

# Optimal Capacitor Allocation Using Metaheuristic Algorithms in Radial Distribution Networks

Kola Sampangi Sambaiah

Email: [sambaiahks@gmail.com](mailto:sambaiahks@gmail.com)

**Abstract:** Capacitor allocation plays a vital role in the planning and operation of distribution networks. Optimal allocation of capacitor provides reactive power compensation, relieves feeder capacity, improves voltage profile, minimizes power losses, annual cost, and maximizes net savings. However, optimal capacitor allocation is a complex combinatorial optimization problem which consists of finding the bus location, number of capacitors to be installed and their respective sizes by satisfying all the distribution network constraints. The present article investigates the effective implementation of two novel metaheuristic algorithms for solving capacitor allocation optimization problems in the radial distribution network (RDN). The first algorithm is inspired by the water cycle process of nature in the real world where streams and rivers flow to the sea known as the water cycle algorithm (WCA). The second algorithm is inspired by salp swarming behavior in oceans for navigating and foraging is known as the salp swarm algorithm (SSA). To crisscross the feasibility, WCA and SSA are tested on standard 9, 33, 34, 69, and 85 – bus RDNs. Both the algorithms require less computational time for evaluating the objective function and only a few parameters need to be tuned. In addition, to show the superiority of the results obtained by WCA and SSA comparison has been made with various existing optimization techniques. The comparison confirms that both algorithms are more effective in minimizing power losses and operating costs and well suitable for solving capacitor allocation problems in RDNs.

**Keywords** — Capacitor allocation, Metaheuristic algorithms, Power loss minimization, Radial distribution networks, Salp swarm algorithm, Water cycle algorithm.

## Nomenclature:

$P_m$	real power flow from bus $m + 1$ connecting bus $m$ and $m + 1$	$C_1$	adaptive parameter
$Q_m$	reactive power flow from bus $m + 1$ connecting bus $m$ and $m + 1$	$C_2$ and $C_3$	random numbers between 0 and 1
$P_{Lm}$	real load at bus $m$	$lb_d$	lower boundary of the $d^{\text{th}}$ dimension
$Q_{Lm}$	reactive load at bus $m$	$ub_d$	upper boundary of the $d^{\text{th}}$ dimension
$R_{m,m+1}$	line resistance between bus $m$ and $m + 1$	RDN	radial distribution network
$X_{m,m+1}$	line reactance between bus $m$ and $m + 1$	WCA	water cycle algorithm
$V_m$	voltage magnitude at bus $m$	SSA	salp swarm algorithm
$P_L(m, m + 1)$	Line loss between bus $m$ and $m + 1$	SA	simulated annealing
$(P_{T,L})$	total line losses in a distribution network	TS	tabu search
$Q_0^{cap}$	smallest capacitor size in kVAR	GA	genetic algorithm
$Q_m^{cap}$	size of the installed capacitor at bus $m$ in kVAR	WSCP	widespread commercial package
$K_P$	equivalent annual cost per unit of power loss in \$(/kW-year)	PSO	particle swarm optimization
$K_m^{cap}$	cost of capacitor installed at bus $m$ (in \$/kVAR-year)	PGSA	plant growth simulation algorithm
$V_{Min}$	minimum permissible limits of voltage in p.u.	DSA	direct search algorithm
$V_{Max}$	maximum permissible limits of voltage in p.u.	HBMO	honeybee mate optimization
$RD$	raindrops	TLBO	teaching learning based optimization
$N_{RD}$	initial population (the number of raindrops)	BA	bat algorithm
$N_{var}$	design variables	CS	cuckoo search
$N_{sr}$	sum of rivers	BFOA	bacterial foraging optimization algorithm
$NS_k$	number of streams	SSO	shark smell optimization
$rand$	random number distributed uniformly between 0 and 1	WOA	whale optimization algorithm
$D$	a value between 1 and 2	FPA	flower pollination algorithm
$d$	distance between stream and river	GWO	grey wolf optimizer
$X_d^1$	location of leader salp in the $d^{\text{th}}$ dimension	SLD	single line diagram
$X_d^i$	location of follower salp in the $d^{\text{th}}$ dimension	ACO	ant colony optimization
$F_d$	food source location in the $d^{\text{th}}$ dimension	MINLP	mixed integer nonlinear programming

## 1. Introduction

The analysis of electric distribution networks is attaining as the most significant area for research over the past few decades. Since the distribution network is the final link between the bulk power generation system and consumers, hence utmost care must be taken for network operation, control, and maintenance. In general, a distribution network carries high currents and operates at low voltages due to the presence of inductive loads. When we move far away from the substation the bus voltages will reduce and power loss will increase. This is due to the lack of reactive power availability throughout the distribution network. Hence the problem of insufficient reactive power in distribution networks can be solved by providing effective reactive power support or reactive power compensation. Capacitor allocation in the distribution network will provide reactive power support and relieves feeder capacity, improve voltage profile, minimize power losses, annual cost, and maximize net savings. It is a known fact that optimal capacitor allocation is a complex combinatorial optimization problem that consists of finding the bus location, the number of capacitors to be installed, and their respective sizes by satisfying all the distribution network constraints. Due to this, the problem of capacitor allocation is considered a most significant challenge in the power system.

Inappropriate capacitor allocation leads to high power losses, unbalanced bus voltages, deteriorates power quality and reliability finally system breakdown. In this context, several researchers have made noteworthy contributions to solving the capacitor allocation problem. A variety of methods have been implemented to solve the capacitor allocation problem in recent years. A survey on capacitor allocation significance in distribution networks for power loss reduction and voltage profile improvement is presented in [1]. Bae has initially implemented an analytical method to identify the capacitor locations for reducing yearly losses through reactive power compensation by considering variable annual reactive power load in [2]. In [3], authors have developed an analytical method for the reduction of power and energy losses to optimize the net savings by considering a generalized procedure using equal area criteria. Salama et al in [4], have developed a step-by-step procedure for calculating variable load by assessing realistic financial data. Baran and Wu initially proposed a mixed integer solution algorithm which is based feasible direction approach for solving the capacitor allocation problem in [5]. Chiang et al in [6], have implemented simulated annealing (SA) with a generalized solution algorithm using a software package in Fortran. In [7], authors have formulated and solved capacitor placement problems with variable load demand by a tabu search-based solution algorithm. Levitin et al in [8], have implemented a fast energy loss computation technique with a genetic algorithm (GA). In [9], the author has applied GA for selecting fixed and switched capacitors to compensate for reactive power in the radial distribution network. The capacitor allocation problem is solved by using GA for loss minimization which has the capability of finding both location and size in [10].

A fuzzy based GA has been developed to identify both location and size for overall savings in [11]. In [12], authors achieved maximum savings through capacitor allocation after an appropriate linearization by mixed-integer linear program technique using a widespread commercial package (WSCP). A two-stage method has been implemented for solving the capacitor sizing problem: Markov chains are used to identify the sensitive buses and a heuristic method for sizing capacitors in [13]. An evolutionary technique has been proposed to minimize annual active power losses through capacitor placement in [14]. In [15], authors have implemented particle swarm optimization (PSO) for fixed capacitor placement and sizing considering current and voltage harmonics in the distribution network. In [16], the authors used a loss sensitivity index for determining capacitor location and a plant growth simulation algorithm (PGSA) for capacitor size. A direct search algorithm (DSA) has been used for determining capacitor locations with fixed sizes in [17]. In [18], authors have implemented PSO algorithm by considering both static and dynamic approaches for capacitor allocation in the distribution network. An integrated evolutionary algorithm has been implemented for solving capacitor placement using differential evolution and pattern search (DE – PS) in [19]. In [20], a novel stochastic technique based on the point estimate method (PEM) known as the self-adaptive modified honeybee mating optimization (SAMHBMO) algorithm has been implemented to solve the capacitor placement problem by considering uncertainty effects in the distribution system. A human ability-based algorithm known as teaching learning-based optimization (TLBO) is used for capacitor placement on various distribution networks in [21]. In [22], a novel method for capacitor placement based on PSO with Gaussian and Cauchy probability distribution function operators for chaotic load sequence in the distribution network. In [23], authors have proposed a hybrid optimization algorithm which is a combination of two bio-inspired algorithms namely bat algorithm (BA) and cuckoo search (CS) algorithm implemented to capacitor allocation where available capacitors are both fixed and variable switching types.

A combinational technique has been used for identifying the capacitor location and for sizes a bio-inspired algorithm called bacterial foraging optimization algorithm (BFOA) is used [24]. A new meta-heuristic algorithm called shark smell optimization (SSO) is used for capacitor location and size in [25]. In [26], the whale optimization algorithm (WOA) has been used for capacitor location and size in distribution networks. Flower pollination algorithm (FPA) has been implemented for capacitor placement in distribution network reduced power loss in [27]. Recently, grey wolf optimizer (GWO) and water cycle algorithm (WCA) have been extensively implemented on practical distribution networks for capacitor allocation with variable load demand in [28]. A global soft computing technique has been used for capacitor location and sizing in standard distribution systems [29]. In [30] authors have implemented a polar bear optimization algorithm for capacitor location and sizing to reduce real power loss and capacitor cost.

A hybrid technique has been used to solve the capacitor allocation problem to minimize total power loss and energy cost in standard and practical distribution systems [31]. In [32] author implemented a genetic algorithm considering daily load demand to achieve cost benefits. The harmony search algorithm is implemented for capacitor sizing and placement in a harmonic polluted distribution network [33]. A multiverse optimizer has been used for evaluating capacitor size and location [34]. Recently, authors have implemented a spotted hyena optimizer and mathematical remora optimization algorithm on a distribution system with various loading conditions for attaining power loss and energy cost minimization [35,36]. Summary of optimization techniques/algorithms used for SCB location and size in distribution networks have been presented in the below **Table 1**.

