is an expression of the fact that we can only observe mean values, because the universe does not consist of a single geometry. Once the notion of classical geometry is appropriately introduced, also the concept of propagation of information, and the fact that the speed of light is a universal constant can be shown to result as a consequence of the fundamental setup.

Till now we have spoken only of energy. Indeed, since what we have called energy is what shapes the geometry, and *anything* in the universe is in the end a deformation of space, we are not separating here a background space in which other degrees of freedom (particles, fields, whatever else) live in. The set of geometries are not the space where other elements live in: they *are* the universe. The universe *consists* of geometries. The sum A.1 can therefore be considered the "partition function", or the functional generating all the observables of the theory. The dynamics is intrinsic in A.1: the time evolution is uniquely given by the entropy-weighted sum as a function of $E \propto \mathcal{T}$. Any type of "force" or interaction is therefore entropic by definition. In this scenario, there is neither reversibility nor conservation of energy. Although not surprising for what we just said, the statement about energy conservation may seem to contradict one of the most established principles of physics. However, here we are talking about the non-conservation of the total energy of the universe, a phenomenon that occurs at the cosmological scale. Indeed, as discussed in [1], this seems to agree with several cosmological observations. The physics at the scale of a typical local experiment is on the other hand approximately described by an effective theory obtained by neglecting the cosmological evolution. In this case the usual energy conservation principle is recovered. Indeed, the sum of A.1 over all energies turns out to be a generalization of the Feynman's path integral, in which the paths are the geometries. Although the measure of the sum is not formulated in terms of exponential of an action but exponential of entropy, it can be shown that the sum $\sum_E \mathcal{Z}(E)$ implies the Lagrangian formulation in the continuum limit, a non-trivial result for which we refer the reader to Ref. [1] (chapter 3, section 5) [10].

The effective theory

At any age of the universe the most entropic geometry turns out to be a 3-sphere with radius proportional (after appropriate conversion of units) to the age of the universe itself. Therefore, what we introduced as classical space, namely the space corresponding to the most entropic geometry, is always of finite extension. From A.1 one derives that the history of the universe progresses toward increasing size of the space, which can be interpreted as a classical space expanding proportionally to the age, to serve as base for a quantum world provided with a minimal distance. On the continuum, this scenario can be approximated by string theory. The physical content of the world implied by A.1 can be investigated by considering string constructions corresponding to a compactification to four dimensions, in which however the space-time coordinates are considered of infinite extension only for convenience of the analysis. Indeed, from a physical point of view, they too are compactified. However, differently from the other string coordinates, they are not frozen at the Planck scale, but extended as much as the classical space of the universe. These coordinates expand therefore as a function of the age of the universe. As it was for the geometries, also in the case of string compactifications one must proceed in analogy to (A.1), and investigate string constructions by introducing an entropy in the space of all string compactifications, allowing to weight the compactifications, and identify those of highest entropy. This can be done, and an entropy can be introduced and put in relation to the symmetry of the string construction [11].

[10] The "weird" formulation of the measure reflects the profound difference between this approach and other based on a discretization of fields and/or the space they live in (such as for instance the spin foam models (see [13], and [14] for a review). In its basic formulation our approach does not rely on any kind of Lagrangian/Hamiltonian formulation, in A.1 there is neither time evolution nor quantization. It is pure statistics, and as such it deeply differentiates also from approaches with which it shares somehow some concepts, such as Wheeler's geometrodynamics [15].

[11] It is not necessary to introduce it in an absolute way: what we are interested in is just the comparison between different constructions, therefore their *relative* degree of symmetry. In the case of orbifold constructions this is simple to find out, because

