Growing Amount of Information inside a Space-Region may cause to expand the Region and to change its Surface-Curvature.

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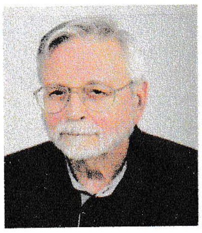

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Abstract.

According to G. t' HOOFT's holographic principle the combination of quantum mechanics and gravity requires that a **3**–dimensional space–region to be projected on the **2**–dimensional bounding surface of the region. In the limit of very large regions the bounding surfaces can be taken as flat planes at infinity, thus phenomena taking place in **3**–dimensional space can be projected onto distant "viewing screens" with no loss of information. The discrete lattice–sites of a screen are "pixels", each one can only store ⟨**1**⟩ bit of information. An analogy with a hologram can be made which stores a **3**–dimensional image on a **2**–dimensional film. As in the case of the hologram the flat **2**–diemsional image must be rich enough to code full rotationally invariant description of a **3**–dimensional object.
All matter is composed of elementary structure–less constituents called partons. The presence of a parton is represented by projecting its location by a light–ray on screen. A space–time–event which combines all partons at a specific instant of time forms a **3**–dimensional light–front. Light–front–quantization of quantum–gravity can be formulated by taking the transverse space as a discrete lattice where the lattice is composed of binary pixels of a spacing in the order of PLANCK–length. No distribution of matter will ever require more than ⟨**1**⟩ bit of information per PLANCK–area on a screen. The quantization in longitudinal direction considered under the parton–concept is familiar from Quantum–Chromo–Dynamics ⟨**QCD**⟩ originated in works on deep inelastic scattering. The Hamiltonians are the classical ones and the eigenvectors are well–defined super–positions of FOCK–space–states. Each parton–momentum simply acts as scale–transformations in longitudinal direction, but classical scale–invariance will usually be destroyed by divergent high frequency effects. Cutoffs in frequency–spectra, models in connection with so–called Fixed–Point–Hamiltonians and derived from string–theory will finally help to find–out that a growing amount–of–information inside a space–region may cause its extension.

Given that the maximal allowable information for each part of space is finite, then it is impossible to localize a particle with infinite precision at a point of the continuum. Therefore one could assume that information is stored in points of a discretized space. In order to get information described holographically, it must exist in some duplicated form, thus it is assumed to be stored on surfaces, or screens. Screens separate points and in this way they are natural places for information about particles that move from one side to the other. Within in this environment gravity will take the form of an entropic–force. An entropic force is an effective macroscopic–force that originates in a system with many degrees–of–freedom by the statistical tendency to increase its entropy. The force–equation is expressed in terms of entropy–differences.
A small piece of a holographic–screen is considered and a particle of a certain mass approaching it from the side at which space–time has already emerged. When the particle merges with the microscopic degrees– of–freedom on screen, it influences the amount of information that is already stored there. Assuming that the change–in–entropy near the screen is linear to displacement–from–screen then it is proportional to particle–mass. Force comes into play from an analogy with osmosis across a semi–permeable membrane and the membrane carries a temperature, the particle will experience an entropic–force. Based on relations from W. G. UNRUH and J. D. BEKENSTEIN temperature and acceleration can finally be brought into close connection which will let the just mentioned force to appear in the form of NEWTON's second law.
Thinking about a piece of a holographic–screen as a storage–device for information, the maximal storage–space or the total number of bits is proportional to this screen–area. Because each fundamental–bit occupies ⟨**1**⟩ unit–cell, the number of used bits can easily be calculated. Additionally the total energy is considered which had already been contained on screen–side when the particle was approaching (evenly distributed over the occupied bits) together with its mass–equivalence due A. EINSTEIN and temperature determined as average–energy per bit. All this combined together will let the above entropic force become NEWTON's gravity –force. The real consequence of this finally is: A growing amount on surface of a space.region may change the surface–curvature.

1 *Introduction.*

Some insights shall be given into the nature of matter–related–information contained in space and its reaction back on space if the amount of information gets increased while processes are considered under a combination of quantum–mechanics and theory–of –gravity in the view of the holographic–principle.

The first parts ⟨1^⟩ of the presentation are mainly qualitatively oriented, while the second parts ⟨2^⟩ shall contribute some appropriate quantitative explanations additionally.

Part ⟨**1^1**⟩ – influence by [1] of L. SUSSKIND – will given an introduction to G. 't HOOFT's holographic principle followed by discussions about problems that quantum–theoretically will come–up together with suggested solutions on how to evade un–surmountable difficulties that may arise.
Part ⟨**1^2**⟩ – modelled after E. VERLINDE [2] – is concerned with gravity and the role entropy plays in this context.

1^1 *The World as a Hologram.*

Most physicists believe that the degrees–of–freedom of the world consist of fields–filling–space, but some theorists believe that a small distance cutoff will be required in order to make sense of quantum–gravity. According to this philosophy the world is about as rich in structure as a **3**–dimensional discrete lattice–theory with a spacing in order of the PLANCK–length ⟨l_P⟩.

One may expect that if the energy–density in a region of space is bounded then the maximum entropy is proportional to the volume ⟨**V**⟩ of the region. But there is a good reason to believe that the correct result in quantum–theory–of–gravity is that the maximum entropy is proportional to the bounding–area of ⟨**V**⟩ and not to ⟨**V**⟩ itself; the appropriate argument is due to J. D. BEKENSTEIN [3]. His considerations are shortly summarized in ⟨**2^1.1**⟩.

G. 't HOOFT [4] has proposed a far more radical interpretation of BEKENSTEIN's understanding.

- According to 't HOOFT it must be possible to describe all phenomena within ⟨**V**⟩ by a set of degrees–of–freedom which reside on the bounding–surface of ⟨**V**⟩. The number of degrees–of–freedom should be no larger than that of a **2**–dimensional lattice with approximately ⟨**1**⟩ binary degree–of–freedom per PLANCK–area. In other words, the world is in a certain sense a **2**–dimensional lattice of spins.
- G. 't HOOFT further imagines that in the limit of a very large region the bounding–surface can be taken to be a flat plane at infinity. In some way, phenomena taking place in a **3**–dimensional space can be projected onto distant ⟨**viewing–screens**⟩ with no loss of information. In what follows, such a **2**–dimensional surface is considered as a screen and its discrete lattice–sites as pixels. A pixel can only store ⟨**1**⟩ bit–of–information and is therefore either lit or dark.

G. 't HOOFT has made the analogy with a hologram which stores a **3**–dimensional image on a **2**–dimensional film. As in the case of hologram the flat **2**–dimensional image must be rich enough to code the full rotationally invariant description of **3**–dimensional objects.

All matter is assumed to be composed of elementary structure–less constituents called partons. Presence of a parton is recorded on screen by projecting it through light– rays passing through the screen at right–angles and lightning a pixel on screen.

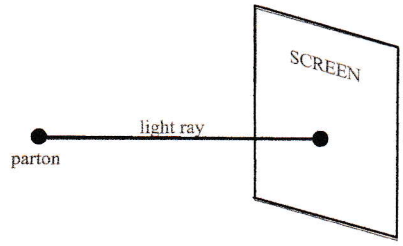

Fig. 1^1
Location ⟨$\mathbf{x_m}$⟩ is mapped to a point ⟨$\mathbf{X_\perp}$⟩. If the screen is oriented perpendicular to the ⟨**z**⟩–axis then the image on screen is ⟨**[X,Y]**⟩. It can be said that the presence of the parton is recorded by lightning the pixel at ⟨**[X,Y]**⟩.

No distribution of matter will ever require more than ⟨**1**⟩ bit–of–information per PLANCK–area on screen. The image of a parton on screen generally will not be unique in which case there will be more than ⟨**1**⟩ light–ray from the parton to the screen, the state on screen is then a quantum–super–position. The image of the parton can be in anyone of the possible locations.

Instead of coding a space–point ⟨$\underline{\mathbf{x}}$⟩ an event ⟨$\underline{\mathbf{x}}$,**t**⟩ shall now be coded on screen, with ⟨**t**⟩ as time–variable. Light–rays passing through the event and intersecting the screen perpendicularly. All incomming light–rays hitting the screen at a certain instant of time will form a **3**–dimensional light–front. One may introduce a set of light–front–coordinates by gauge–fixing the metric of space–time, as shown in ⟨**2^1.2**⟩.

With this in mind one will arrive at the following statement of ‘t HOOFT’s holographic–principle:

- Light–front–quantization of quantum–gravity can be formulated without direct reference to the longitudinal direction $\langle x^- \rangle$ and the transverse space may be taken to be discrete lattice. The lattice is composed of binary pixels with spacing in the order of the PLANCK–length. One may find it convenient to consider larger lattice–spacing and to have a larger number of configurations on each side but the net–result will be again $\langle 1 \rangle$ bit per PLANCK–cell.