**Table 1**

Summary of optimization techniques/algorithms used for capacitor allocation in distribution networks

Author Year [Reference]	Technique / Algorithm	Objective	Capacitor type Switched / fixed	Merits	Demerits	Test system
Y. G. Bae 1978 [2]	Analytical	yearly loss reduction	fixed	simple mathematical equations. different reactive power levels.	loss due to reactive power only considered and voltage regulation not considered.	CPSL 23 kV feeder
Grainger J. J., and Lee, S. H., 1981 [3]	Analytical	power and energy loss minimization	fixed	unrealistic assumption not considered. capacitor installation economic analysis has been considered.	not suitable for the network with laterals branches and voltage regulation is not considered.	9 – bus
Salama M. M. A et al., 1985 [4]	Analytical	reducing power and energy loss	fixed	step by step calculation for variable load and considered realistic financial data.	complex mathematical equations.	11 – bus
Baran M. E., and Wu, F. F., 1989 [5]	Analytical	real power loss minimization	fixed	ac power model, incorporated capacitor cost into problem formulation, and considered bus voltage magnitudes as constraints.	the cost factor is oversimplified.	9 – bus and 69 – bus
Chiang et al., 1990 [6]	SA	real power and energy loss minimization	both fixed and switched	cost analysis suitable for variable capacitor locations.	voltage regulation not considered.	69 – bus
Yann-Chang Huang et al., 1996 [7]	TS	capacitor cost investment and energy loss minimization	fixed	reduces search with priority list obtained by sensitivity analysis method.	the solution obtained is near-optimal.	69 – bus
Levitin G., et al., 2000 [8]	GA	power and energy loss minimization	fixed	four different loads have been considered instead of three.	convergence characteristics have not been discussed.	CPSL 23 kV feeder

Author Year [Reference]	Technique / Algorithm	Objective	Capacitor type Switched / fixed	Merits	Demerits	Test system
Das D., 2002 [9]	GA	energy loss minimization	both fixed and switched	both fixed and marginal costs are considered.	energy loss minimization compromises overall savings.	69 – bus
Swarup K. S., 2005 [10]	GA	loss minimization	fixed	the algorithm has the capability of finding both location and size.	obtained results not validated with existing literature.	Practical Indian 29 – bus, standard IEEE 33-bus and 34-bus 15 – bus and 69 – bus
Prasad P. V., et al., 2007 [11]	Fuzzy – GA	total loss minimization	fixed	the algorithm has the capability of finding both location and size.	convergence characteristics and computational time have not been discussed.	15 – bus, 33 – bus, and Canadian 141 – bus
Khodr H. M., et al., 2008 [12]	WSCP	maximize savings	both fixed and switched	the method is computationally efficient.	convergence characteristics have not been discussed.	9 – bus and 69 – bus
Hamouda A., et al., 2010 [13]	Two-stage Heuristic Method	maximize net savings	fixed	Markov chains are used to identify the sensitive buses for capacitor placement.	convergence characteristics and computational efficiency of the algorithm have not been discussed.	10 – bus and 34 – bus
Elmaouhab A. et al., 2011 [14]	Evolutionary technique	the annual cost of active power loss minimization	fixed	bus voltage magnitudes are maintained within the limits.	net injected reactive power is more compared to existing techniques to minimize losses.	18 – bus and 33 – bus
Taher S. A et al., 2011 [15]	PSO	cost of power and energy loss minimization	fixed	network harmonics and power quality has been considered.	convergence characteristics and application of an algorithm for large networks have not been discussed.	10, 34, and 85 – bus
Rao R. S et al., 2011 [16]	PGSA	voltage improvement and loss minimization	fixed	the algorithm does not need any external control parameters and it handles the optimization problem objective and constraints separately.	convergence characteristics of cost function have not been discussed.	22 – practical, standard 69 – and 85 – bus
Raju M. R. et al., 2012 [17]	DSA	maximization of net savings and voltage stability improvement	both fixed and switched	the algorithm is methodic and sequential.	convergence characteristics and computational efficiency of the algorithm have not been discussed.	70 and 135 – bus
Singh, S. P., and Rao A. R., 2012 [18]	PSO	power and energy losses minimization	both fixed and switched	power flow is computed with a simple iterative method and search space is reduced through the dynamic sensitivity technique.	convergence characteristics and computational efficiency of the algorithm have not been discussed.	

Author Year [Reference]	Technique / Algorithm	Objective	Capacitor type Switched / fixed	Merits	Demerits	Test system
El-Fergany A. A., 2013 [19]	DE – PS	annual operating cost minimization	both fixed and switched	search space is reduced through pre-identification of bus locations.	convergence characteristics and application of an algorithm for large networks have not been discussed.	34 – bus and 69 – bus
Kavousi-Fard, A., and Niknam, T. 2013 [20]	SAMHBMO	power quality improvement, power, and energy losses minimization	fixed	repository stores Pareto optimal solutions and clustering technique with fuzzy provides pre- determined bus locations.	the computational efficiency of the algorithm has not been discussed.	18 and 33 – bus
Sultana, S., and Roy, P. K. 2014 [21]	TLBO	power loss and energy costs minimization	fixed	the algorithm is capable of handling constrained optimization problems with few parameters tuning.	slow convergence and solution obtained are near- optimal for large networks.	22, 69, 85, and 141 – bus
Lee C. S et al., 2015 [22]	PSO	voltage improvement, loss minimization, and feeder reliever	fixed	convergence characteristics show the efficiency and reliability of the algorithm.	algorithm control parameters have not been discussed.	9 – bus and 33 – bus
Injeti S. K et al., 2015 [23]	BA and CS	maximization of net savings and power loss minimization	both fixed and switched	the high quality solution is obtained with hybridization.	more parameter tuning is needed.	34 – bus and 85 – bus
Devabalaji K. R et al., 2015 [24]	BFOA	power loss minimization and voltage improvement	fixed	the algorithm has been implemented for variable load levels.	the solution obtained if near-optimal solution.	34 and 85 – bus
Gnanasekaran N et al., 2016 [25]	SSO	energy loss and reactive power compensation cost minimization	both fixed and switched	the algorithm has the capability of finding both location and size.	the computational efficiency of the algorithm has not been discussed.	34 and 118 – bus
Prakash, D. B., and Lakshminaraya na C. 2017 [26]	WOA	line losses and improve bus voltage minimization	switched	the algorithm requires less parameter tuning.	multi-objective optimization objective functions must be contradictory.	34 and 85 – bus
Tamilselvan V et al., 2018 [27]	FPA	power loss and reactive power compensation cost minimization	switched	the algorithm has been extensively implemented on various test systems.	the solution obtained is near-optimal.	33, 34, 69, and 85 – bus
Kola Sampangi, S. and Thangavelu, J 2020 [28]	GWO and WCA	voltage deviation and power loss minimization	both fixed and switched	algorithms have been extensively implemented on various practical test systems.	capacitor rated values are not standard available.	Practical Indian 28 – bus, 47 – bus, 52 – bus, and 85 – bus

In the present article, two novel metaheuristic algorithms namely water cycle algorithm (WCA) and salp swarm algorithm (SSA) are implemented for solving capacitor allocation problems in the distribution networks. The aim of the formulated objective function is to minimize power losses, improve voltage profile and maximize net savings, subject to operating constraints. Both the algorithms are implemented in MATLAB environment and tested on various standard radial distribution networks.

In the present study, the assumptions considered are:

- The capacitor losses associated cost is negligible compared to the system's losses cost.
- The impact of harmonic's is neglected.
- The test system is assumed to be balanced or is within tolerable limits.
- Fixed transformers tap settings values.
- The distribution substation (slack bus) is not capable of injecting reactive power.
- Always bus 1 is considered as reference/slack.
- The stray capacitor's effect on the line is neglected.
- The capacitor size (kVAR) and cost are linearly proportional.
- All loads are constant power.

The main contributions of the present article are:

- Summary of optimization techniques/algorithms has been implemented in the literature for solving capacitor allocation problems in distribution networks.
- Two novel meta-heuristic algorithms WCA and SSA are effectively employed for solving the problem of capacitor allocation in distribution networks.
- Both the algorithms are tested on standard radial distribution networks.
- The two main objectives of power loss minimization and net savings maximization are attained without violating the system constraints.

The remaining of this paper is structured as follows: Section 2 explains the distribution network load flow, Section 3 illustrates the problem formulation, Section 4 gives the overview and implementation of the water cycle algorithm for capacitor allocation, Section 5 gives the overview and implementation of salp swarm algorithm for capacitor allocation, Section 6 comprises the simulation results and discussions, Section 7 presents the conclusion and future scope.

## **2. Distribution Network Load Flow**

In general, distribution networks are simple radial structured with several lateral branches. Radial distribution network (RDN) has high line resistance to inductive reactance ratio ( $R/L$ ) makes load flow studies complex. Conventional load flow analysis methods like Newton-Rapson and Gauss-Seidel are not much efficient in solving radial load flow due to convergence problems. Several simplified methods are proposed for normal and variable load distribution in networks. A direct approach method for solving load flow in RDN using topological structure is presented in [37, 38]. The present method avoids the convergence problem, hence in the present study, the same load flow method has been used.

### 3. Problem formulation

Reactive power compensation in distribution networks can be advantageous only if it is applied appropriately. Appropriate application means identifying the proper location and size of reactive power aid. In general, attaining zero power loss in a power system is impossible, however, it is feasible to retain them at a minimum. Reactive power compensation is used most often for power loss minimization with additional benefits such as relieving feeder capacity, improving equipment utilization, and avoiding equipment aging. It is acquitting that loss minimization does not ensure the benefits of operational cost minimization or net savings maximization unless the efficiency of all units must be identical.

#### 3.1. Power loss equations

The problem formulation for the present problem can be obtained by **Fig. 1**. Single line diagram (SLD) of a sample distribution network [5]. The realized equations for load flow calculation are

$$P_{m+1} = P_m - P_{Lm+1} - \left( \frac{P_m^2 + Q_m^2}{|V_m|^2} \right) * R_{m,m+1} \quad (1)$$

$$Q_{m+1} = Q_m - Q_{Lm+1} - \left( \frac{P_m^2 + Q_m^2}{|V_m|^2} \right) * X_{m,m+1} \quad (2)$$

$$|V_{m+1}|^2 = |V_m|^2 - 2 * (R_{m,m+1} * P_m + X_{m,m+1} * Q_m) + (R_{m,m+1}^2 + X_{m,m+1}^2) * \left( \frac{P_m^2 + Q_m^2}{|V_m|^2} \right) \quad (3)$$

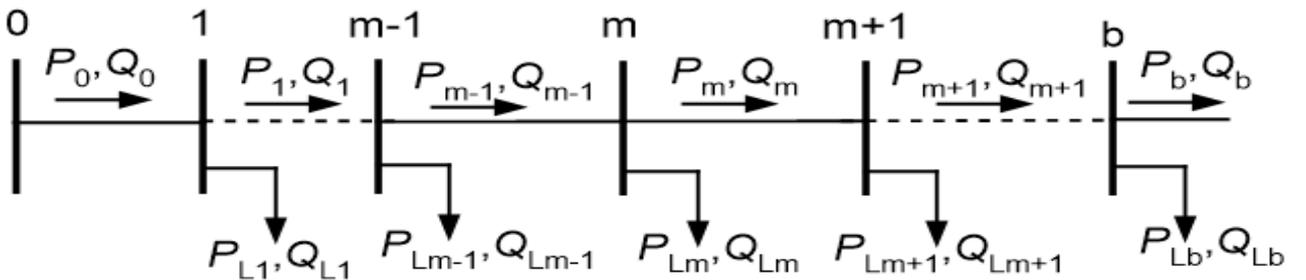
where  $P_m$  and  $Q_m$  are real and reactive power flow from bus  $m + 1$  connecting bus  $m$  and  $m + 1$ ;  $P_{Lm}$  and  $Q_{Lm}$  are real load and reactive load at bus  $m$ ;  $R_{m,m+1}$  and  $X_{m,m+1}$  are line resistance and reactance between bus  $m$  and  $m + 1$ , respectively.