There is no single string compactification that, alone, exactly accounts for the physical content, and allows to investigate the spectrum of elementary particles and interactions. The latter can be investigated by looking at the class of string compactifications whose geometries corresponds to the highest amount of symmetry reduction, because they are those that contribute the most. Indeed, as we said, what are usually called compactifications to four dimensions, namely constructions in which four coordinates are infinitely extended, do not appropriately correspond to the real physical situation, in which also space-time is compact. This point is not a negligible aspect: precisely the compactness of space gives rise to non-vanishing sub-Planckian masses as lowest momenta [12]. It should by now be clear that, like field theory, in this context also string theory is only a useful tool for investigating certain properties, but not the whole story. Anyway, although complicated, the analysis of the physical content can nevertheless be done. What comes out is that supersymmetry is broken at the Planck scale [13], and the spectrum of the elementary particles and their interactions is the already known one of the Standard Model, except for the Higgs field, absent here. Not only masses, but also couplings, and as a consequence all physical quantities, turn out to depend on the extension, and therefore on the age, of the universe, which is the only independent parameter of the theory. At any chosen age of the universe, the entire physical content, the masses and the interaction couplings, are uniquely determined. This theoretical framework is therefore extremely predictive. Its predictive power is a "double-edged sword": on one side any mismatch with the experimental results can potentially rule out the entire scenario; on the other side, any agreement is a strong support of the entire construction.

In this scenario, reversibility only exists as an approximation, in an effective action derived for any point of the evolution of the universe, in which one deals with masses and couplings as they were constant. At a fundamental level, everything is ruled by entropy: the concept of causality itself is here substituted by an evolution toward more entropic states. Once the cosmological evolution is neglected it is possible to approximate the dynamics with a field theoretical action built out of the degrees of freedom of the spectrum. In principle, any instant of the age of the universe should corresponds to a different effective action. However, since the cosmological evolution is encoded in the dependence of masses and couplings on the age of the universe, while the fundamental degrees of freedom remain the same, it is possible to consider an effective action in which masses and couplings are inserted as external parameters, without caring about their origin, and neglecting their dependence on the age of the universe. Since at present time the dependence of these quantities on the age of the universe is very mild, as long as one is not interested in phenomena occurring at the cosmological scale, the approximation obtained by freezing them to a fixed, present age, makes sense. The dynamics implied in the effective action will approximately describe the time evolution in a neighbour of the present age, with a time parameter that does not encode the whole time evolution. As a consequence, such an effective theory does not need to be consistent as gauge theory in the massive sector (no Higgs mechanism). It does not need to provide a fundamental description of time-reversal and therefore also of CP violation either: in this theoretical framework, these phenomena must be investigated outside of field theory. The effective theory only works as a truncated theory, in which the computation of amplitudes through Feynman diagrams makes only sense once endowed with a different interpretation, that implements the statistical approach derived from the sum (A.1).

---

they are derived by modding out flat space by different symmetry groups. Luckily, orbifolds turn also out to be enough for the purpose of investigating the spectrum.

[12] Compactness of space does not necessarily imply non-vanishing ground modes of the momenta. However, here the space is a bit more complicated: in the string orbifold language, it is "shifted".

[13] Differently from what one would naively expect, despite of being supersymmetry broken at the Planck scale, the vacuum energy is not of order of the Planck mass, but has a mild dependence on the inverse of the age of the universe, leading to the correct estimation of the cosmological constant (see Ref. [1]).

Appendix B: Deriving the CKM matrix

The CKM matrix is built as product of three matrices:

$$\begin{aligned} CKM &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\vartheta_{sb} & \sin\vartheta_{sb} \\ 0 & -\sin\vartheta_{sb} & \cos\vartheta_{sb} \end{pmatrix} \\ &\times \begin{pmatrix} \cos\vartheta_{db} & 0 & \sin\vartheta_{db} \\ 0 & 1 & 0 \\ -\sin\vartheta_{db} & 0 & \cos\vartheta_{db} \end{pmatrix} \\ &\times \begin{pmatrix} \cos\vartheta_{ds} & \sin\vartheta_{ds} & 0 \\ -\sin\vartheta_{ds} & \cos\vartheta_{ds} & 0 \\ 0 & 0 & 1 \end{pmatrix} ; \end{aligned} \tag{B.1}$$

where the entries, derived from quark and meson phase space volumes, are [1]:

$$\sin\vartheta_{sb} \approx \delta \sim 0.046 \tag{B.2}$$

$$\sin\vartheta_{ds} \approx \frac{m_\pi}{m_K} \sim 0.22 \tag{B.3}$$

$$\sin\vartheta_{db} \approx \delta^{\frac{2}{3}} \times \frac{m_\pi}{m_K} \sim 0.009 \, , \tag{B.4}$$

$$\tag{B.5}$$

where $\delta \sim 100$ is the inverse of the coupling strength of the $SU(2)$ group at the base of the rotations among particle's families, evaluated at the appropriate quark scale. For a discussion of the dependence of matter symmetries on this group we refer the reader to [1].