The obvious question now arises, how one may code the longitudinal location of a parton? Before being able to answer this question it is necessary to get more information about light–front–quantization and the parton–concept. These concepts are familiar from QCD (Quantum–Chromo–Dynamics) and originated in work on deep inelastic–scattering. Some very basic concepts for light–front–quantization for conventional field–theories like QCD is given in a **⟨2^1.3⟩**.

The boost–problem has a close analogy with the problem of scale–invariance in ordinary–quantum–field–theory. Classical scale–invariance will usually be destroyed by divergent high–frequency–effects and even if it is possible to preserve it, the invariance is realized in some anomalous forms. Before discussing boosts in more details, the subtleties of scale–invariance in the language of Hamiltonian–quantum–mechanics shall be reviewed in **⟨2^1.4⟩**.

The momentum–space–integrals of light–front–perturbation–theory can diverge at high frequencies in two ways:

- The most obvious way is at large values of the transverse momentum which can be neglected when it is assumed that transverse–momentum–integrals are finite and dominated by the values of $\langle P_\perp \rangle$ in order of some characteristic scale; in quantum–theory–of–gravity the PLANCK–mass would be the appropriate scale.
- The second form of high–frequency–divergence that can occur involves integrals over longitudinal momenta $\langle p_- \rangle$. The range of such integrals is always restricted to values $\langle p_- < p_-(tot) \rangle$; more information about this kind of high–frequency–divergence can be found in **⟨2^1.5⟩**.

If one now considers scale–transformation by which the infrared–cutoff is rescaled while ultraviolet–cutoff remains fixed . In general the energy–levels will not simply rescale and one will say that the scale–invariance is broken. However there may be special Hamiltonians (**call fixed–point–Hamiltonians**) which preserve certain features of scale–invariance. In particular for such Hamiltonians when the infrared–cutoff $\langle \kappa \rangle$ is rescaled the energy–spectrum scales just as in ***equ.[2^1.4^2]*** then such a theory is still said to be scale–invariant.
The problem of determining fixed–point–Hamiltonians with behaviour of wave–functions under the boost in QCD is not a well developed subject. Some simple possible behaviours of boost–operations having discussed in the past will now be described in **⟨2^1.6⟩**. The behaviours do not represent established fixed–points but are simplified models.

Trying in doing numerical QCD–work by using light–front–methods involves a method of regulating the low longitudinal momentum–divergences. It consists of replacing the infinite axis of longitudinal momentum by a periodic box of size $\langle 1/\epsilon \rangle$. This has the effect of replacing the $\langle p_- \rangle$–axis by a discrete lattice with spacing $\langle \epsilon \rangle$, the smallest allowable momentum is $\langle \epsilon \rangle$.

In order to code the longitudinal motion of systems on screen it will be assumed that longitudinal momentum comes in discrete units of size $\langle \epsilon \rangle$.

- One may consider a single pixel at a transverse position $\langle \underline{X}_\perp \rangle$. Since the pixel can only record a single bit–of–information it can at most record the presence or absence of a parton but not its state of longitudinal–motion. Thus it may be assumed that a lit–pixel at $\langle \underline{X}_\perp \rangle$ represents a parton of minimal–longitudinal–momentum $\langle \epsilon \rangle$.
- If now the momentum is increased to twice the minimum. Since a pixel cannot be lit twice, one is forced to light $\langle 2 \rangle$ pixels. In general a system with a longitudinal momentum $\langle N\epsilon \rangle$ will be identified by $\langle N \rangle$ lit–pixels. Thus a system is boosted to a large momentum not by boosting its partons but by increasing their number. In other words all partons are **⟨wee⟩** and not $\langle 2 \rangle$ of them may occupy the same pixel.

Crudely speaking when the momentum is doubled each parton must be replaced by $\langle 2 \rangle$ with an average–separation $\langle l_s \rangle$. Scale $\langle l_s \rangle$ could be as small as the PLANCK–length $\langle l_P \rangle$ or larger. The ratio is a dimensionless number $\langle g \rangle$. If $\langle g \rangle$ is small the evolution of the parton–distribution – as it is boosted – is unconstraint by the condition that the density will not exceed $\langle l_P^{-2} \rangle$ and develops according to the Free–String–behaviour. However, since the number of partons increases like the longitudinal–momentum $\langle p_- \rangle$ but the radius of the distribution increases only logarithmically, a point will be reached at which the density becomes $\langle l_P^{-2} \rangle$. At this point the system–area must increase more rapidly. More details presented in **⟨2^17⟩**.

1^2 *On the Origin of Gravity and the Laws of NEWTON.*

The central notation needed to derive gravity is information. More precisely, it is the amount of information associated with matter and its location, in whatever form a microscopic theory likes to have it, is measured in terms of entropy. The most important assumption will be that information associated with space obeys the holographic–principle. Space is a storage for information associated with matter. Given that the maximal allowable information for each part of space is finite, then it is impossible to localize a particle with infinite precision at a point of the continuum. Thus points and coordinates arise as derived concepts. Therefore one could assume that information is stored in points of a discretized space. In order to get information described holographically, it must exist in some duplicated form, thus it is assumed to be stored on surfaces, or screens. Screens separate points and in this way they are natural places for information about particles that move from one side to the other. Within in this environment gravity will get the form of an entropic–force.

An entropic force is an effective macroscopic–force that originates in a system with many degrees–of–freedom by the statistical tendency to increase its entropy. The force–equation is expressed in terms of entropy–differences and is independent of details of the microscopic dynamics.

- One of the best–known examples is the elasticity of a polymer–molecule. A single polymer–molecule can be modelled by joining–together many monomers of fixed–length, where each monomer can freely rotate around the points–of–attachments and direct itself in any spatial direction. Each of these configurations has the same energy. When the polymer–molecule is immerged into a heath–bath with temperature ⟨**T**⟩, it likes to put itself into a randomly coiled configuration, which is entropically favoured. When the polymer–molecule is now stretched to an extended length–⟨$\mathbf{\Delta x}$⟩, the statistical tendency to bring it back to the entropic–equilibrium is translated into a macroscopic–force ⟨**F**⟩. The Force requires to keep the polymer in stretched–position at temperature ⟨**T**⟩ of the heat–bath can be deduced in its magnitude and will be directed in opposition to the entropic–force ⟨**F**⟩ which tries to restore the polymer back to its equilibrium–position. This entropic–force is recognized by the fact that it points in the direction of original entropy and is proportional to the temperature ⟨**T**⟩.

One considers a small piece of a holographic–screen and a particle of mass ⟨**m**⟩ approaching it from the side at which space–time has already emerged. When the particle merges with the microscopic degrees–of–freedom on screen, it influences the amount of information that is already stored on screen. Assuming that the change–in–entropy ⟨$\mathbf{\Delta S}$⟩ near the screen is linear to displacement–from–screen ⟨$\mathbf{\Delta x}$⟩ then it is proportional to mass ⟨**m**⟩. The question comes up, what is the entropic–force resulting herewith?
The basic–idea how force comes into play can be derived from an analogy with osmosis across a semi–permeable membrane. When a particle of mass ⟨**m**⟩ has entropic reason to be on one side of the membrane and the membrane carries a temperature ⟨**T**⟩, the particle will experience an entropic–force ⟨**F**⟩ which will be obtained from relation ⟨$\mathbf{F \cdot \Delta x = T \cdot \Delta S}$⟩.

- It is assumed, ⟨$\mathbf{\Delta S}$⟩ growth linearly with ⟨$\mathbf{\Delta x}$⟩ from membrane and thus ⟨$\mathbf{\Delta S / \Delta x = [k = const] \cdot m}$⟩ is proportional to ⟨**m**⟩.
- On the other side W.G. UNRUH [5] showed that there exists a close relation between a temperature ⟨**T**⟩ and the acceleration ⟨$\underline{\mathbf{a}}$⟩ of an observer in an accelerated frame which can be expressed by ⟨$\mathbf{T = k^{-1} \cdot \underline{a}}$⟩.

From both statements one finally recovers ⟨$\mathbf{F = m \cdot \underline{a}}$⟩ and thus the second–law of NEWTON will be fulfilled by the entropic–force ⟨**F**⟩ mentioned above.

The boundary is now supposed to be a sphere. One may think about the boundary as a storage–device for information, the maximal storage–space or the total number of bits is proportional to the area ⟨**A**⟩ of the sphere. Each fundamental–bit occupies ⟨**1**⟩ unit–cell.