The line loss between bus  $m$  and  $m + 1$  is given by

$$P_L(m, m + 1) = \left( \frac{P_m^2 + Q_m^2}{|V_m|^2} \right) * R_{m,m+1} \quad (4)$$

Therefore, total line losses ( $P_{T,L}$ ) in a distribution network is evaluated as the summation of all line losses and it is given by

$$P_{T,L} = \sum_{m=0}^{b-1} P_L(m, m + 1) \quad (5)$$



**Fig. 1.** SLD of a sample distribution network.

In distribution networks capacitor allocation minimize power losses. The available standard sizes of capacitors in the market with the smallest size is  $Q_0^{cap}$ . Also, from size-to-size annual cost per kVAR differs. The list of capacitor sizes and their associated annual cost per kVAR are illustrated in **Table 2**.

**Table 2**

Capacitor sizes available and their associated costs

Number,( $n$ )	1	2	3	4	5	6	7	8	9
Size, $Q_n^{cap}$ (in kVAR)	150	300	450	600	750	900	1050	1200	1350
Cost, $K^{cap}$ (in \$/kVAR-year)	0.500	0.350	0.253	0.220	0.276	0.183	0.223	0.170	0.207
Number,( $n$ )	10	11	12	13	14	15	16	17	18
Size, $Q_n^{cap}$ (in kVAR)	1500	1650	1800	1950	2100	2250	2400	2550	2700
Cost, $K^{cap}$ (in \$/kVAR-year)	0.201	0.193	0.187	0.211	0.176	0.197	0.170	0.189	0.187
Number,( $n$ )	19	20	21	22	23	24	25	26	27
Size, $Q_n^{cap}$ (in kVAR)	2850	3000	3150	3300	3450	3600	3750	3900	4050
Cost, $K^{cap}$ (in \$/kVAR-year)	0.183	0.180	0.195	0.174	0.188	0.170	0.183	0.182	0.179

### 3.2. Objective function

The main objective of the capacitor allocation problem is to minimize power loss, capacitor integration cost, and improve bus voltage profile. To ease the newly facilitated network, allocated capacitor operation and maintenance cost is excluded. Also, peak power loss minimization benefits of power transmitted in lines/cables and transformers are ignored. The present objective function of capacitor allocation to achieve power loss and annual operating cost minimization is represented mathematically as follows

$$F = \min\{cost\} = K_P * P_{T,L} + \sum_{m=1}^b K_m^{cap} * Q_0^{cap} \quad (6)$$

### 3.3. Constraints

Bus voltage limits

$$V_{Min} \leq V_m \leq V_{Max} \quad \forall m = 1, 2, \dots, b \quad (7)$$

Capacitor allocation limits

$$Q_m^{cap} \leq \sum_{i=1}^b Q_{Lm} \quad (8)$$

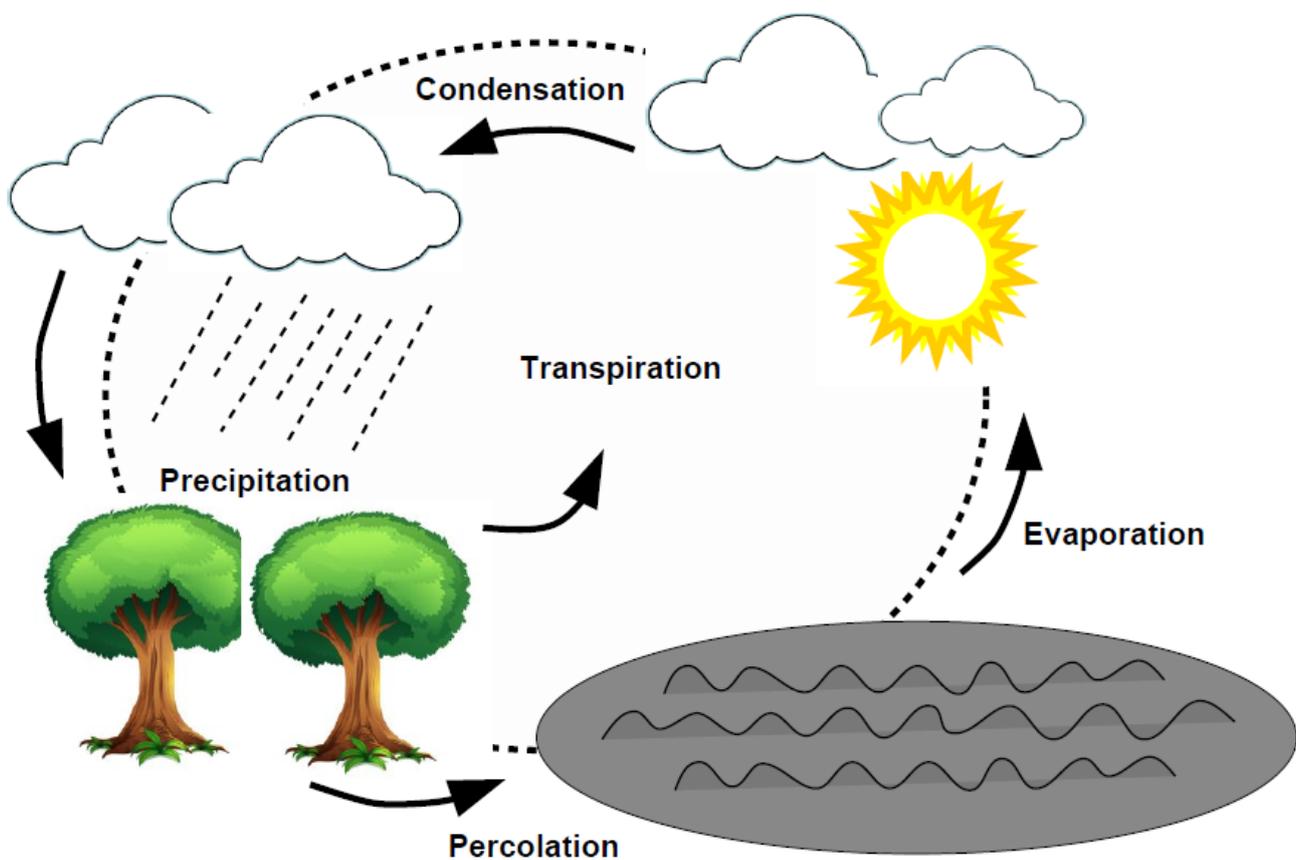
where  $V_{Min}$  and  $V_{Max}$  are minimum and maximum permissible limits of voltages, respectively.  $Q_{Lm}$  is the reactive load at bus  $m$ .

## 4. Water Cycle Algorithm

Eskandar *et al.* [39] has proposed a water cycle algorithm (WCA) in 2012. The algorithm is inspired by the water cycle process of nature in the real world. When water flows downwards from one place to another a stream or a river will be formed. This implies that rivers are formed mostly above the mountains due to the melting of snow or ancient glaciers. The rivers naturally flow downwards. During

the downwards flow of water gathered from rain and additional streams finally end up at sea. Where this journey of streams and rivers flow to the sea is generally known as the water cycle or hydrologic cycle. The illustration of the simplified hydrologic cycle is represented in **Fig. 2**.

The algorithm is employed to solve various constrained and engineering design problems. The effective ability of WCA has been assessed and reported in terms of solution accuracy and computational effort in the literature. In [40], authors have implemented an efficient chaotic WCA by considering the chaos in real water cycle behavior in which random process is replaced with chaotic operators improved the optimization strategies of intensification and diversification. In [41], authors have developed a Gaussian bare-bones WCA to solve the reactive power compensation problem. Even though improved or modified WCA varieties attain better solutions for various complex optimization problems still the standard version of WCA has significant ability in finding global optimum via appropriate parameter tuning. Hence for solving the capacitor allocation optimization problem in a standard distribution network WCA has been effectively implemented.



**Fig. 2.** Illustration of the simplified hydrologic cycle

#### 4.1 Procedure of WCA

The detailed WCA open-source code has been provided to demonstrate the step-by-step process and to assess the performance and efficiency in solving optimization problems. The algorithm starts with an assumption of rain or precipitation phenomena. Initialization begins in WCA with a raining process. This process involves the generation of the initial population randomly which are known as decision parameters. The initially generated population is known as raindrops (RD). The optimization problem of

$N_{var}$  dimensions can be represented as an array of  $1 \times N_{var}$  with single raindrop (single solution set) is as follows:

$$RD = [r_1, r_2, r_3, \dots, r_N] \quad (9)$$

The matrix RD has random solutions generated for the first iteration

$$RD = \{r_i^k : k = 1:N_{var} \text{ and } i = 1:N_{RD}\} \quad (10)$$

The optimization algorithm begins with the generation of the random number (i.e., raindrops population) forms a matrix, Y of  $N_{RD} \times N_{var}$  (rows represents population and column represents decision parameters, respectively):

$$Y = \begin{bmatrix} r_1^1 & r_1^2 & \dots & r_1^{N_{var}} \\ r_2^1 & r_2^2 & \dots & r_2^{N_{var}} \\ r_3^1 & r_3^2 & \dots & r_3^{N_{var}} \\ \vdots & \vdots & \vdots & \vdots \\ r_{N_{RD}}^1 & r_{N_{RD}}^2 & \dots & r_{N_{RD}}^{N_{var}} \end{bmatrix} \quad (11)$$

From the above equation, the cost function ( $CF_j$ ) can be formulated as

$$CF_j = Cost_j = f(r_i^1, r_i^2, r_i^3, \dots, r_i^{N_{var}}), \quad i = 1, 2, 3, \dots, N_{RD} \quad (12)$$

Later, the streams, rivers, and sea can be evaluated using equation (13), which mainly depends on the potential of flow. Here, the sea is the destination (optimal solution) which is selected from the best RD, the river is formed from good RD and the streams are formed due to the remaining RD.

$$NS_k = \text{round} \left\{ \left| \frac{Cost_k}{\sum_{i=1}^{N_{SR}} Cost_i} \right| \times N_{RD} \right\}, \quad k = 1, 2, 3, \dots, N_{SR} \quad (13)$$

It is observed that with a random distance (Y) streams flow to the river. Similarly, the river flows to the sea, thus the new location for streams and rivers are

$$Y_{stream}^{i+1} = Y_{stream}^i + rand * D * (Y_{river}^i - Y_{stream}^i) \quad (14)$$

$$Y_{river}^{i+1} = Y_{river}^i + rand * D * (Y_{sea}^i - Y_{river}^i) \quad (15)$$