Appendix C: Age of the universe, SU(2) coupling

In our framework every parameter is a function of the age of the universe. In particular, so is also the neutron's mass. Therefore, a precise determination of the age, needed in order to compute couplings and masses, can be obtained from the experimental value of the neutron's mass. In this way we obtain:

$$\mathcal{T} \approx 5.038816199 \times 10^{60} \, \mathrm{M_P^{-1}} \quad (= 12.6202827 \times 10^9 \text{ yr}) \, . \tag{C.1}$$

The inverse of the strength of the $SU(2)$ coupling at energy scale $M_0 = 1/2\sqrt{\mathcal{T}}$ in units of c^2 times the Planck mass M_P is:

$$\alpha^{-1}_{SU(2)} = 147.2 \, . \tag{C.2}$$

In order to run its inverse to the 0.1 GeV scale, we multiply it by the logarithmic fraction of the two energy scales, thereby obtaining:

$$\alpha^{-1}_{SU(2)}|_{E=10^{-20} \, M_P c^2} = \alpha^{-1}_{SU(2)}|_{E=M_0} \times \frac{\log_{10}(10^{-20})}{\log_{10} M_0} \tag{C.3}$$

and:

$$\alpha^{-1}_{SU(2)}|_{E=10^{-18} \, M_P c^2} = \alpha^{-1}_{SU(2)}|_{E=M_0} \times \frac{\log_{10}(10^{-18})}{\log_{10} M_0} \tag{C.4}$$

for energies of 0.1 and 10 GeV respectively (here we approximate the GeV scale as $\sim 10^{-19}$ times the Planck mass scale).

References

[1] A. Gregori, *"The Universe of Codes, Beyond General Relativity and Quantum Mechanics"*. Lambert Academic Publishing, ISBN-13: 978-613-9-45238-5, 2019. Content available also on ResearchGate, `https://bit.ly/universeofcodes`.

[2] A. Aguilar-Arevalo, B. Brown, L. Bugel, G. Cheng, J. Conrad, R. Cooper, R. Dharmapalan, A. Diaz, Z. Djurcic, D. Finley, R. Ford, F. Garcia, G. Garvey, J. Grange, E.-C. Huang, W. Huelsnitz, C. Ignarra, R. Johnson, G. Karagiorgi, T. Katori, T. Kobilarcik, W. Louis, C. Mariani, W. Marsh, G. Mills, J. Mirabal, J. Monroe, C. Moore, J. Mousseau, P. Nienaber, J. Nowak, B. Osmanov, Z. Pavlovic, D. Perevalov, H. Ray, B. Roe, A. Russell, M. Shaevitz, J. Spitz, I. Stancu, R. Tayloe, R. Thornton, M. Tzanov, R. V. de Water, D. White, D. Wickremasinghe, and E. Z. and, *Significant Excess of Electronlike Events in the MiniBooNE Short-Baseline Neutrino Experiment,* Physical Review Letters **121** (nov, 2018).