- The number of used bits ⟨**N**⟩ will be proportional to ⟨**A**⟩ in the form ⟨$\mathbf{N = A \cdot [\mathit{q = const}] \cdot G^{-1}}$⟩ where ⟨**G**⟩ denotes the NEWTON–constant.
- It may be supposed that the total energy ⟨**E**⟩ is evenly distributed over the occupied bits ⟨**N**⟩.
- Temperature ⟨**T**⟩ is now determined by ⟨$\mathbf{E \sim T = [\mathit{r = const}] \cdot N \cdot T}$⟩ as the average–energy per bit. Additionally ⟨**E**⟩ can now be identified by EINSTEIN's–formular ⟨$\mathbf{E = M \cdot c^2}$⟩, where ⟨**M**⟩ represents the mass that would emerge in the space–part enclosed by the spherical screen; even though ⟨**M**⟩ is not directly visable in emerged space enclosed by the screen, its presence can be noticed via its energy ⟨**E**⟩.

The rest is now straightforward:

► One eliminates ⟨**E**⟩ and inserts the expression for the number of bits to determine ⟨**T**⟩.

► Next one uses ⟨$\mathbf{\Delta S / \Delta x = k \cdot m}$⟩ for the change of entropy to determine the force.

► Finally one inserts the measure ⟨$\mathbf{A = 4\pi \cdot R^2}$⟩ for the sphere and identifies all the mentioned constants.

After having done all this one will finally obtain NEWTON's law–of–gravity ⟨$\mathbf{F = G \cdot M \cdot m / R^2}$⟩ for the entropic–force ⟨$\mathbf{F = m \cdot \underline{a}}$⟩ from above. More details about contents of this chapter can be found in ⟨**2^(2.)**⟩.

2 *Quantitative Additions to the more qualitative Statements from above.*

Many of previous statements had been specified more or less qualitatively and shall now be further supported in a more quantitative way. In trying this, new agreements should be remembered in advance:

- Space is a **3**–dimensional lattice of spin–like degrees–of–freedom with a lattice–spacing equal to the PLANCK–length.
- The conclusion of J. D. BEKENSTEIN that the maximum–entropy of a spatial region in quantum–gravity is not proportional to the region but to its bounding surface.
- Based on this t' HOOFT's proposal that it must be possible to describe all phenomena in a space–region by a set of degrees–of–freedom which will reside on the region's boundary. The number of degrees–of–freedom should be that of a **2**–dimensional lattice with ⟨**1**⟩ binary degree–of–freedom per PLANCK–area. In other words the world is a **2**–dimensional lattice of spins. In the limit of very large region the bounding–surface can be taken as a flat plane at infinity. In some way, the phenomena taking place in **3**–dimensional space can be projected onto a distant ⟨**viewing–screen**⟩.
- Mapping of **4**–dimensional events and their images on distant screen have to be described quantum–mechanically. All matter is supposed to be composed of elementary structure–less constituents called partons . A parton is represented by projecting it with light–rays intersecting perpendicularly a distant screen, its presence by lightning an information–pixel on screen. No distribution of matter will require no more than ⟨**1**⟩ bit per PLANCK–area on screen. All light–rays passing through an event at same time–instant and hit the screen perpendicularly form a 3–dimensional light–front.

2^1.1 *BEKENSTEIN's Considerations.*

It may be temporarily supposed the world is a **3**–dimensional lattice of spin–like degree–of–freedom. For definiteness may be assumed the lattice–spacing is the PLANCK–length ⟨l_P⟩ and each site is equipped with a spin which can be in one of ⟨**2**⟩ states. The number of distinct orthogonal quantum–states ⟨**N**⟩ may now be considered in a space–region of volume ⟨**V**⟩ as:

$$[2\hat{\ }1.1\hat{\ }1] \quad N(V) = 2^n,$$

where ⟨**n**⟩ is the number of sites in ⟨**V**⟩. The logarithm of ⟨**N(V)**⟩ is the maximal possible entropy in ⟨**V**⟩ and is equal to:

$$[2\hat{\ }1.1\hat{\ }2] \quad S = \log\{N(V)\} = n\cdot\log\{2\} = V\cdot\log\{2\}/l_P^3,$$

More generally one expects that if the energy–density is bounded then maximum entropy is proportional to the volume of the spatial–region. But the correct result in quantum–theory–of–gravity is that the maximum–entropy is proportional to the bounding–area and not to the volume of the region. The argumentation goes as follows:

- A black–hole is a maximally dense object whose entropy is assumed to be found on its horizon. This entropy is equal to:

  $$[2\hat{\ }1.1\hat{\ }3] \quad S = [\text{horizon}]\cdot\log\{2\}/l_P^2 = [\text{horizon}]/4G,$$

  due to the BEKENSTEIN–HAWKING–formula with ⟨**G**⟩ as NEWTON–constant. It may be assumed that a certain state exist with an entropy described by ***eq.[2^1.1^2]*** and of an energy equal to a black–hole of size ⟨**>V**⟩. If now a region ⟨**<V**⟩ inside ⟨**V**⟩ could be found to have an entropy of a black–hole with size ⟨=**V**⟩ – which can be achieved by adding more mass into ⟨**V**⟩ – a contradiction will arise. Since the entropy of the latter black–hole would be smaller than the original entropy, second law of thermo–dynamics would be violated.

Therefore BEKENSTEIN concluded that the maximal entropy of ⟨**V**⟩ must be given by ***eq.[2^1.1^3]*** instead of ***eq.[2^1.1^2]***.

2^1.2 *Introduction of Light–Front–Coordinates by Gauge–Fixing the Space–Time.*

A set of light–front–coordinates are introduced:

$$[2\hat{\ }1.2\hat{\ }1] \quad ds^2 = g_{(+,-)}\cdot dx^+\cdot dx^- + g_{(+,i)}\cdot dx^+\cdot dx^i + g_{(i,j)}\cdot dx^i\cdot dx^j,$$

where components ⟨x^i⟩ refer to transverse–space and ⟨x^+,x^-⟩ are light–like linear combinations of time ⟨**t**⟩ and ⟨**z**⟩–coordinates. If ⟨$x^- = \infty$⟩ then the metric is flat with ⟨$g_{(+,-)} = 1,\ g_{(+,i)} = 0$⟩ and ⟨$g_{(i,j)} = \delta_{ij}$⟩. One may further identify ⟨$x^+ = z+t,\ x^- = t-z$⟩. The screen will be identified by ⟨$x^- = \infty$⟩, the trajectories:

$$[2\hat{\ }1.2\hat{\ }2] \quad x^+ \wedge x^i = const,$$

can be seen as light–like geodesics and the surface ⟨x^+ = **const** ⟩ is a light–front. The standard–practice in doing light–front–quantization is to use ⟨x^+⟩ as time–on–screen; as the time– coordinate in general.
Mapping from ⟨4⟩–dimensional space–time to screen is now transferring a point ⟨x^+, x^-, x^i⟩ to the point ⟨x^+, x^i⟩.

2^1.3 *Light–Front–Quantization and Partons.*

Light–front–coordinates are introduced as ***eq.[2^1.2^1]*** for flat–metric. The coordinate ⟨x^+⟩ is used as time–coordinate. Light–like–surfaces ⟨x^+ = **const**⟩ play the role of instantaneous hyper–surfaces. Partons such quarks or gluons are described by a transverse position ⟨$\underline{X}^{\perp}$⟩ or transverse momentum ⟨$\underline{Q}_{\perp}$⟩ and a positive longitudinal momentum ⟨p_-⟩. In addition a parton may carry internal quantum–numbers and spin– degress–of–freedom which are irrelevant in current considerations.

Light–front creation– and annihilation–operators ⟨$a^+(X_\perp,k_-) \wedge a^-(X_\perp,k_-)$⟩ create and annihilate particles in the usual way. The classical free Hamiltonian has the form:

$$[2\hat{}1.3\hat{}1] \quad H_0 = \int\{a^+(p)[P_\perp{}^2+M^2]a^-(p)/2p_-\}d^2P_\perp dp_-,$$

where ⟨**M**⟩ is the mass of the parton. Classical interactions take the form:

$$[2\hat{}1.3\hat{}2] \quad H_I = \lambda\cdot\int\{a^+(p)a^+(q)a^-(p+q)F(p_-/q_-)/\sqrt{[(p_-)(q_-)(p_-+q_-)]}\}dp^3dq^3+h.c.,$$

where ⟨**F**⟩ are simple rational functions that depend on the spins of involved particles; for scalar particles they are constant. Additional terms may exist with higher powers of the creation– and annihilation–operators. The operators ⟨$a^\pm$⟩ have canonical communication– or anti–communication–relations:

$$[2\hat{}1.3\hat{}3] \quad [a^-(p),a^+(q)] = \delta^2(\underline{P}_\perp-\underline{Q}_\perp)\delta(p_- - q_-).$$

Of special interest are the properties of the theory under longitudinal boots which act according to:

$$[2\hat{}1.3\hat{}4] \quad (p_- \rightarrow \exp\{\omega\}\cdot p_-) \wedge (\underline{P}_\perp \rightarrow \underline{P}_\perp),$$

where ⟨ω⟩ is the hyperbolic–boost–angle. The operators–⟨$a^\pm$⟩ naively transform like:

$$[2\hat{}1.3\hat{}5] \quad a(\underline{P}_\perp,p_-) \rightarrow \exp\{\omega/2\}\cdot a(\underline{P}_\perp,\exp\{\omega\}\cdot p_-).$$

The Hamiltonian transforms as:

$$[2\hat{}1.3\hat{}6] \quad H \rightarrow H\cdot\exp\{-\omega\}.$$

An important advantage of the light–front–method is that the vacuum is the naive FOCK–space–vacuum annihilated by all the ⟨a^-⟩'s. On top of the vacuum is a space of parton–states by acting with he the ⟨a^+⟩–opereators:

$$[2\hat{}1.3\hat{}7] \quad |k_1k_2\ldots\rangle = a^+(k_1)a^+(k_2)\ldots|0\rangle.$$

The Hamiltonian acts within this space.