The significance of the evaporation process is effectively implemented to avoid immature convergence and local optimal trap. This process will begin after verifying the condition in equation (16)

$$|Y_{sea}^i - Y_{river}^i| < d_{max} \quad i = 1, 2, 3, \dots, N_{SR} - 1 \quad (16)$$

The  $d_{max}$  value decreases adaptively with the evaporation process every time,

$$d_{max}^{i+1} = d_{max}^i - \left( \frac{d_{max}^i}{iter_{max}} \right) \quad (17)$$

Once the process of evaporation is completed, the new raindrops formed due to the raining process. Hence new streams will be formed by the equation (18).

$$Y_{stream}^{new} = Y_{sea} + \sqrt{\mu} \times randn(1, N_{var}) \quad (18)$$

where  $\mu$  is the co-efficient with value 0.1 which represents searching region near the sea.

```

Set the WCA parameters:  $N_{RD}$ ,  $N_{var}$ ,  $N_{sr}$ ,  $d_{max}$ ,  $Iter_{max}$ .
Evaluate the number of individual streams flows towards rivers and sea using Equation (14) and (15).
Generate the intial population with random variables for feature selection.
Define the flow intensity using Equation (16).
While (t <  $Iter_{max}$ ) or (other termination criteria)
    for i = 1 :  $N_{RD}$  (Population Size)
        Stream flows to its corresponding rivers and sea using Equation (18).
        Evaluate the Cost Function or Objective Function of the generated stream using Equation (6).
            if  $Y_{stream}^{new} > Y_{river}$ 
                river = new stream;
            if  $Y_{stream}^{new} < Y_{sea}$ 
                sea = new stream;
            end if
        end if
        River flows to the sea using Equation (15)
        Evaluate the Cost Function of the generated stream
            if  $Y_{river}^{new} < Y_{sea}$ 
                sea = new river;
            end if
        end for
        for i = 1:  $N_{sr}$  (number of rivers)
            if  $(Y_{sea}^1 - Y_{river}^1) < d_{max}$  or (rand < 0.1)
                New streams are created using Equation (18)
            end if
        end for
        Reduce the  $d_{max}$  using Equation (17).
    end While

```

Fig. 3. Pseudo code of Water Cycle Algorithm

#### 4.2 Implementation of WCA for optimal capacitor allocation

To solve the capacitor allocation problem in the distribution network the process of WCA is implemented step-by-step is presented below.

Step 1: The load flow program is supplied with the necessary bus and line [37].

Step 2: Realize the WCA parameters

Step 3: Generate the solution set RD randomly

Step 4: The solution set generated must satisfy system constraints in subsection 3.3.

Step 5: The cost or objective function can be evaluated by running the implemented WCA for each RD.

Step 6: Retain the solution obtained so far from raindrops.

Step 7: The new solutions set must generate.

Step 8: Verify the stopping criteria, if it is fulfilled stop and display the results. else, repeat steps 1 – 7.

Flowchart of WCA implemented for optimal capacitor allocation in RDN is shown in Fig. 4.

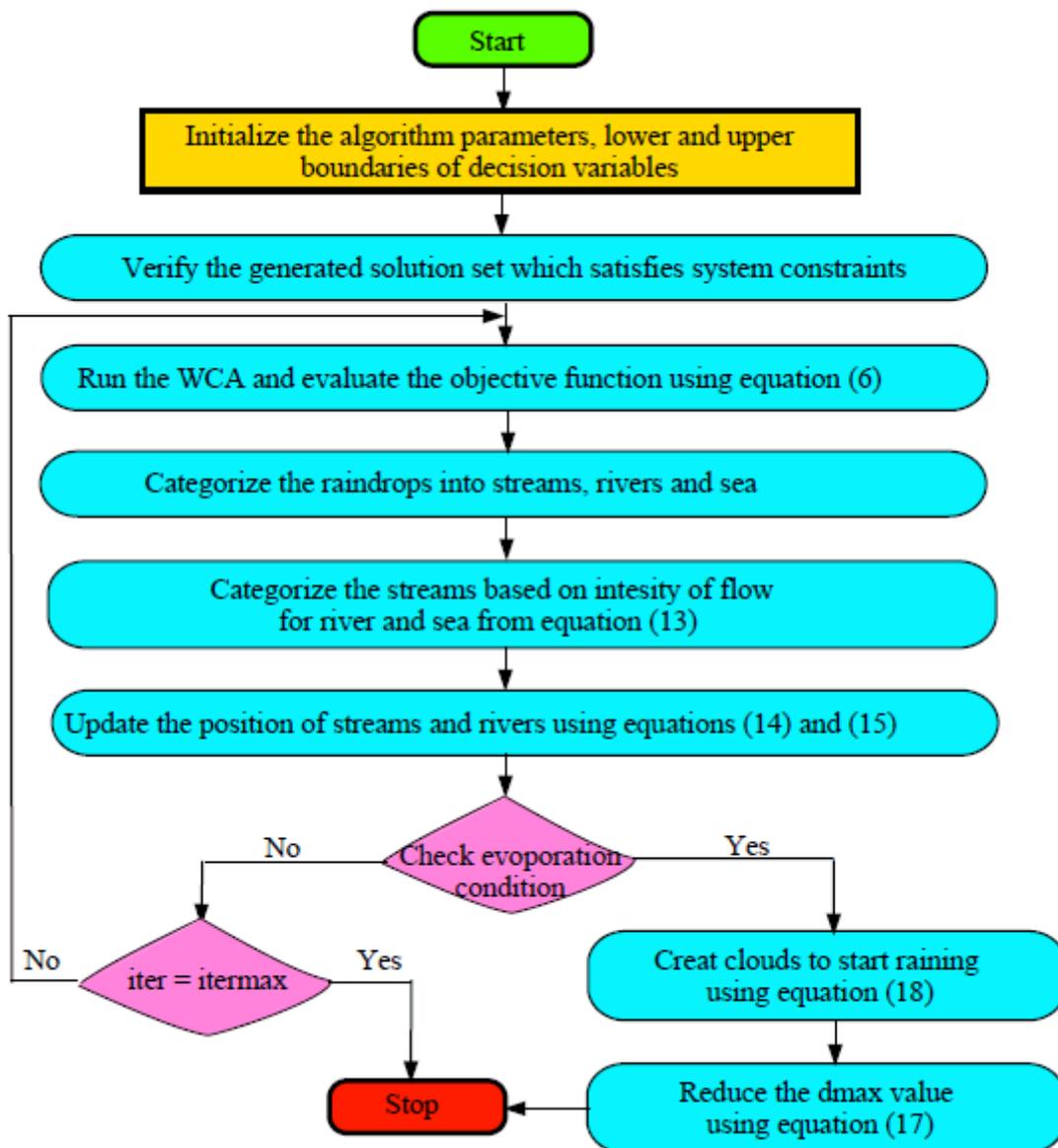
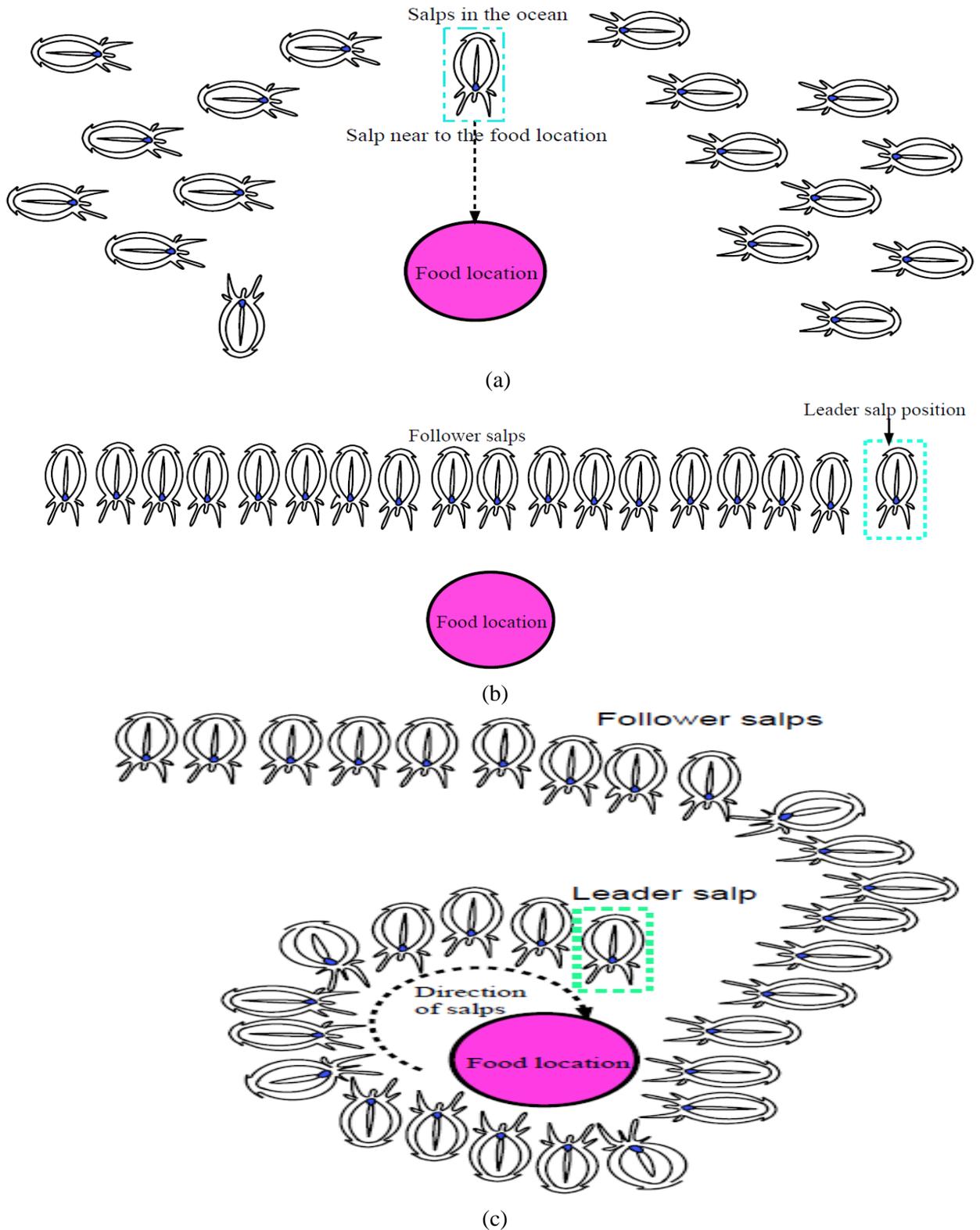


Fig. 4. Flowchart of WCA for solving capacitor allocation problem.