[3] MicroBooNE Collaboration, P. Abratenko, R. An, J. Anthony, L. Arellano, J. Asaadi, A. Ashkenazi, S. Balasubramanian, B. Baller, C. Barnes, G. Barr, V. Basque, L. Bathe-Peters, O. B. Rodrigues, S. Berkman, A. Bhanderi, A. Bhat, M. Bishai, A. Blake, T. Bolton, J. Y. Book, L. Camilleri, D. Caratelli, I. C. Terrazas, F. Cavanna, G. Cerati, Y. Chen, D. Cianci, G. H. Collin, J. M. Conrad, M. Convery, L. Cooper-Troendle, J. I. Crespo-Anadon, M. Del Tutto, S. R. Dennis, P. Detje, A. Devitt, R. Diurba, R. Dorrill, K. Duffy, S. Dytman, B. Eberly, A. Ereditato, L. Escudero-Sanchez, J. J. Evans, R. Fine, G. A. F. Aguirre, R. S. Fitzpatrick, B. T. Fleming, N. Foppiani, D. Franco, A. P. Furmanski, D. Garcia-Gamez, S. Gardiner, G. Ge, V. Genty, S. Gollapinni, O. Goodwin, E. Gramellini, P. Green, H. Greenlee, W. Gu, R. Guenette, P. Guzowski, L. Hagaman, O. Hen, C. Hilgenberg, G. A. Horton-Smith, A. Hourlier, R. Itay, C. James, X. Ji, L. Jiang, J. H. Jo, R. A. Johnson, Y. J. Jwa, D. Kalelo, D. Kalra, N. Kamp, N. Kaneshige, G. Karagiorgi, W. Ketchum, M. Kirby, T. Kobilarcik, I. Kreslo, R. LaZur, I. Lepetic, K. Li, Y. Li, K. Lin, A. Lister, B. R. Littlejohn, W. C. Louis, X. Luo, K. Manivannan, C. Mariani, D. Marsden, J. Marshall, D. A. M. Caicedo, K. Mason, A. Mastbaum, N. McConkey, V. Meddage, T. Mettler, K. Miller, J. Mills, K. Mistry, T. Mohayai, A. Mogan, J. Moon, M. Mooney, A. F. Moor, C. D. Moore, L. M. Lepin, J. Mousseau, M. Murphy, D. Naples, A. Navrer-Agasson, M. Nebot-Guinot, R. K. Neely, D. A. Newmark, J. Nowak, M. Nunes, O. Palamara, V. Paolone, A. Papadopoulou, V. Papavassiliou, S. F. Pate, N. Patel, A. Paudel, Z. Pavlovic, E. Piasetzky, I. Ponce-Pinto, S. Prince, X. Qian, J. L. Raaf, V. Radeka, A. Rafique, M. Reggiani-Guzzo, L. Ren, L. C. J. Rice, L. Rochester, J. R. Rondon, M. Rosenberg, M. Ross-Lonergan, B. Russell, G. Scanavini, D. W. Schmitz, A. Schukraft, W. Seligman, M. H. Shaevitz, R. Sharankova, J. Shi, J. Sinclair, A. Smith, E. L. Snider, M. Soderberg, S. Soldner-Rembold, S. R. Soleti, P. Spentzouris, J. Spitz, M. Stancari, J. S. John, T. Strauss, K. Sutton, S. Sword-Fehlberg, A. M. Szelc, W. Tang, K. Terao, M. Thomson, C. Thorpe, D. Totani, M. Toups, Y. T. Tsai, M. A. Uchida, T. Usher, W. Van De Pontseele, B. Viren, M. Weber, H. Wei, Z. Williams, S. Wolbers, T. Wongjirad, M. Wospakrik, K. Wresilo, N. Wright, W. Wu, E. Yandel, T. Yang, G. Yarbrough, L. E. Yates, H. W. Yu, G. P. Zeller, J. Zennamo, and C. Zhang, *Search for an Excess of Electron Neutrino Interactions in MicroBooNE Using Multiple Final State Topologies,* 2021.