According to the most naive view of the theory the Hamiltonian is the classical one and the eigen–vectors are well–defined convergent superpositions of FOCK–space–states. Boosting a system along ⟨z⟩–axis is straightforward. Each parton–momentum transforms according to ***eq.[2^1.3^4].*** The transformation simply acts as a scale–transformation of the longitudinal–momentum–axis. According to this naive view a longitudinal boost preserves the transverse dimensions of the system and rescales or LORENTZ–contracts all longitudinal dimensions. However, the correct description is far more complicated due to various kinds of divergences.

2^1.4 *Scale–Invariance in the Language of Hamiltonian–Quantum–Mechinics.*

The degrees–of–freedom are the spatial FOURIER–modes ⟨$\phi(\mathbf{p})$⟩ where the spatial– momentum ⟨$\underline{\mathbf{p}}$⟩ may be any real **3**–dimensional vector. For current purposes it will be important to cut–off the theory in infrared. This may be done by eliminating modes with momenta less than some cutoff ⟨κ⟩ or alternatively putting the system in a finite box of size ⟨κ^{-1}⟩. If one proposes now the classical theory has a scale–invariance under which all momenta including the infrared–regulator ⟨κ⟩ rescale as:

$$[2\hat{}1.4\hat{}1] \quad p \rightarrow \lambda\cdot p,$$

then the naive invariance would imply, the spectrum of the Hamiltonian rescales like:

$$[2\hat{}1.4\hat{}2] \quad E_i \rightarrow \lambda\cdot E_i,$$

and the wave–functionals of the eigen–vectors transform in a way like:

$$[2\hat{}1.4\hat{}3] \quad \Psi_i[\phi(\underline{p})] \rightarrow \Psi_i[\lambda\cdot\phi(\lambda\cdot\underline{p})].$$

Each fluctuation of ⟨$\underline{\mathbf{p}}$⟩ stretches to ⟨$\lambda\cdot\underline{\mathbf{p}}$⟩.
The problem with this naive view is now that the low and high frequencies are not at all decoupled. This

causes divergences in integrations over the large momenta which make everything infinite. To define the theory again an ultraviolet–cutoff ⟨ν⟩ must also be supplied. From a operational point–of–view the use of the ultraviolet cutoff is always correlated with a particular experimental setup. The mathematical description of the cutoff–theory will not have modes with momentum higher as ultraviolet–cutoff ⟨ν⟩. It will have Hamiltonians ⟨H_ν⟩, energy–levels ⟨E_ν⟩ and wave–functionals ⟨Ψ_ν⟩.

2^1.5 *Second Form of high Frequency–Divergence in Momentum–Space–Integrals of Light–Front–Perturbation.*

From ***eq.[2^1.3^1]*** one can see that energy of a parton diverges when ⟨$p_- \rightarrow 0$⟩. This region describes large distances in the ⟨x_-⟩–coordinate but small distances in ⟨x^+⟩. The divergences in this region are not connected with scale– transformation–anomalies but rather anomalies in the behaviour of LORENTZ–boosts.

An example of such a divergence involves the probability for a single particle of momentum ⟨p_-+q_-⟩ to be a pair of partons of momenta ⟨p_-⟩ and ⟨q_-⟩, which in second–order perturbation theory is given by:

$$[2^\wedge 1.5^\wedge 1] \quad \int dq_- |\langle p+q|H_I|p,q\rangle|^2 [E(p+q)-E(p)-E(q)]^{-2}.$$

Using ***eq.[2^1.3^2]*** one will find that the integrand behaves like:

$$[2^\wedge 1.5^\wedge 2] \quad (q_-)F^2(p_-/q_-),$$

as ⟨$(q_-) \rightarrow 0$⟩. Furthermore, if the low–momentum–parton has spin ⟨J⟩ the function ⟨F⟩ has the form ⟨$(q_-)^{-J}$⟩ in this limit. Evedently the probability diverges for ⟨$J \geq 1$⟩ and this divergences indicate that the population of partons can become infinite at low longitudinal momentum thus invalidating the naive picture.

It should be imagined that the degrees–of–freedom ⟨$a^\pm(p)$⟩ being laid–out on ⟨p_-⟩–axis. The length of the axis is finite and given by ⟨$p_-(tot)$⟩. A longitudinal boost is represented as scale–transformation of this axis. LORENTZ–invariance requires perfect scale–invariance with Hamiltonian–transformation according to ***eq.[2^1.3^6]***. The divergences at low ⟨p_-⟩ can potentially disrupt the classical–invariance in a manner similar to the wav ordinary–divergences can ruin scale–invariance. As in that case one must introduce a cutoff on low–values ⟨p_-⟩. Thus one introduces a cutoff which eliminates all ⟨$p_- < \epsilon$⟩ and searches for a fixed of the renormalization–group. Those fixed–points classify the possible boost–invariant–theories.

2^1.6 *Simple possible Models of the Fixed–Point–Hamiltonians.*

The EINSTEIN/LORENTZ–Fixed–Point:

- In very simple field–theories with no divergences the wave–function of a particle or system–of–particles is a convergent FOCK–space–state with a finite average–number of partons. If the cutoff ⟨ϵ⟩ is sufficiently small the probability to find a parton with ⟨$p_- < \epsilon$⟩ is negligibly small. Thus boosting the system is trivial. Each parton shifts to its boosted position, transverse sizes keep invariant and longitudinal sizes contract.

The FEYMAN/BJORKEN–Fixed–Point:

- In case of gauge–theories longitudinal divergences induce a divergent distribution of partons at low ⟨p_-⟩ and the number of partons per unit–⟨p_-⟩ behaves like:

  $$[2^\wedge 1.6^\wedge 1] \quad dn/d(p_-) \sim 1/(p_-).$$

  Introducing a high–frequeny–cutoff ⟨$p_- < \epsilon$⟩ is more serious this time since it eliminates degrees–of–freedom which are present in the hadron. In this case when a boost doubles the longitudinal momentum of each parton, a hole is left in the region ⟨$\epsilon < p_- < 2\epsilon$⟩ and new partons must be added in order to fill this region. In other words, the boost–operator must contain an extra–term which acts as a source for partons of low–⟨p_-⟩; these partons are called ⟨**wee**⟩–partons. The ⟨**wee**⟩–partons create anomalies of matter under boosts. For example, because they always carry low longitudinal–momenta they contribute a cloud which does not LORENTZ–contract. Furthermore they tend to be found at progressively larger transverse distances from the centre–of–mass.

  $$[2^\wedge 1.6^\wedge 2] \quad R_\perp{}^2(wee) \sim \log\{p_-(tot)/\epsilon\}.$$

  In this model the quantum–numbers of a hadrons are carried by ⟨**"valence"**⟩–partons which carry finite fractions of the total momentum and therefore behave as in the EINSTEIN/LORENTZ–case. Thus spatial distributions of charge, baryon–number and angular–momentum LORENTZ–contract and do not transversely spread.

  The cutoff ⟨ϵ⟩ is arbitrary but as in ordinary renormalization–theory, physical applications may make a particular choice most convenient. For example, in case of a high–energy–particle colliding with a fixed target the target determines some range of frequencies for which it is sensitive. Retaining higher frequencies in the description only leads to unnecessary complexity.