## 5. Salp Swarm Algorithm

Mirjalili *et al* [42] has proposed a salp swarm algorithm (SSA) in 2017. It is a bio-inspired optimization algorithm that mimics the behavior of salp in deep oceans for navigating and foraging. The

algorithm has initially tested on standard benchmark function and later implemented on different design and complex engineering problems. The swarming behavior of salp is illustrated in **Fig. 5**. **Fig. 5(a)** illustrates salps in the ocean. And the illustration of salp swarming in the ocean is shown in **Fig. 5(b)**. The salps are used to form a chain known as a salp chain. The structure of the salp chain is illustrated in **Fig. 5(c)**. The main reason behind the behavior of chain formation is to attain better movement through fast shifting and foraging.



**Fig. 5.** (a) Salps in the ocean (b) Salp swarming (c) Salp chain.

Similar to other swarm-based techniques, salps location is distinct in  $n$  – dimension search space. Hence salp location is saved in a matrix with two dimensions called  $X$ . The food source present in the search space called  $F$  is considered as the swarm's target. The position of the leader salp is obtained from the first iteration and this equation is used to update its position.

$$X_d^1 = \begin{cases} F_d + C_1((ub_d - lb_d)C_2 + lb_d), & C_3 \geq 0 \\ F_d - C_1((ub_d - lb_d)C_2 + lb_d), & C_3 < 0 \end{cases} \quad (19)$$

$$C_1 = 2 * e^{-\left(\frac{t}{T}\right)^2} \quad (20)$$

$$X_d^i = \frac{1}{2}(X_d^i + X_d^{i-1}) \quad (21)$$

where,  $X_d^1$  and  $F_d$  are the leader (first salp) position and the food source location in the  $d^{\text{th}}$  dimension,  $lb_d$  and  $ub_d$  are lower and upper boundaries of the  $d^{\text{th}}$  dimension,  $C_2$  and  $C_3$  are random numbers.

```

Initialize the population size of the salps  $N_s$  ( $s = 1, 2, 3, \dots, n$ ) considering lb and ub.
Generated population has to satisfy all the constraints.
While (iter < itermax) or (termination criteria is not met)
    Evalaute the Fitness of each search agent (salp)
    Determine the non-dominated salps
    Update the repository considering the attained non-dominated salps
    if the repositroy becomes full
        Inform to repositry maintenance procedure to remove one repositroy resident
        Add the non-dominated salp to the repository
    end if
Choose a source food from reopsitory;
 $F_d = \text{SelectFood}(\text{repository})$ 
Update  $c_1$  by Equation (20)
for each salp ( $N_s$ )
    if ( $i == 1$ )
        Update the position of the leader salp by using Equation (19)
    else
        Update the position of the follower salp by Equation (21)
    end if
end for
Adjust the salps based on the upper and lower bounds of the variables.
end While
return repository

```

**Fig. 6.** Pseudo code of Salp Swarm Algorithm

### 5.1. Implementation of SSA for capacitor allocation

*Step 0:* In the initial step, initialize the population size of salps  $N_s$ , this salps population is considered a feasible solution that satisfies all the constraints in subsection 3.3. Thus, the solution set of capacitors allocation (i.e., location (l) and size (C)) is formulated as follows:

$$X = \begin{bmatrix} l_1^1 & l_2^1 & l_3^1 & C_1^1 & C_2^1 & C_3^1 \\ l_1^2 & l_2^2 & l_3^2 & C_1^2 & C_2^2 & C_3^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ l_1^n & l_2^n & l_3^n & C_1^n & C_2^n & C_3^n \end{bmatrix} \quad (22)$$

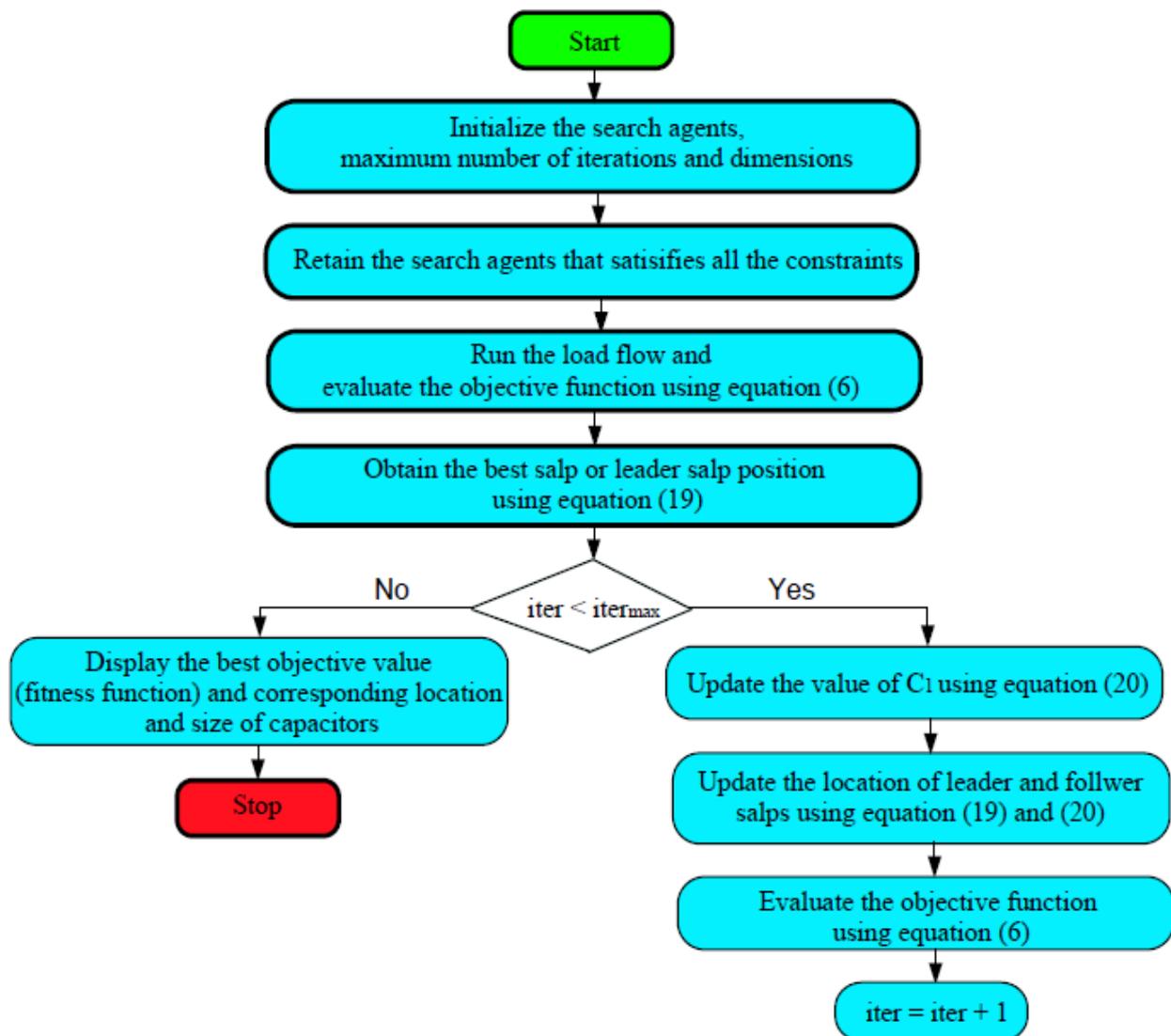
*Step 1:* Feed the required input bus and line data for the load flow program [30]. Run the SSA and evaluate the fitness or objective function (6). For the first iteration, compute the leader salp position using Eq. (19).

*Step 2:* In steps of iterations the parameter  $C_1$  is updated using Eq. (20) and leader salp and follower salp positions are updated by Eq. (19) and Eq. (21).

*Step 3:* Evaluate the fitness or objective function value (cost minimization) for each iteration.

*Step 4:* Check for the stopping criteria, if it is satisfied stop and display the results. Otherwise, repeat steps 1 – 3.

The flow chart of SSA for capacitor allocation in distribution networks is illustrated in **Fig. 7**.



**Fig. 7.** Flow chart of the proposed SSA for allocation of SCB

## 6. Simulation results and discussions

To crisscross the feasibility, WCA and SSA are tested on standard 9, 33, 34, 69, and 85 – bus radial distribution networks. Both the algorithms have been implemented in MATLAB. The entities considered for WCA and SSA are presented in **Table 3**. The simulation results obtained from WCA and SSA depend on random variables generated. Hence, it is necessary to run both algorithms for 50 trails for attaining the optimal solution set. The value of the equivalent annual cost per unit of power loss is 168 \$/kW [20]. The bus voltage limits considered for the present analysis are  $V_{min} = 0.9 p.u.$  and  $V_{max} = 1.1 p.u.$

**Table 3**

WCA and SSA parameters

Entity	WCA	SSA
Population size	100	100
Maximum iterations ( $iter_{max}$ )	500	500
Trail runs	50	50

### 6.1. Test systems

#### 6.1.1. 9-bus system

The SLD of the 9-bus distribution network is shown in **Fig. 8**. The network line voltage rating is 23 kV. The bus and line data of the network is taken from **Table A1**. The total load demand of real and reactive on the network is 12,368 kW and 4186 kVAR, respectively. The power loss of the network at the base case is 783.63 kW. WCA gives the optimal bus locations as 4, 5, 8 with optimal capacitor bank sizes of 3450, 900, and 600 kVAR, respectively. Whereas SSA gives the optimal bus locations as 2, 4, 6 and optimal capacitor bank sizes as 4050, 3150, and 1350 kVAR, respectively. The results obtained by both algorithms are presented in **Table 4**. It is noticed that after capacitor allocation the power loss reduction obtained by WCA and SSA are 682.99 kW and 678.845 kW, respectively. The percentage power loss reduction obtained by WCA and SSA compared to the base case is 12.77% and 13.371%, respectively. **Fig. 9**, illustrates the comparison of obtained power loss reduction by WCA and SSA with other algorithms. In addition, it is observed from Table 4, that annual cost is reduced from 131,675 \$ to 115,688 \$ by WCA and 131,675 \$ to 115,664 \$, by SSA. It is noticed that subsequent net saving is achieved by SSA of 16,011 \$ outperforms all other algorithms. The minimum bus voltage before capacitor allocation is 0.8375 p.u. However, after capacitor allocation, the minimum bus voltage obtained is 0.9000 p.u. It can be concluded that both algorithms have the capability of minimizing power loss and operating costs. SSA is more capable to handle complex combinatorial optimization problems of capacitor allocation.