[4] P. Abratenko, R. An, J. Anthony, L. Arellano, J. Asaadi, A. Ashkenazi, S. Balasubramanian, B. Baller, C. Barnes, G. Barr, V. Basque, L. Bathe-Peters, O. B. Rodrigues, S. Berkman, A. Bhanderi, A. Bhat, M. Bishai, A. Blake, T. Bolton, J. Book, L. Camilleri, D. Caratelli, I. C. Terrazas, F. Cavanna, G. Cerati, Y. Chen, D. Cianci, G. Collin, J. Conrad, M. Convery, L. Cooper-Troendle, J. Crespo-Anadón, M. D. Tutto, S. Dennis, P. Detje, A. Devitt, R. Diurba, R. Dorrill, K. Duffy, S. Dytman, B. Eberly, A. Ereditato, J. Evans, R. Fine, G. F. Aguirre, R. Fitzpatrick, B. Fleming, N. Foppiani, D. Franco, A. Furmanski, D. Garcia-Gamez, S. Gardiner, G. Ge, V. Genty, S. Gollapinni, O. Goodwin, E. Gramellini, P. Green, H. Greenlee, W. Gu, R. Guenette, P. Guzowski, L. Hagaman, O. Hen, C. Hilgenberg, G. Horton-Smith, A. Hourlier, R. Itay, C. James, X. Ji, L. Jiang, J. Jo, R. Johnson, Y.-J. Jwa, D. Kalra, N. Kamp, N. Kaneshige, G. Karagiorgi, W. Ketchum, M. Kirby, T. Kobilarcik, I. Kreslo, I. Lepetic, K. Li, Y. Li, K. Lin, B. Littlejohn, W. Louis, X. Luo,

K. Manivannan, C. Mariani, D. Marsden, J. Marshall, D. M. Caicedo, K. Mason, A. Mastbaum, N. McConkey, V. Meddage, T. Mettler, K. Miller, J. Mills, K. Mistry, A. Mogan, T. Mohayai, J. Moon, M. Mooney, A. Moor, C. Moore, L. M. Lepin, J. Mousseau, M. Murphy, D. Naples, A. Navrer-Agasson, M. Nebot-Guinot, R. Neely, D. Newmark, J. Nowak, M. Nunes, O. Palamara, V. Paolone, A. Papadopoulou, V. Papavassiliou, S. Pate, N. Patel, A. Paudel, Z. Pavlovic, E. Piasetzky, I. Ponce-Pinto, S. Prince, X. Qian, J. Raaf, V. Radeka, A. Rafique, M. Reggiani-Guzzo, L. Ren, L. Rice, L. Rochester, J. R. Rondon, M. Rosenberg, M. Ross-Lonergan, G. Scanavini, D. Schmitz, A. Schukraft, W. Seligman, M. Shaevitz, R. Sharankova, J. Shi, J. Sinclair, A. Smith, E. Snider, M. Soderberg, S. Söldner-Rembold, P. Spentzouris, J. Spitz, M. Stancari, J. John, T. Strauss, K. Sutton, S. Sword-Fehlberg, A. Szelc, W. Tang, K. Terao, C. Thorpe, D. Totani, M. Toups, Y.-T. Tsai, M. Uchida, T. Usher, W. V. D. Pontseele, B. Viren, M. Weber, H. Wei, Z. Williams, S. Wolbers, T. Wongjirad, M. Wospakrik, K. Wresilo, N. Wright, W. Wu, E. Yandel, T. Yang, G. Yarbrough, L. Yates, H. Yu, G. Zeller, J. Zennamo, and C. Z. and, *Search for an anomalous excess of charged-current quasielastic ν_e interactions with the MicroBooNE experiment using Deep-Learning-based reconstruction,* Physical Review D **105** (jun, 2022).