The KLEBANOV/SUSSKIND–Fixed–Point:

- Divergences at large transverse momenta can alter the boost–operation. In order to understand this one must generalize the cutoff–procedure in a way that all modes with frequencies greater than some value ⟨ν⟩:

  [2^1.6^3] $(K_\perp^2+m^2)/2(k_-) > \nu$

  will be cut–off. Once again one can eliminate the effects of the cutoff by either letting ⟨($\nu \rightarrow 0$) or ($p_- \rightarrow \infty$)⟩. Now ⟨ν⟩ may kept fixed. The phase–space for partons looks like:

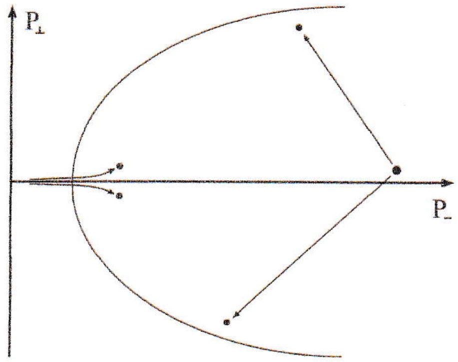

Fig. 2^1.6
Parton–phase–space.

A low longitudinal–momentum–parton is now considered. The interaction–terms in Hamiltonian ⟨**H**⟩ allows the parton to become splitted into ⟨**2**⟩–partons, each one with a smaller ⟨$\mathbf{p_-}$⟩ than the original one. From ***Fig. 2^1.6*** will be obvious that the transverse momentum of the pair must be small. This insures that the probability for the original parton to become a pair is small; therefore divergences at low ⟨$\mathbf{p_-}$⟩ can be ignored. If one boosts the system so that the parton gets a much higher ⟨$\mathbf{p_-}$⟩, the transverse–phase–space for a splitting becomes much bigger. And as far as the theory produces transverse–divergences the probability for such an event will become larger as the system is boosted. Eventually the parton will be replaced by a number of partons closely spaced in the transverse–space. If the system is further boosted the effect will continue and the partons reveal transverse–fine–structure within structures ad infinitum. Thus the boost–operator contains terms which continuously create ⟨**wee**⟩–partons migrating to larger ⟨$\mathbf{p_-}$⟩. In doing this they split into short–distance pairs which continue to migrate and split.

Next a behaviour will be described that cannot be found in ordinary–field–theory but occurs in string–theory, nevertheless can be described in parton–terms.

The Free–String–Fixed–Point.

- A single parton of a longitudinal momentum of order ⟨ϵ⟩ is boosted to twice of its original–momentum. Instead of finding a parton of a double momentum one will find ⟨**2**⟩–partons each with original–momentum ⟨ϵ⟩. The ⟨**2**⟩–partons have a transverse–wave–function which is rotationally symmetric and peaked at a transverse radial–distance ⟨$l_s = \sqrt{\alpha'}$⟩, where ⟨α'⟩ is the usual dimensional string–constant. If one now doubles the momentum again, each of the ⟨**2**⟩–partons is resolved into ⟨**2**⟩ more ones with a similar wave–function. Unlike the KLEBANOV/SUSSKIND–case the evolution does not create pairs of smaller and smaller transverse–size, moreover no parton is ever found with ⟨$\mathbf{p_- > \epsilon}$⟩ and all partons are ⟨**wee**⟩. After ⟨**n**⟩ iterations the total number of partons is ⟨$\mathbf{2^n}$⟩ and the total longitudinal momentum is:

  [2^1.6^4] $p_-(tot) = \epsilon \cdot 2^n$.

  It can be shown that after many iterations the transverse–density of partons relative to the centre–of–mass is Gaussian with a radius satisfying:

  [2^1.6^5] $R_\perp^2 = l_s^2 \cdot n = l_s^2 \cdot \log\{p_-/\epsilon\}$.

  One might also expect that no LORENTZ–contraction will occur since all partons are ⟨**wee**⟩.
  The implication of such a parton–model for high–energy cross–sections are interesting. One may assume that a particle collides with a fixed target while scattering is weak enough that the total cross–section is additive in the constituent–partons. Since the number of partons is proportional to the momentum of the incident particle the cross–section grows linearly with the laboratory–energy. However, this behaviour together with ***eq.[2^1.6^5]*** cannot persist indefinitely without violating unitarity. Either addition of partons will eventually lead to shadowing corrections or the geometric size of the parton–cloud will have to grow more quickly. The latter option is correct.

The string–like parton–model shall now be compared with the real string–theory in the light–front–frame:

- The work will be done in units with ⟨ $l_s = 1$⟩.

- A free string is described by a transverse coordinate ⟨$\underline{X}_\perp(\sigma,x^+)$⟩ which is a function of the parameter ⟨σ⟩ and the light–front–time ⟨x^+⟩. It is convenient to allow ⟨σ⟩ to run ⟨$0 \rightarrow p_-(tot)$⟩. A longitudinal–boost can then be seen as a rescaling ⟨σ⟩. The operator ⟨$\underline{X}_\perp$⟩ can be expanded into normal–modes ⟨$\underline{X}_\perp$⟩ and the centre–of–mass ⟨$\underline{X}_\perp(cm)$⟩.

  $$[2^\wedge 1.6^\wedge 6]\quad \underline{X}_\perp(\sigma,x_-)-\underline{X}_\perp(cm) = {}_{[1]}\Sigma^{[\infty]}\langle \underline{X}_l\cdot exp\{il(\sigma-x^+)/p_-\}-\underline{X}^*_l\cdot exp\{il(\sigma+x^+)/p_-\}\rangle.$$

  If one considers the mean–square–size of the string:

  $$[2^\wedge 1.6^\wedge 7]\quad R_\perp{}^2 = [\underline{X}_\perp(\sigma)-\underline{X}_\perp(cm)]^2.$$

  The matrix–element gets a contribution from each mode and the result diverges:

  $$[2^\wedge 1.6^\wedge 8]\quad R_\perp{}^2 = {}_{[1]}\Sigma^{[\infty]}\langle 1/l\rangle = log\{\infty\}.$$

  The problem is, a high–frequency–cutoff had not been introduced. From ***eq.[2^1.6^6]*** one can see that the frequency of l–th mode is ⟨l/p_-⟩. The highest allowable frequency when the cutoff is in place is of order ⟨$1/\epsilon$⟩ so that the highest allowable mode is:

  $$[2^\wedge 1.6^\wedge 9]\quad l_{max} = p_-/\epsilon.$$

  The logarithmic divergence can now be replaced by ***eq.[2^1.6^5]***.

The string–theory does not endow the string with an independent longitudinal–coordinate. The coordinate ⟨$x^-(\sigma)$⟩ is determined in terms of the transverse–coordinates. The longitudinal–size could be computed and it was found to diverge if no high–frequency–cutoff had been used. This time the divergence is quadratic and gives:

$$[2^\wedge 1.6^\wedge 10]\quad R^2 = 1/\epsilon^2.$$

The longitudinal size does not LORENTZ–contract as ⟨p_-⟩ increases.

Another feature of the string–wave–function involves the length of the transverse–projection of the string as ⟨p_-⟩ increases. It could be shown that as the number of string–modes increases the transverse–string–length increases linearly. This means that the string–length is proportional to ⟨p_-⟩. In order to understand the connection with the free–string–parton–model, the string is thought to be made of string–bits with a length ⟨l_s⟩. One will realize that the number of bits is proportional to ⟨p_-⟩ and each bit is ⟨**wee**⟩.

2^1.7 *Growth of Particles with Momentum.*

Eventually the area ⟨**A**⟩ occupied by a particle will grow like:

$$[2^\wedge 1.7^\wedge 1]\quad A \sim l_P{}^2\cdot(p_-/\epsilon).$$

No matter how small ⟨**g**⟩ is, interactions will become important when the number of modes becomes so large that the density gets ⟨g^{-2}⟩ in string–units. The basic–requirement for consistency with the bound on area–density is now that the effect of interaction is so repulsive that the partons create an incompressible fluid.

2^2.1 *Information about Particles stored on Screens and Direction of emergent Space.*

Space in first place is a device introduced to describe e.g. positions and movements of particles. Thus it is literally a storage for information and this information is associated with matter. Given that, the maximal allowed information is finite for any part–of–storage then it is impossible to locate a particle with infinite precision at a point of the continuum. In fact, points and coordinates arise as derived concepts. One may assume that information is stored in points of discretizied space (like in a lattice–model). But if all the associated information would be stored without duplication one would not get a holographic description and this means, one would be unable to recover the new aspects of gravity.

Therefore is assumed that information is stored on surfaces or screens. Screens separate points and thus they are natural places to store information about particles moving from one side of the screen to its alternate side.

- One may suppose that this information about particles–locations in this way are stored in discrete information–bits on screens.
- Dynamics on each screen is given by some unknown rules which can be thought as a way of processing the information that is stored on the screen. Hence it does not have to be given by a local field–theory or anything else familiar. The microscopic details are irrelevant so far.