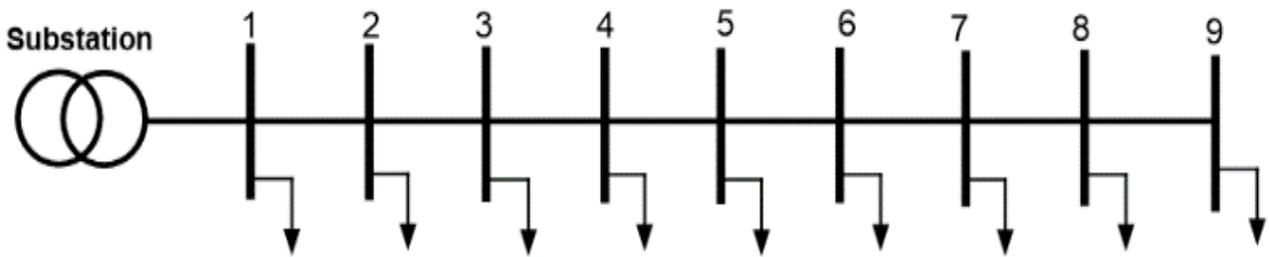


Fig. 8. SLD of 9-bus distribution network.

Table 4

Comparison of results obtained by WCA and SSA with other techniques (9 – bus)

Parameter	Base case	GA [22]	PSO [22]	ES [22]	PSGA [16]	WCA	SSA
Power loss (kW)	783.63	701.478	698.10	698.10	694.93	<b>682.99</b>	<b>678.845</b>
Loss reduction (%)	-	10.48	10.91	10.91	11.33	<b>12.77</b>	<b>13.371</b>
Annual cost (\$)	131,675	118,916	118,538	118,538	118,340	<b>115,688</b>	<b>115,664</b>
Net saving (\$)	-	12,759	13,137	13,137	13,334	<b>15,987</b>	<b>16,011</b>
Capacitor size (kVAR)	-	3300 @ 4	4050 @ 4	4050 @ 4	1200 @ 4	<b>3450 @ 4</b>	<b>4050 @ 2</b>
@ bus location		1800 @ 5	1650 @ 5	1650 @ 5	1200 @ 5	<b>900 @ 5</b>	<b>3150 @ 4</b>
		900 @ 9	750 @ 9	750 @ 9	200 @ 8	<b>600 @ 8</b>	<b>1350 @ 6</b>
					407 @ 9		
Net kVAR injected	-	6000	6450	6450	3007	<b>4950</b>	<b>8550</b>
Min. Voltage (p.u.)	0.8375	0.9007	0.9000	0.9000	NA	<b>0.9000</b>	<b>0.9000</b>
Max. Voltage (p.u.)	0.9929	0.9992	1.001	1.001	NA	<b>0.9969</b>	<b>1.004</b>

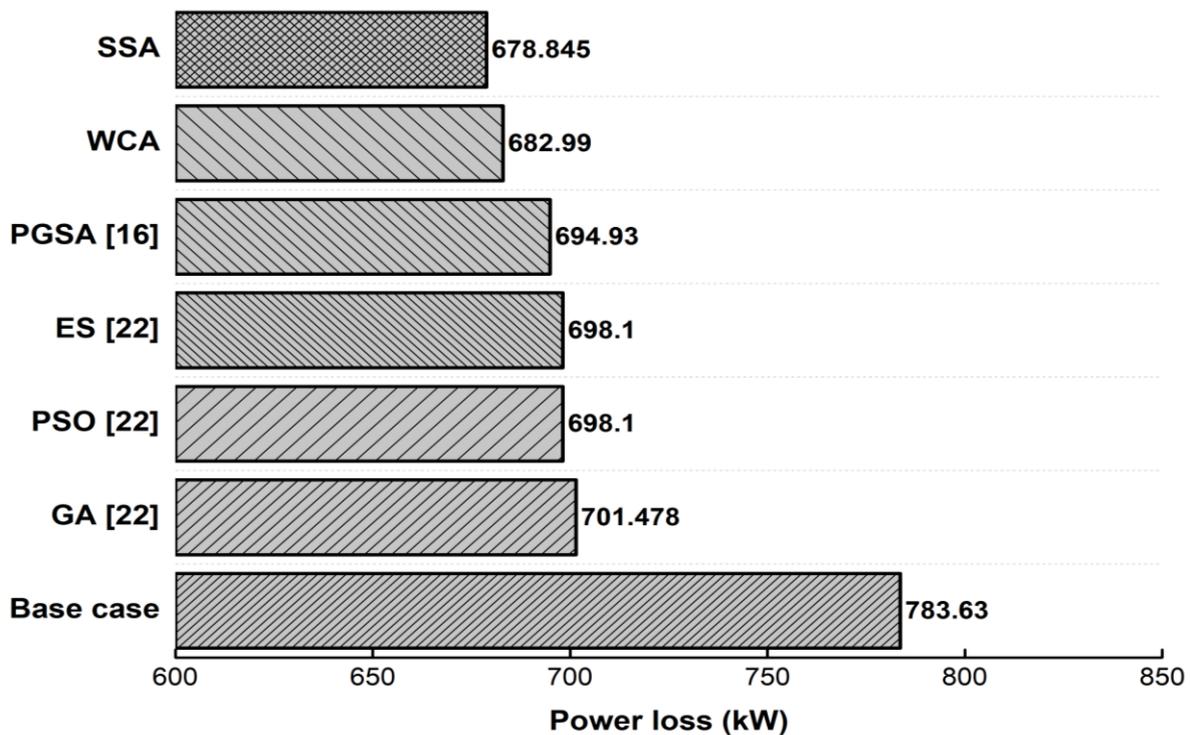
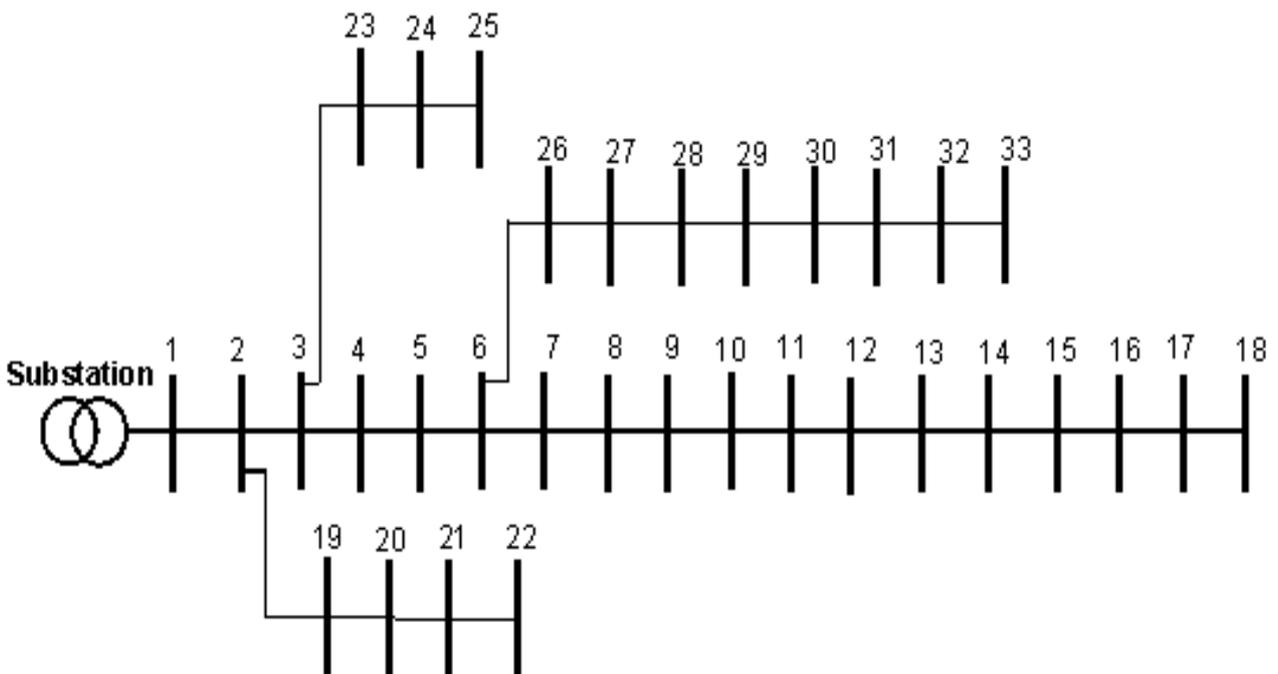


Fig. 9. Comparison of WCA and SSA with different algorithms based on loss reduction for 9 – bus system

### 6.1.2. 33-bus system

The SLD of the 33-bus distribution network is shown in **Fig. 10**. The network line voltage rating is 12.66 kV. The bus and line data of the network is taken from **Table A2**. The total load demand of real and reactive on the network is 3715 kW and 2300 kVAR, respectively. The power loss of the network at the base case is 202.66 kW. WCA gives the optimal bus locations as 11, 23, 29 with optimal capacitor bank sizes of 450, 450, and 900 kVAR, respectively. Whereas SSA gives the optimal bus locations as 11, 23, 29 and optimal capacitor bank sizes like 450, 450, and 1050 kVAR, respectively. The results obtained by both algorithms are presented in **Table 5**. It is noticed that after capacitor allocation the power loss reduction obtained by WCA and SSA are 132.66 kW and 132.35 kW, respectively. The percentage power loss reduction obtained by WCA and SSA compared to the base case is 34.54% and 34.69%, respectively. **Fig. 11**, Illustrates the comparison of obtained power loss reduction by WCA and SSA with other algorithms. In addition, it is observed from Table 4, that annual cost is reduced from 34,047 \$ to 22,698.88 \$ by WCA and 34,047 \$ to 22,696.65 \$, by SSA. It is noticed that subsequent net saving is achieved by SSA of 11,350.35 \$, outperforming all other algorithms. The minimum bus voltage before capacitor allocation is 0.9131 p.u. However, after capacitor allocation the minimum bus voltage obtained is 0.9366 p.u. **Fig. 12** Illustrates the improved voltage profile obtained by WCA and SSA compared with the base case. It can be concluded that both algorithms have the capability of minimizing power loss and operating costs. SSA is more capable to handle complex combinatorial optimization problems of capacitor allocation.