[5] MicroBooNE Collaboration, P. Abratenko, R. An, J. Anthony, L. Arellano, J. Asaadi, A. Ashkenazi, S. Balasubramanian, B. Baller, C. Barnes, G. Barr, V. Basque, L. Bathe-Peters, O. B. Rodrigues, S. Berkman, A. Bhanderi, A. Bhat, M. Bishai, A. Blake, T. Bolton, J. Y. Book, L. Camilleri, D. Caratelli, I. C. Terrazas, F. Cavanna, G. Cerati, Y. Chen, D. Cianci, J. M. Conrad, M. Convery, L. Cooper-Troendle, J. I. Crespo-Anadon, M. Del Tutto, S. R. Dennis, P. Detje, A. Devitt, R. Diurba, R. Dorrill, K. Duffy, S. Dytman, B. Eberly, A. Ereditato, L. Escudero-Sanchez, J. J. Evans, R. Fine, G. A. F. Aguirre, R. S. Fitzpatrick, B. T. Fleming, N. Foppiani, D. Franco, A. P. Furmanski, D. Garcia-Gamez, S. Gardiner, G. Ge, S. Gollapinni, O. Goodwin, E. Gramellini, P. Green, H. Greenlee, W. Gu, R. Guenette, P. Guzowski, L. Hagaman, O. Hen, C. Hilgenberg, G. A. Horton-Smith, A. Hourlier, R. Itay, C. James, X. Ji, L. Jiang, J. H. Jo, R. A. Johnson, Y. J. Jwa, D. Kalra, N. Kamp, N. Kaneshige, G. Karagiorgi, W. Ketchum, M. Kirby, T. Kobilarcik, I. Kreslo, R. LaZur, I. Lepetic, K. Li, Y. Li, K. Lin, A. Lister, B. R. Littlejohn, W. C. Louis, X. Luo, K. Manivannan, C. Mariani, D. Marsden, J. Marshall, D. A. M. Caicedo, K. Mason, A. Mastbaum, N. McConkey, V. Meddage, T. Mettler, K. Miller, J. Mills, K. Mistry, T. Mohayai, A. Mogan, J. Moon, M. Mooney, A. F. Moor, C. D. Moore, L. M. Lepin, J. Mousseau, M. Murphy, D. Naples, A. Navrer-Agasson, M. Nebot-Guinot, R. K. Neely, D. A. Newmark, J. Nowak, M. Nunes, O. Palamara, V. Paolone, A. Papadopoulou, V. Papavassiliou, S. F. Pate, N. Patel, A. Paudel, Z. Pavlovic, E. Piasetzky, I. Ponce-Pinto, S. Prince, X. Qian, J. L. Raaf, V. Radeka, A. Rafique, M. Reggiani-Guzzo, L. Ren, L. C. J. Rice, L. Rochester, J. R. Rondon, M. Rosenberg, M. Ross-Lonergan, G. Scanavini, D. W. Schmitz, A. Schukraft, W. Seligman, M. H. Shaevitz, R. Sharankova, J. Shi, J. Sinclair, A. Smith, E. L. Snider, M. Soderberg, S. Soldner-Rembold, S. R. Soleti, P. Spentzouris, J. Spitz, M. Stancari, J. S. John, T. Strauss, K. Sutton, S. Sword-Fehlberg, A. M. Szelc, W. Tang, K. Terao, M. Thomson, C. Thorpe, D. Totani, M. Toups, Y. T. Tsai, M. A. Uchida, T. Usher, W. Van De Pontseele, B. Viren, M. Weber, H. Wei, Z. Williams, S. Wolbers, T. Wongjirad, M. Wospakrik, K. Wresilo, N. Wright, W. Wu, E. Yandel, T. Yang, G. Yarbrough, L. E. Yates, H. W. Yu, G. P. Zeller, J. Zennamo, and C. Zhang, *Search for an anomalous excess of charged-current ν_e interactions without pions in the final state with the MicroBooNE experiment,* 2021.

[6] MicroBooNE Collaboration, P. Abratenko, R. An, J. Anthony, L. Arellano, J. Asaadi, A. Ashkenazi, S. Balasubramanian, B. Baller, C. Barnes, G. Barr, V. Basque, L. Bathe-Peters, O. B. Rodrigues, S. Berkman, A. Bhanderi, A. Bhat, M. Bishai, A. Blake, T. Bolton, J. Y. Book, L. Camilleri, D. Caratelli, I. C. Terrazas, F. Cavanna, G. Cerati, Y. Chen, D. Cianci, J. M. Conrad, M. Convery, L. Cooper-Troendle, J. I. Crespo-Anadon, M. Del Tutto, S. R. Dennis, P. Detje, A. Devitt, R. Diurba, R. Dorrill, K. Duffy, S. Dytman, B. Eberly, A. Ereditato, J. J. Evans, R. Fine, G. A. F. Aguirre, R. S. Fitzpatrick, B. T. Fleming, N. Foppiani, D. Franco, A. P. Furmanski, D. Garcia-Gamez, S. Gardiner, G. Ge, S. Gollapinni, O. Goodwin, E. Gramellini, P. Green, H. Greenlee, W. Gu, R. Guenette, P. Guzowski, L. Hagaman, O. Hen, C. Hilgenberg, G. A. Horton-Smith, A. Hourlier, R. Itay, C. James, X. Ji, L. Jiang, J. H. Jo, R. A. Johnson, Y. J. Jwa, D. Kalra, N. Kamp, N. Kaneshige, G. Karagiorgi,