Further shall be assumed that there exists ⟨1⟩ specific direction corresponding to scale– or coarse–graining–variable of the microscopic theory.

- In this direction space will be emergent.
- So screens that store information are like stretched horizons. One side there is space on the alternate

side is nothing yet.

Additionally may be assumed that the actual microscopic theory has a well–defined notion of time and its dynamics keeps invariant with respect to time–transitions. This allows one to define energy and temperature by employing techniques of statistical–physics.

These will be the basic ingredients together with the entropy associated with the amount of information for further discussions.

2^2.2 *Acceleration associated with Temperature and NEWTON's second Law.*

The starting assumption is directly motivated by BEKENSTEIN's thoughts initially mentioned and finally expressed by:

[2^1.1^3] $S = [\text{horizon}]\cdot\log\{2\}/l_P^2 = [\text{horizon}]/4G.$

But now it is not used near a black–hole–horizon but in flat, non–relativistic space. For this reason a small part of a holographic screen is now considered and a particle of mass ⟨**m**⟩ approaching it on the screen–side of already emerged space–time. Eventually the particle merges with the microscopic degrees–of–freedom on the screen. But before doing so, it already influences the amount of information that was originally stored on screen.

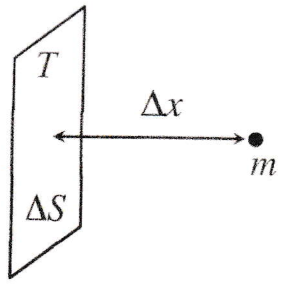

Fig. 2^2.2
*Particle with mass ⟨**m**⟩ approaches part of holographic screen. Screens bounds emerged part of space which contains the particle, and stores data that describe part–of–space that has not yet emerged, as well as some part of the emerged space.*

Motivated by BEKENSTEIN's argument may be postulated that change–of–entropy associated with the information on screen–boundary equals:

[2^2.2^1] $\{\Delta S = 2\pi\cdot k_B\} \leftarrow \{\Delta x = \hbar/(m\cdot c)\},$

where ⟨$\hbar$⟩ is the PLANCK–constant, ⟨k_B⟩ the BOLTZMANN–constant and ⟨**c**⟩ the vacuum–speed–of–light. This may be written in a slightly more general form by assuming that the change–of–entropy near screen is linear in displacement ⟨**Δx**⟩:

[2^2.2^2] $\Delta S = 2\pi\cdot k_B\cdot m\cdot c\cdot\Delta x/\hbar.$

- Why ⟨**ΔS**⟩ is also proportional to the mass ⟨**m**⟩? This can be answered if one imagine the particle to be splitted into lighter sub–particles, each of those carries its own change–in–entropy after a shift of ⟨**Δx**⟩. Because entropy and mass are both additive, therefore it's natural that the above mentioned proportionality must also hold for ⟨**m**⟩ in total.

How does now force come into the discussion?

- The basic idea is to use the analogy with osmosis across the semi–permeable membrane. When the particle has an entropic–reason to be on one side of the membrane and the membrane has a temperature ⟨**T**⟩, it will experience an effective force ⟨**F**⟩ equal to:

  [2^2.2^3] $F\cdot\Delta x = T\cdot\Delta S,$

  where ⟨**F**⟩ is an entropic–force.

From NEWTON's second law one knows that force needs to have a non–zero acceleration. As W. G. UNRUH [5] showed, acceleration ⟨$\underline{a}$⟩ and temperature ⟨**T**⟩ can be closely related.

- An observer in acceleration ⟨$\underline{a}$⟩ will experience a temperature of:

  [2^2.2^4] $k_B\cdot T = \hbar\cdot\underline{a}/(2\pi\cdot c).$

  This equation is to be read as temperature ⟨**T**⟩ to cause an acceleration ⟨$\underline{a}$⟩ and may be taken as the temperature associated with the information–bits on screen.

With the requirements of ***equ.**[2^2.2^2]* and *[2^2.2^4]* one finally will obtain from ***equ.**[2^2.2^3]* NEWTON's second law in the form:

[2^2.2^5] $F = m\cdot\underline{a}.$

2^2.3 *Law of Gravity.*

The boundary is now supposed not to be infinitely extended, it shall become a closed surface, here especially the surface ⟨**A**⟩ of a sphere. This boundary is now considered as a device to store information and under the assumption that the holographical principle holds:

- The total number of bits is proportional to the surface ⟨A⟩ and each fundamental bit occupies ⟨1⟩–unit–cell of ⟨A⟩.

If the number of bits proportional to ⟨A⟩ are denoted by ⟨N⟩, one may write:

[2^2.3^1] $N = A \cdot c^3/(G \cdot \hbar)$,

where ⟨G⟩ will already be identified with NEWTON's–constant. There may be a total energy ⟨E⟩ in the system supposed to be evenly distributed over the bits ⟨N⟩ and the temperature ⟨T⟩ of the system is then determined by the equipartition–rule:

[2^2.3^2] $E = N \cdot k_B \cdot T/2$,

as the average–energy per bit. After this EINSTEIN's mass–energy–equation:

[2^2.3^3] $E = M \cdot c^2$

is also needed, where ⟨M⟩ represents the mass that would be emerged in the space enclosed inside the sphere. Although the mass ⟨M⟩ cannot be seen directly in the emerged space, it is effectively noticed due to its energy. If one eliminates ⟨E⟩ from *equ.[2^2.3^2]* and *equ.[2^2.3^3]* and in the resulting relation between ⟨N⟩ and ⟨T⟩ replaces ⟨N⟩ with *equ.[2^2.3^1]* one will get an expression for ⟨T⟩. This inserted into *equ.[2^2.2^5]* together with *equ.[2^2.2^2]* and:

[2^2.3^4] $A = 4\pi \cdot R^2$.

(for the surface ⟨A⟩ of the sphere) will get a final expression for ⟨F⟩ in the form of:

[2^2.3^5] $F = G \cdot M \cdot m/R^2$.

This is nothing else as NEWTON's law of gravity Thus in summary can be said that by the recent assumptions the entropic–force ⟨F⟩ reveals itself as NEWTON's force of gravity.

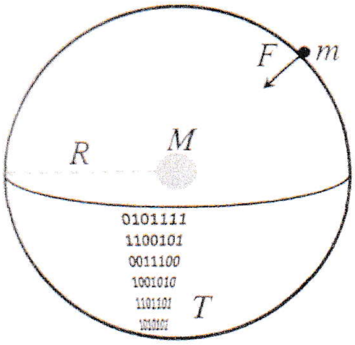

Fig. 2^2.3
Particle with mass ⟨m⟩ near a spherical, holographic screen. The energy is evenly distributed over the occupied bits, and is equivalent to the mass ⟨M⟩ that would emerge in the space enclosed by the screen.

From actual and previous chapters one may summarize:

[2^2.3^2] $E = N \cdot k_B \cdot T/2$ $\quad\quad$ $\Delta S = 2\pi \cdot k_B \cdot m \cdot c \cdot \Delta x/\hbar$ [2^2.2^2]

$\cup$ $\quad\quad$ $\cup$

[2^2.3^3] $E = M \cdot c^2$ $\quad\quad$ $\Delta S = F \cdot \Delta x/T$ ⇐ $F \cdot \Delta x = T \cdot \Delta S$ [2^2.2^3]

⇩ $\quad\quad$ ⇩

$M \cdot c^2 = N \cdot k_B \cdot T/2$ $\quad\quad$ $F/T = 2\pi \cdot k_B \cdot m \cdot c/\hbar$

⇩ $\quad\quad$ ⇩

$2M \cdot c^2/N = k_B \cdot T$ $\cup$ $F = 2\pi \cdot k_B \cdot T \cdot m \cdot c/\hbar$

⇩

$F = 4\pi \cdot M \cdot c^3 \cdot m/(N \cdot \hbar)$ $\cup$ $N = A \cdot c^3/(G \cdot \hbar)$ [2^2.3^1]

⇩

$F = 4\pi \cdot G \cdot M \cdot m/A$

⇩

$F = 4\pi \cdot G \cdot M \cdot m/A$ $\cup$ $A = 4\pi \cdot R^2$ [2^2.3^4]

⇩

[2^2.3^6] $F = G \cdot M \cdot m/R^2$

The final equation *[2^2.3^6]* is NEWTON's law of gravity. It had been derived so far partly for dimensional reasons and also due to NEWTON's law of forces (from previous chapter), contains ingredients in its evolution that lead to black–hole thermodynamics and holographic–principle. All this together casts a completely new light on the origin of gravity, which has become now the character of an entropic force.