**Fig. 10.** SLD of 33 – bus distribution network.

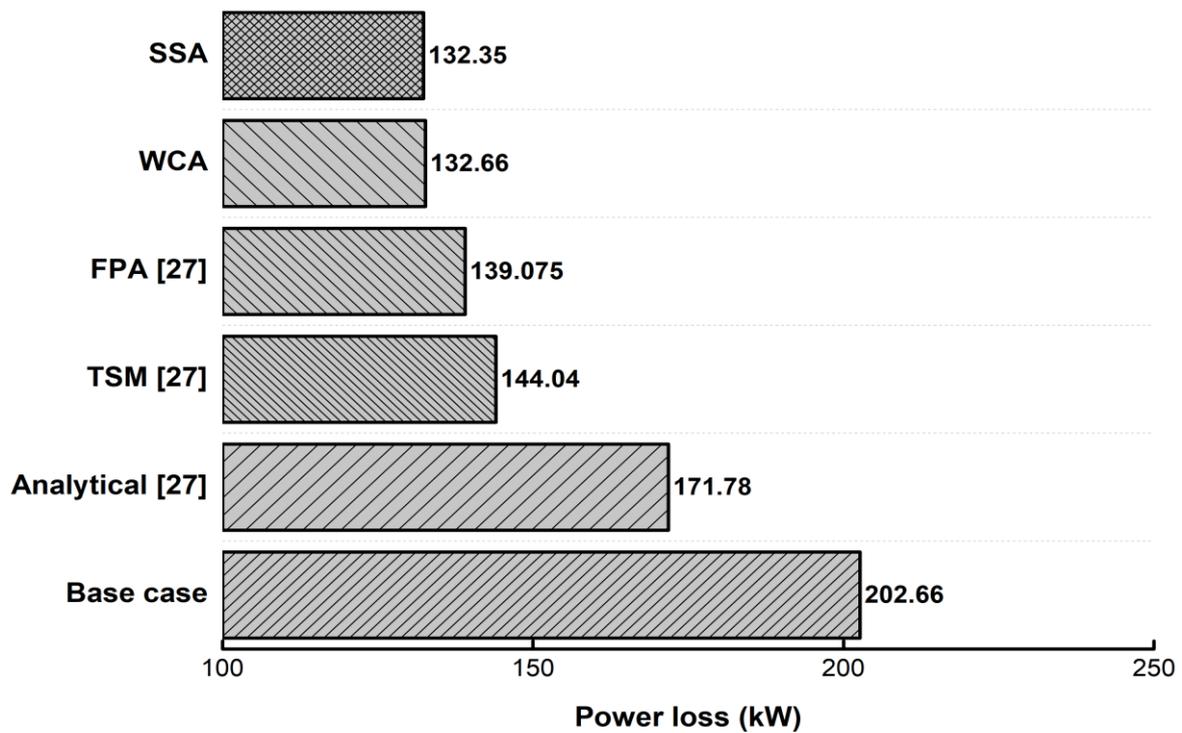
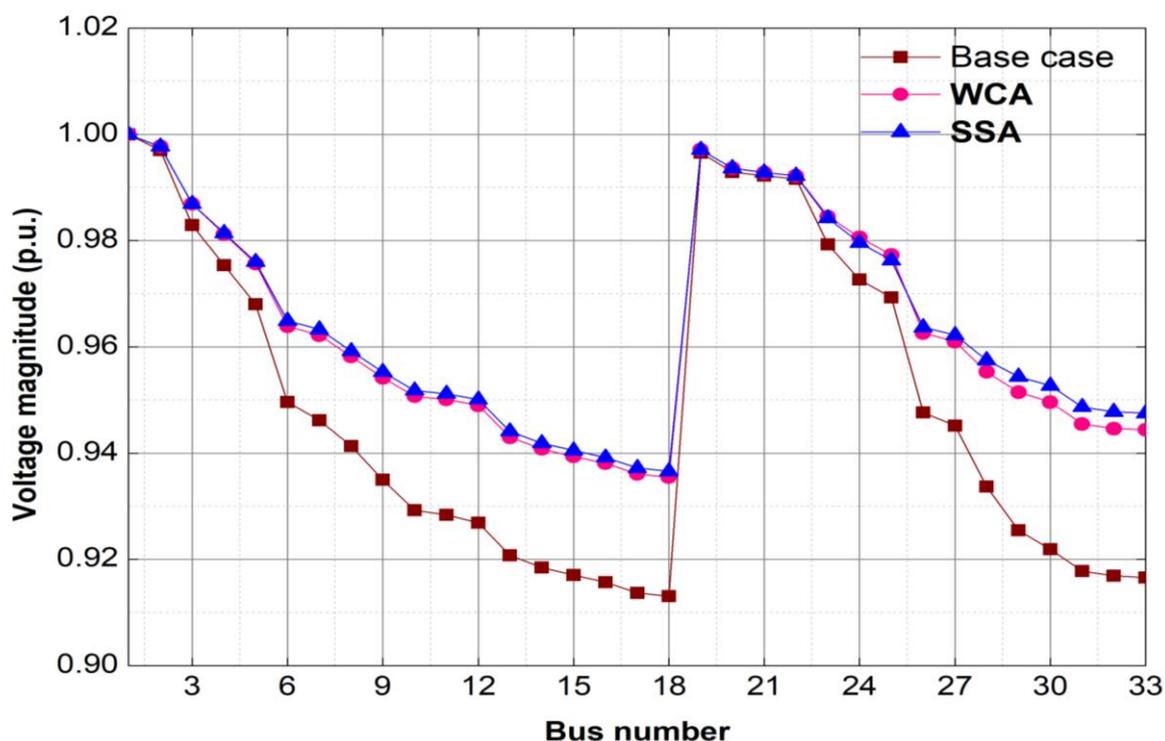


Fig. 11. Comparison of SSA with different algorithms based on loss reduction for 33 – bus system

Table 5

Comparison of results obtained by the proposed algorithm with other techniques (33 – bus)

Parameter	Base case	Analytical [27]	Two-stage method [27]	FPA [27]	WCA	SSA
Power loss (kW)	202.66	171.78	144.04	139.075	<b>132.66</b>	<b>132.35</b>
Loss reduction (%)	-	15.23	28.92	31.37	<b>34.54</b>	<b>34.69</b>
Annual cost (\$)	34,047	29,358.39	24,705.87	23,757	<b>22,698.88</b>	<b>22,696.65</b>
Net saving (\$)	-	4688.61	9341.13	10290	<b>11348.12</b>	<b>11350.35</b>
Capacitor size (kVAR) @ bus location	-	450 @ 9 800 @ 29 900 @ 30	850 @ 7 25 @ 29 900 @ 30	450 @ 13 450 @ 24 900 @ 30	<b>450 @ 11</b> <b>600 @ 23</b> <b>900 @ 29</b>	<b>450 @ 11</b> <b>450 @ 23</b> <b>1050 @ 29</b>
Net kVAR injected	-	2150	1775	1800	<b>1950</b>	<b>1950</b>
Min. Voltage (p.u.)	0.9131	0.9501	0.9251	0.9327	<b>0.9355</b>	<b>0.9366</b>
Max. Voltage (p.u.)	0.9970	NA	NA	NA	<b>0.9976</b>	<b>0.9977</b>



**Fig. 12.** Improved voltage profile obtained after OSCB allocation using SSA for 33 – bus system

### 6.1.3. 34-bus system

The SLD of the 34-bus distribution network is shown in **Fig. 13**. The network line voltage rating is 11 kV. The bus and line data of the network is taken from **Table A3**. The total load demand of real and reactive on the network is 4636.5 kW and 2873.5 kVAR, respectively. The power loss of the network at the base case is 221.73 kW. WCA gives the optimal bus locations as 8, 18, 23 with equal optimal capacitor bank sizes of 750 kVAR at each location. Whereas SSA gives the optimal bus locations as 8, 18, 23 and optimal capacitor bank sizes like 750, 900, and 750 kVAR, respectively. The results obtained by both algorithms are presented in **Table 6**. It is noticed that after capacitor allocation the power loss reduction obtained by WCA and SSA are 160.80 kW and 160.58 kW, respectively. The percentage power loss reduction obtained by WCA and SSA compared to the base case is 27.47% and 27.57%, respectively. **Fig. 14**, Illustrates the comparison of obtained power loss reduction by WCA and SSA with other algorithms. In addition, it is observed from Table 6, that annual cost is reduced from 37,250 \$ to 27,636.50 \$ by WCA and 37,250 \$ to 27,556.14 \$, by SSA. It is noticed that subsequent net saving is achieved by SSA of 9,693.86 \$, outperforming all other algorithms. The minimum bus voltage before capacitor allocation is 0.9417 p.u. However, after capacitor allocation the minimum bus voltage obtained is 0.9502 p.u. **Fig. 15** Illustrates the improved voltage profile obtained by WCA and SSA compared with the base case. It can be concluded that both algorithms have the capability of minimizing power loss and operating costs. SSA is more capable to handle complex combinatorial optimization problems of capacitor allocation.

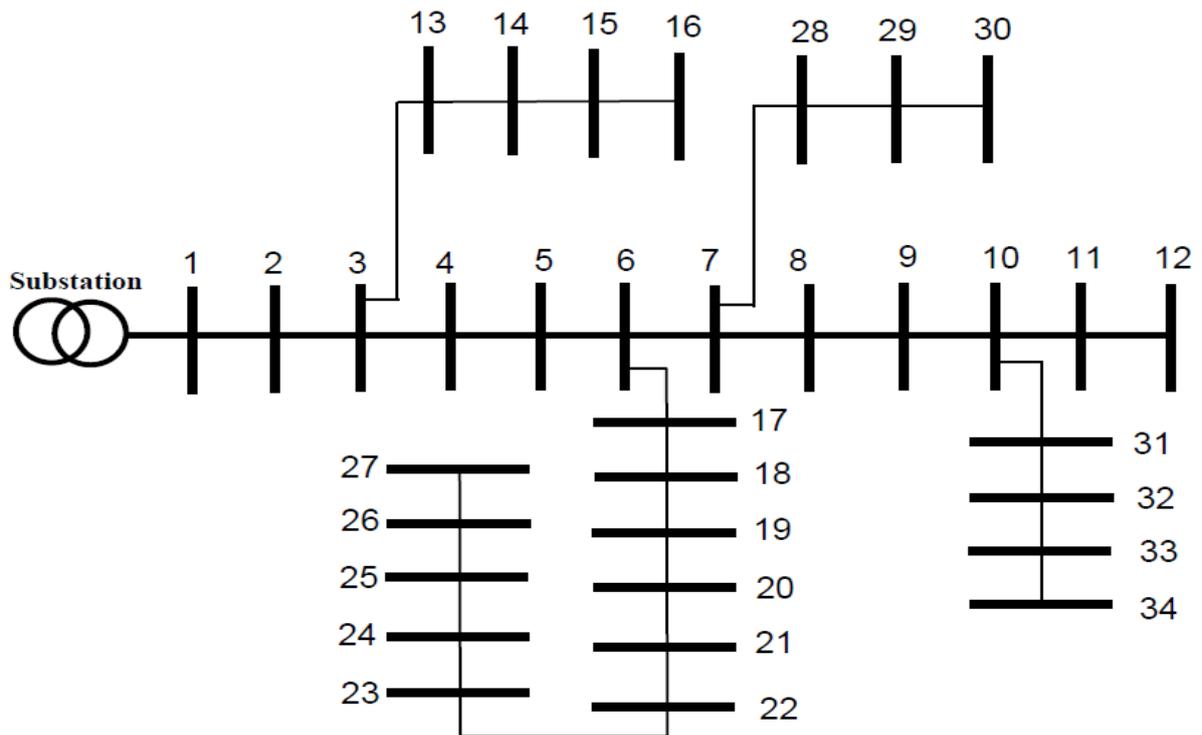


Fig. 13. SLD of 34 – bus distribution network.