W. Ketchum, M. Kirby, T. Kobilarcik, I. Kreslo, I. Lepetic, K. Li, Y. Li, K. Lin, B. R. Littlejohn, W. C. Louis, X. Luo, K. Manivannan, C. Mariani, D. Marsden, J. Marshall, D. A. M. Caicedo, K. Mason, A. Mastbaum, N. McConkey, V. Meddage, T. Mettler, K. Miller, J. Mills, K. Mistry, T. Mohayai, A. Mogan, J. Moon, M. Mooney, A. F. Moor, C. D. Moore, L. M. Lepin, J. Mousseau, M. Murphy, D. Naples, A. Navrer-Agasson, M. Nebot-Guinot, R. K. Neely, D. A. Newmark, J. Nowak, M. Nunes, O. Palamara, V. Paolone, A. Papadopoulou, V. Papavassiliou, S. F. Pate, N. Patel, A. Paudel, Z. Pavlovic, E. Piasetzky, I. Ponce-Pinto, S. Prince, X. Qian, J. L. Raaf, V. Radeka, A. Rafique, M. Reggiani-Guzzo, L. Ren, L. C. J. Rice, L. Rochester, J. R. Rondon, M. Rosenberg, M. Ross-Lonergan, B. Russell, G. Scanavini, D. W. Schmitz, A. Schukraft, W. Seligman, M. H. Shaevitz, R. Sharankova, J. Shi, J. Sinclair, A. Smith, E. L. Snider, M. Soderberg, S. Soldner-Rembold, P. Spentzouris, J. Spitz, M. Stancari, J. S. John, T. Strauss, K. Sutton, S. Sword-Fehlberg, A. M. Szelc, W. Tang, K. Terao, C. Thorpe, D. Totani, M. Toups, Y. T. Tsai, M. A. Uchida, T. Usher, W. Van De Pontseele, B. Viren, M. Weber, H. Wei, Z. Williams, S. Wolbers, T. Wongjirad, M. Wospakrik, K. Wresilo, N. Wright, W. Wu, E. Yandel, T. Yang, G. Yarbrough, L. E. Yates, H. W. Yu, G. P. Zeller, J. Zennamo, and C. Zhang, *Search for an anomalous excess of inclusive charged-current ν_e interactions in the MicroBooNE experiment using Wire-Cell reconstruction,* 2021.

[7] A. Gregori, *A Note on the Muon's Anomalous Magnetic Dipole Moment,* `viXra:2110.0157`.

[8] A. Gregori, *The spectrum of the universe of codes,* `viXra:1301.0102`.