2^2.4 *Further Additions to Contents of Sections 2^(2.1–2.3).*

Final remarks referring the previous context shall be given in addition now:

- Equipartition in general holds only for free systems. But how essential is this? Energy usually spreads over the microscopic degrees–of–freedom according to some non–trivial distribution–function. When lost information–bits are randomly chosen among all bits, one expects the energy–change associated with ⟨ΔS⟩ still to be proportional to the energy–per–unit–area ⟨E/A⟩. This fact could therefore be true even when equipartition is not strictly obeyed.

- It may appear somewhat counter–intuitive that in ***equ.[2^2.2^4]*** the scalar temperature ⟨T⟩ is related to the vector–quantity ⟨$\underline{a}$⟩, while from ***equ.[2^2.2^2]*** the vector–gradient ⟨$\Delta S/n$⟩ can be derived which is obviously related to scalar ⟨**m**⟩.
  In order to understand this, a particle with mass ⟨**m**⟩ is to be considered when it approaches the screen. Here it will merge with the microscopic degrees–of–freedom on the screen and hence it is made–up of same information–bits as those live on screen already. Since each bit carries an energy ⟨$½\cdot k_B\cdot T$⟩, the number of bits follow from:

  [2^2.4^1] $m\cdot c^2 = ½\cdot n\cdot k_B\cdot T.$

  With this one will get further:

[2^2.2^4]	$k_B\cdot T = \hbar\cdot \underline{a}/(2\pi\cdot c)$	∪	$m\cdot c^2 = ½\cdot n\cdot k_B\cdot T$ ⇓	*[2^2.4^1]*
[2^2.2^2]	$\Delta S = 2\pi\cdot k_B\cdot m\cdot c\cdot \Delta x/\hbar$	∪	$m\cdot c = (1/4\pi)\cdot n\cdot \underline{a}\cdot \hbar/c^2$	

$$\Downarrow$$
$$\Delta S = 2\pi\cdot k_B\cdot(\Delta x/\hbar)\cdot n\cdot \underline{a}\cdot \hbar/c^2$$
$$\Downarrow$$
$$\Delta S = k_B\cdot n\cdot \underline{a}\cdot \Delta x\,/2c^2$$
$$\Downarrow$$

[2^2.4^2] $\Delta S/n = k_B\cdot \underline{a}\cdot \Delta x\,/2c^2$

By combining the above equations one of course will again recover ⟨F⟩ in ***equ.[2^2.2^5]*** as an entropic–force. But by introducing number–of–bits ⟨n⟩ associated with the particle, one succeeded in making identifications more natural in terms of their scalar– versus vector–character.
One may conclude that the acceleration ⟨$\underline{a}$⟩ is related to an entropic–gradient ⟨**ΔS/n**⟩, thus a particle will stay in rest because there is no entropy–gradient. This fact makes in natural to introduce the NEWTON–potential ⟨Φ⟩ and write the acceleration as ⟨$\underline{a} = -\nabla\Phi$⟩ and this allows to express change–of–entropy in the following way:

[2^2.4^3] $\Delta S/n = -k_B\cdot \nabla\Phi/2c^2.$

The important conclusion has now been obtained that the NEWTON–potential keeps track of the depletion–of–entropy per Information–bit.

3 *All of this can be condensed into a coherent Picture.*

Obviously two different aspects about how information behaves inside a **3**–dimensional space–region or on its bounding **2**–dimensional surface were significant in the previous presentation.

- The first one was concerned with **3**–dimensional light–fronts of events inside a space–region. Holographic principle and light–front–quantization took the crucial parts during these discussions. According to this final reasoning in this context one will be forced to realize that a space–region may be expanded by an increasing amount of information inside the region.
- While by the second aspect the influence of entropy stood in foreground especially due to the role it plays in connection with gravity. Due to the fact that gravity reveals itself as directly connected to an amount of entropy or information one will have to take into consideration that information may change the curvature of a space–region.

The endeavour to condense the former results in a condensed and coherent manner can finally summarized by the following statements:

- A space–region will be expanded caused by an increasing amount of information inside.
- Information is responsible for gravity and thus causes the curvature of space–time due to EINSTEIN's general theory of relativity.

3^1 *Space will expand due to growing Amount of Information inside.*

BEKENSTEIN (when finished his thought–experiment) had been stated that if the maximal number of distinct orthogonal quantum–states in a **3**–dimensional region is bounded then the maximal entropy must be proportional to the bounding surface of the region. Thereafter G. 't HOOFT had extended BEKENSTEIN's interpretation by his holographic–principle wherein he combined quantum–mechanics

and gravity. According to ‘t HOOFT all phenomena taking place in **3**–dimensional region ⟨**V**⟩ can be projected onto the **2**–dimensional bounding surface of ⟨**V**⟩ without loss of information. He declared the **3**–dimensional world as ⟨**2**⟩–dimensional lattice with ⟨**1**⟩ binary degree–of–freedom per PLANCK–area and he imagined that in the limit of very large regions the bounding surfaces can be taken to be flat planes at infinity. Thus phenomena taking place in **3**–dimensional space will be projected on distant viewing–screens. Their discrete lattice–sites are referred as pixels. A pixel can only store ⟨**1**⟩ of information and is either lit or dark.
Points in space are represented by light–rays intersecting the screens in right–angles and all points of a certain instant in time connected in a space–time–event passing perpendicularly through a screen will form a **3**–dimensional light–front. Light–fronts are the subjects of G. ‘t HOOFTs holographic–principle were he further stated in:

● Light–front–quantization in quantum–gravity can be formulated with no direct reference to the longitudinal direction and the transverse space may be taken to be a discrete lattice. This lattice is composed of binary pixels with spacing in order of PLANCK–length.

The obvious question may then arise as to how one may code longitudinal locations of partons. An answer can be given:

► If mapping of 4–dimensional space–time–events and their images on screens can initially be described in semi–classical terms with keeping in mind to extend it further quantum–mechanically.

► In viewing all matter being composed of elementary structure–less constituents called partons.

Light–front–quantization is started with some basic concepts of a conventional field–theory like QCD. According to the most naive view of the theory the Hamiltonian is the classical one and the eigen– vectors are well–defined convergent super–positions of FOCK–space–states. A longitudinal boost is represented as a scale–transformation along the axis of the longitudinal momentum. By boosting a system the transformation simply acts as a scale–transformation along the longitudinal–momentum–axis, transverse– dimensions keep preserved, all longitudinal dimensions will be rescaled or LORENTZ–contracted.

The boost–problem has a close analogy with the problem of scale–invariance in ordinary quantum–field–theory. Classical scale–invariance will usually be destroyed by divergent high–frequency–effects, the invariance is realized in some anomalous forms. The problem with the naive view is that the low and high frequencies are not at all decoupled. This may cause divergences in integrations over large momenta which make everything infinite. The momentum–space–integrals of light–front–perturbation theory can diverge in ⟨**2**⟩ ways:

► The first one occurs at large values of the transverse momentum and reflects divergences of the short distance–singularities which are familiar already from covariant perturbation theory.

► The second form that can occur involves integrals over longitudinal momenta. Divergences in this region are not connected with scale–transformation–anomalies but rather anomalies in the behaviour of LORENTZ–boosts.

For reasons to avoid again difficulties during further considerations it seems to be advantageous to restrict the theory by defining cutoffs ⟨$\boldsymbol{\kappa}$⟩ against infrared, ⟨$\boldsymbol{\nu}$⟩ with respect to ultraviolet. The mathematical description of the cutoff–theory will be not to allow modes with momenta less than ⟨$\boldsymbol{\kappa}$⟩ and higher than ⟨$\boldsymbol{\nu}$⟩.

By rescaling ⟨$\boldsymbol{\kappa}$⟩ and leaving ⟨$\boldsymbol{\nu}$⟩ fixed in general the energy–levels will not simply rescale so that one is forced to say, the scale–invariance is broken. However in this situations there may exist special Hamiltonians (so– called Fixed–Point–Hamiltonians) which preserve certain features of the scale–invariance. In case of such specials the theory is said to be still scale–invariant.

But even now determing fixed–points and the behaviour of the wave–functions under boost in QCD is not a well–developed subject. Therefore only some possible behaviours of the boost–operation that have been discussed will enable further considerations now. These behaviours do not represent established fixed–points but can be discussed as simplified models instead of and in this position they will show some very interesting properties:

● The FEYMAN/BJORKEN–model in the case of gauge–theories longitudinal–divergences introduce a divergent distribution of partons of low longitudinal–momenta. In this case when a boost doubles the longitudinal–momentum of each parton a hole is left in the region between a cutoff ⟨$\boldsymbol{\epsilon}$⟩ and ⟨$\boldsymbol{2\epsilon}$⟩. New partons must be added to fill the region. The boost–operator must contain an extra–term which acts as source of low–momentum–partons. These partons R. P. FEYNMAN called ⟨**wee**⟩–partons. This kind of partons always carry low longitudinal–momenta and contribute a cloud which does not LORENTZ–contract. Furthermore they tend to be found at progressively larger transverse–distances from centre–of–mass.

Or:

- In the KLEBANOV/SUSSKIND–model a low–longitudinal–momentum parton is considered and the interaction–term of the Hamiltonian allows the parton to split into ⟨2⟩ partons, each one with a smaller longitudinal–momentum than the original one. If the transverse–momentum of the pair is small it insures that the probability for the original parton to become a pair is small and therefore divergences due to low longitudinal–momenta can be ignored. However if one boosts the system so that the parton gets a much larger longitudinal–momentum, the transverse–phase–space for it to split has become much bigger. If the theory has transverse– divergences the likelyhood for something like this will even more increase as the system is further boosted. Eventually the original parton will be replaced by more partons closely spaced in transverse–space. The effect continues as the system is stronger boosted so the partons reveal transverse–fine–structure within structure ad infinitum. Thus the model–boost–operator contains terms which continuously create ⟨**wee**⟩–partons which continue to migrate to larger longitudinal–momenta. As they do so they split into short–distance–pairs which continue to migrate and split.

Or:

- In Free–String–model (which can be shown even being able to described in parton–terms) where a single parton of longitudinal–momentum of order–⟨ϵ⟩ is boosted to twice its original momentum. Instead of finding a parton of twice the momentum one will find ⟨2⟩ partons each of the original–momentum–⟨ϵ⟩. The partons have transverse rotationally–symmetric wave–function peaked at transverse radial distances ⟨$l_s = \pm\sqrt{\alpha'}$⟩, with ⟨α'⟩ the usual dimensional string–constant. If the momentum is doubled again each of the ⟨2⟩ is resolved into ⟨2⟩ more with similar wave–function. Unlike in KLEBANOV/SUSSKIND–case the evolution does not create pairs of smaller and smaller size. Moreover no parton is ever found with ⟨$p_- > \epsilon$⟩ and all partons are ⟨**wee**⟩. After ⟨**n**⟩ iterations the total number of partions has become ⟨2^n⟩, the total longitudinal momentum tends to ⟨$\epsilon 2^n$⟩ and the transverse density of partons relative to the centre–of–mass gets Gaussian with a radius ⟨$R^2 = nl_s^2$⟩. However, this behaviour cannot persist indefinitely, either the addition of partons will eventually lead to shadowing corrections or the geometric–size of the partons will have to grow more quickly.

Finally another idea that originated in attempts to do numerical QCD–work using light–front–methods should also be mentioned.

- In order to code longitudinal–motions of systems on screen it will be assumed that longitudianal–momentum comes in discrete units of size ⟨ϵ⟩. If one considers a single pixel at transverse–position ⟨$X_\perp$⟩. Since a pixel can record only a single bit–of–information it can at most record the presence or absence of a parton but not its state of longitudianal–motion. Therefore one will assume a lit–pixel at ⟨$X_\perp$⟩ represents a parton of minimal longitudinal–motion ⟨ϵ⟩. By increasing the momentum to twice the minimum, one is forced to light ⟨2⟩ pixels, since a pixel cannot be lit–twice. In general a system with momentum ⟨$p_- = N\epsilon$⟩ will be identified with ⟨N⟩ lit–pixels. Thus a system is boosted to larger momentum not by boosting its partons but by increasing their number. In other words, all partons are ⟨**wee**⟩ and no two of them may occupy the same pixel.

Having followed this reasoning one probably cannot avoid to accept that if the entropy on the bounding surface of space region reaches a maximum (and with it the appropriate number of information–pixels on that surface), either further information will be prevented from being stored on this surface or the surface is forced to expand its area. In case of an increasing bounding surface of a space–volume, the volume itself must grow. That this can happen becomes obvious from the path of arguments just given above, especially by the final models.
Therefore, based on all this just described statements it is fair to say that:

- An 3–dimensional space–region will become expanded due to a growing amount–of–information inside.

3^2 *Gravity will turn out to be an entropic Force.*

The amount of information associated with matter and its locations (measured in terms of entropy) is needed to derive gravity. Space is the device introduced to describe positions and movements of particles while it obeys the holographic–principle. Thus it is literally a storage for information associated with matter. Given that the maximal allowed information is finite for any part–of–storage then it is not possible to locate a particle with infinite precision at a point of the continuum. Thus it is favourable to assume that information is stored in points of a discretizied space (like in a lattice–model).
In order to get information described holographically, it must exist bit–wise in some duplicated form on surfaces (screens). A Screen separates points between its front– and back–side and thus is the natural

place to store information about particles moving from one side of the screen to its alternate side. One side there is space on the alternate side is nothing yet. This is because there will exist ⟨1⟩ specific direction where space is emergent to. Within this environment gravity will proved to be an entropic–force.
An entropic force is an effective macroscopic–force that originates in a system with many degrees–of–freedom by the statistical tendency to increase its entropy. The force–equation is expressed in terms of entropy–differences and is independent of details of the microscopic dynamics.

One may consider a small piece of a holographic–screen and a particle of mass ⟨**m**⟩ approaching it from the side at which space–time has already emerged. When the particle merges with the microscopic degrees–of–freedom on screen, it influences the amount of information that is already stored on screen.

- ► Assuming that the change–in–entropy ⟨ΔS⟩ near the screen is linear to displacement–from–screen ⟨Δx⟩ then ⟨$\Delta S/\Delta x = [2\pi \cdot k_B/(\hbar \cdot c)] \cdot m$⟩ is proportional to the mass, with ⟨k_B⟩ as BOLTZMANN–constant,⟨$\hbar$⟩ as PLANCK–constant and ⟨**c**⟩ as vacuum–light–speed.

The question now comes up, what is the entropic–force resulting herewith? The basic–idea how force comes into play can be derived from an analogy with osmosis across a semi–permeable membrane.

- ► When a particle of mass ⟨**m**⟩ has entropic reason to be on one side of the membrane and the membrane carries a temperature ⟨**T**⟩, the particle will experience an entropic–force ⟨$\underline{F}$⟩ which fulfills relation ⟨$F = [2\pi \cdot k_B/(\hbar \cdot c)] \cdot m \cdot T$⟩.
- ► W.G. UNRUH showed now that there exists a close relation ⟨$T = [\hbar \cdot c/(2\pi \cdot k_B)] \cdot a$⟩ between a temperature ⟨**T**⟩ and an acceleration ⟨**a**⟩ an observer will experience in an accelerated frame. Thus one can finally be obtained ⟨$\underline{F} = m \cdot \underline{a}$⟩.

This reasoning is assumed to be valid in general and thus the entropic–force ⟨**F**⟩ in the above context obeys NEWTONS's second law.

The viewing screen (bounding surface) mentioned above as being a sphere, the maximal storage–space or the total number of bits is then proportional to the area ⟨**A**⟩ of the sphere. Each fundamental bit occupies ⟨**1**⟩ unit–cell.

- ► The number of bits proportional to ⟨**A**⟩ can be expressed by ⟨$N = [c^3/(G \cdot \hbar)] \cdot A$⟩ with ⟨**G**⟩ as NEWTON–constant. The total energy in the system is supposed to be ⟨**E**⟩ evenly distributed over the bits ⟨**N**⟩. The temperature ⟨**T**⟩ of the system can then be deduced via the equipartition–rule through ⟨$E = N \cdot k_B \cdot T/2$⟩ which defines the average–energy per bit.
- ► ⟨**E**⟩ is now eliminated by EINSTEIN's energy– mass–relation ⟨$E = M \cdot c^2$⟩ where **M**⟩ represents the mass emerged into the space enclosed by the screen. Although ⟨**M**⟩ cannot be seen directly in the emerged space, it is effectively noticed due to its energy.
- ► In the resulting relation between ⟨**N**⟩ and ⟨**T**⟩ one may replace ⟨**N**⟩ with the number of bits proportional to ⟨**A**⟩ and one will get a for expression for ⟨**T**⟩ alone. This last expression inserted into ⟨$\underline{F} = [2\pi \cdot k_B/(\hbar \cdot c)] \cdot m \cdot T$⟩ together with ⟨$A = 4\pi \cdot R^2$⟩ for the surface of the sphere with its radius ⟨**R**⟩ will result in the final expression ⟨$\underline{F} = G \cdot M \cdot m/R^2$⟩ for the entropic–force ⟨**F**⟩ in the form of NEWTON's law of gravity.

Thus from this summary it can be stated that:

- ● By recent assumptions the entropic–force ⟨**F**⟩ reveals itself as NEWTON's force of gravity influencing the curvature of the screen by A. EINSTEIN's general relativity.

4 *References.*

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