[9] Y. Fukuda, T. Hayakawa, E. Ichihara, K. Inoue, K. Ishihara, H. Ishino, Y. Itow, T. Kajita, J. Kameda, S. Kasuga, K. Kobayashi, Y. Kobayashi, Y. Koshio, M. Miura, M. Nakahata, S. Nakayama, A. Okada, K. Okumura, N. Sakurai, M. Shiozawa, Y. Suzuki, Y. Takeuchi, Y. Totsuka, S. Yamada, M. Earl, A. Habig, E. Kearns, M. D. Messier, K. Scholberg, J. L. Stone, L. R. Sulak, C. W. Walter, M. Goldhaber, T. Barszczxak, D. Casper, W. Gajewski, P. G. Halverson, J. Hsu, W. R. Kropp, L. R. Price, F. Reines, M. Smy, H. W. Sobel, M. R. Vagins, K. S. Ganezer, W. E. Keig, R. W. Ellsworth, S. Tasaka, J. W. Flanagan, A. Kibayashi, J. G. Learned, S. Matsuno, V. J. Stenger, D. Takemori, T. Ishii, J. Kanzaki, T. Kobayashi, S. Mine, K. Nakamura, K. Nishikawa, Y. Oyama, A. Sakai, M. Sakuda, O. Sasaki, S. Echigo, M. Kohama, A. T. Suzuki, T. J. Haines, E. Blaufuss, B. K. Kim, R. Sanford, R. Svoboda, M. L. Chen, Z. Conner, J. A. Goodman, G. W. Sullivan, J. Hill, C. K. Jung, K. Martens, C. Mauger, C. McGrew, E. Sharkey, B. Viren, C. Yanagisawa, W. Doki, K. Miyano, H. Okazawa, C. Saji, M. Takahata, Y. Nagashima, M. Takita, T. Yamaguchi, M. Yoshida, S. B. Kim, M. Etoh, K. Fujita, A. Hasegawa, T. Hasegawa, S. Hatakeyama, T. Iwamoto, M. Koga, T. Maruyama, H. Ogawa, J. Shirai, A. Suzuki, F. Tsushima, M. Koshiba, M. Nemoto, K. Nishijima, T. Futagami, Y. Hayato, Y. Kanaya, K. Kaneyuki, Y. Watanabe, D. Kielczewska, R. A. Doyle, J. S. George, A. L. Stachyra, L. L. Wai, R. J. Wilkes, and K. K. Young, *Evidence for Oscillation of Atmospheric Neutrinos,* Physical Review Letters **81** (aug, 1998) 1562–1567.

[10] A. A. Aguilar-Arevalo, A. O. Bazarko, S. J. Brice, B. C. Brown, L. Bugel, J. Cao, L. Coney, J. M. Conrad, D. C. Cox, A. Curioni, Z. Djurcic, D. A. Finley, B. T. Fleming, R. Ford, F. G. Garcia, G. T. Garvey, C. Green, J. A. Green, T. L. Hart, E. Hawker, R. Imlay, R. A. Johnson, P. Kasper, T. Katori, T. Kobilarcik, I. Kourbanis, S. Koutsoliotas, E. M. Laird, J. M. Link, Y. Liu, Y. Liu, W. C. Louis, K. B. M. Mahn, W. Marsh, P. S. Martin, G. McGregor, W. Metcalf, P. D. Meyers, F. Mills, G. B. Mills, J. Monroe, C. D. Moore, R. H. Nelson, P. Nienaber, S. Ouedraogo, R. B. Patterson, D. Perevalov, C. C. Polly, E. Prebys, J. L. Raaf, H. Ray, B. P. Roe, A. D. Russell, V. Sandberg, R. Schirato, D. Schmitz, M. H. Shaevitz, F. C. Shoemaker, D. Smith, M. Sorel, P. Spentzouris, I. Stancu, R. J. Stefanski, M. Sung, H. A. Tanaka, R. Tayloe, M. Tzanov, R. V. de Water, M. O. Wascko, D. H. White, M. J. Wilking, H. J. Yang, G. P. Zeller, and E. D. Zimmerman, *Search for Electron Neutrino Appearance at the $\Delta m^2 \sim 1eV^2$ Scale,* Physical Review Letters **98** (jun, 2007).

[11] A. Gregori, *On the Critical Temperatures of Superconductors: a Quantum Gravity Approach,* arXiv e-prints (July, 2010) `arXiv:1007.3731`.

[12] A. Gregori, *A physical universe from the universe of codes,* `arXiv:1206.0596`.

[13] J. C. Baez, *Spin foam models,* IOP Publishing **15** (July, 1998) 1827–1858.

[14] D. Oriti, *Spin Foam Models of Quantum Spacetime,* arXiv:gr-qc/0311066.

[15] J. A. Wheeler, *Geometrodynamics.* Academic Press, New York, 1962.