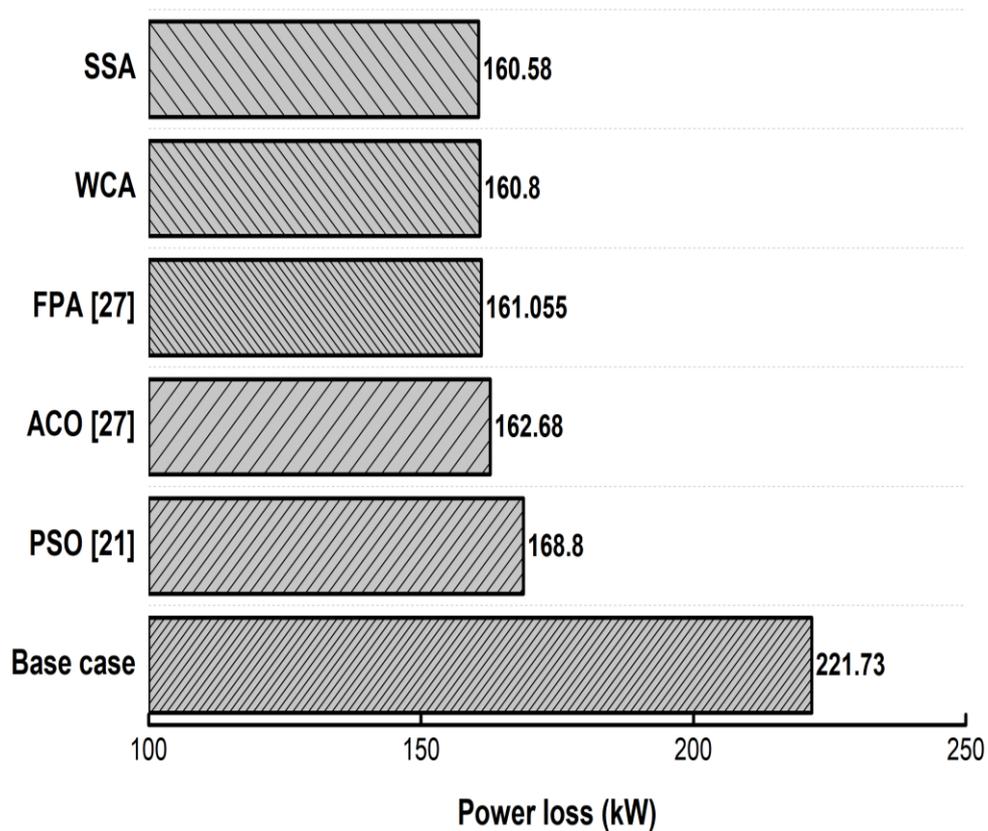
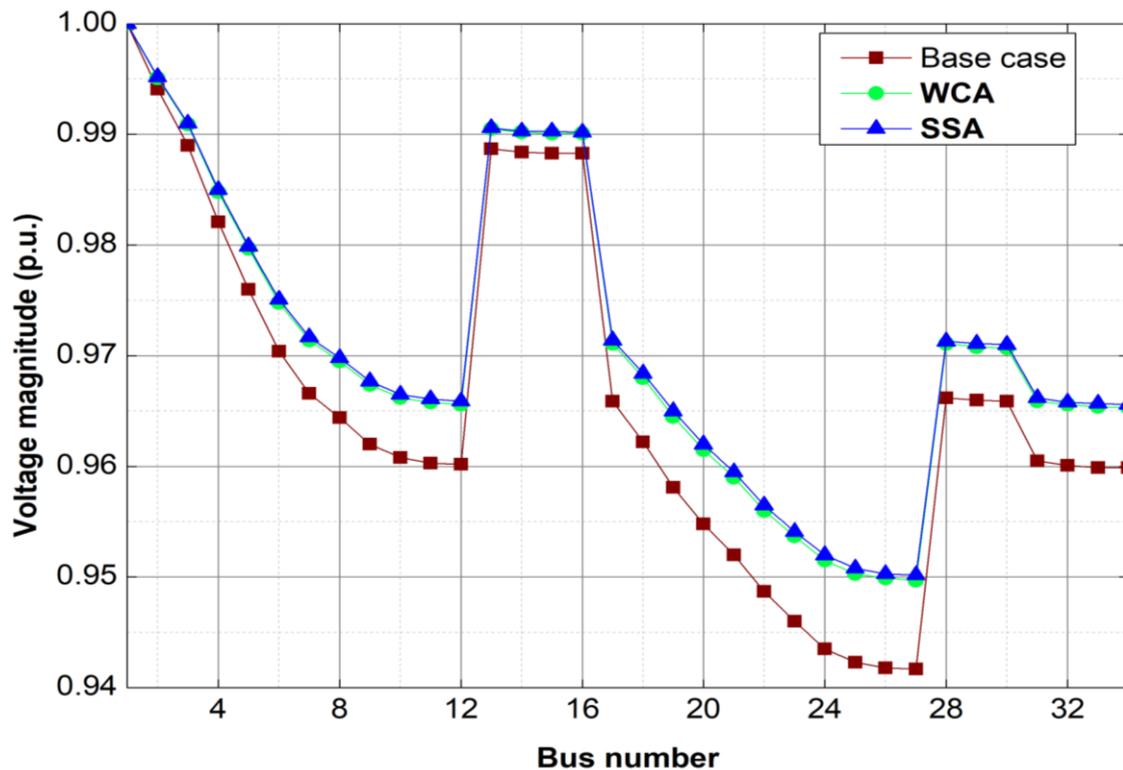


Fig. 14. Comparison of SSA with different algorithms based on loss reduction for 34 – bus system

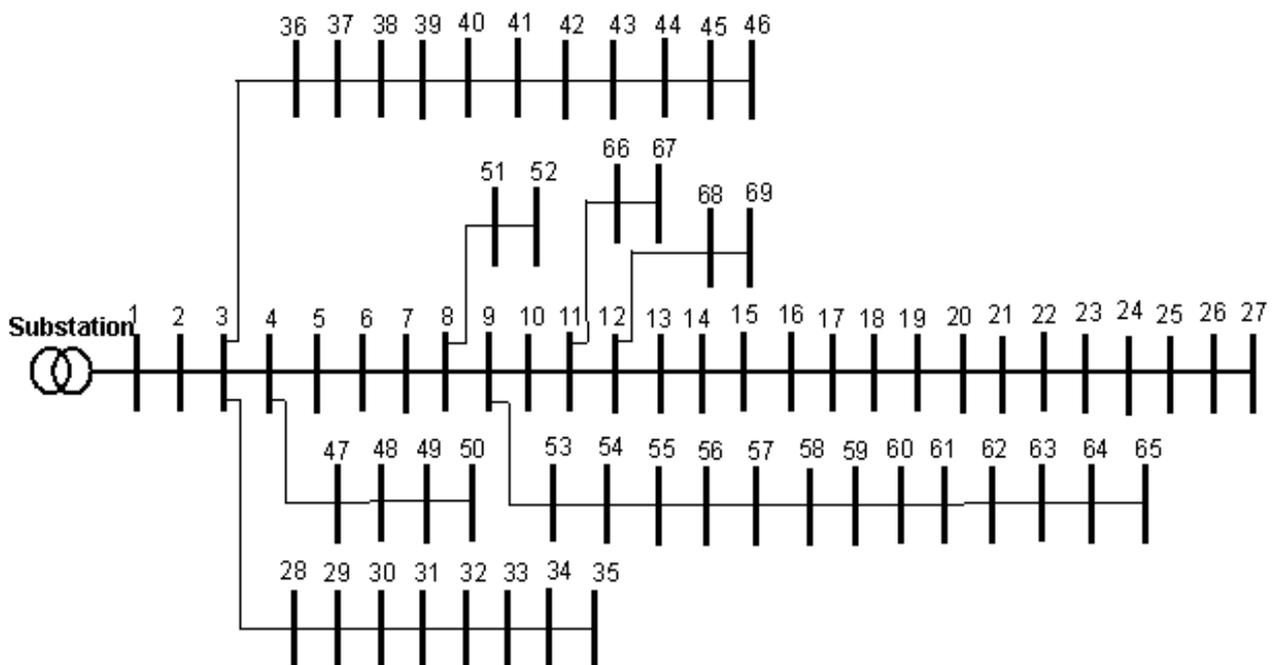
**Table 6**Comparison of results obtained by the proposed algorithm with other techniques (**34 – bus**)

Parameter	Base case	PSO [21]	ACO [27]	FPA [27]	WCA	SSA
Power loss (kW)	221.73	168.8	162.68	161.055	<b>160.80</b>	<b>160.58</b>
Loss reduction (%)	-	23.87	26.63	27.36	<b>27.47</b>	<b>27.57</b>
Annual cost (\$)	37,250	29,936	28,334.5	27,592	<b>27,636.50</b>	<b>27,556.14</b>
Net saving (\$)	-	7314	8915.5	9658	<b>9613.5</b>	<b>9693.86</b>
Capacitor size (kVAR) @ bus location	-	781 @ 19 479 @ 20 803 @ 22	645 @ 9 719 @ 22 665 @ 25	600 @ 10 1050 @ 18 900 @ 24	<b>750 @ 8</b> <b>750 @ 18</b> <b>750 @ 23</b>	<b>750 @ 8</b> <b>900 @ 18</b> <b>750 @ 23</b>
Net kVAR injected	-	2063	2029	2550	<b>2250</b>	<b>2400</b>
Min. Voltage (p.u.)	0.9417	0.9486	0.9501	0.9496	<b>0.9497</b>	<b>0.9502</b>
Max. Voltage (p.u.)	0.9941	NA	NA	NA	<b>0.9951</b>	<b>0.9952</b>

**Fig. 15.** Improved voltage profile obtained after OSCB allocation using SSA for 34 – bus system

#### 6.1.4. 69-bus system

The SLD of the 69-bus distribution network is shown in **Fig. 16**. The network line voltage rating is 12.66 kV. The bus and line data of the network is taken from **Table A4**. The total load demand of real and reactive on the network is 3801.89 kW and 2694.1 kVAR, respectively. The power loss of the network at the base case is 225 kW. WCA gives the optimal bus locations as 17, 60, 65 with optimal capacitor bank sizes of 300, 1200, 300 kVAR, respectively. Whereas SSA gives the optimal bus locations as 10, 17, 60 and optimal capacitor bank sizes like 300, 300, and 1200 kVAR, respectively. The results obtained by both algorithms are presented in **Table 7**. It is noticed that after capacitor allocation the power loss reduction obtained by WCA and SSA are 145.36 kW and 145.26 kW, respectively. The percentage power loss reduction obtained by WCA and SSA compared to the base case is 35.39% and 35.44%, respectively. **Fig. 17**, Illustrates the comparison of obtained power loss reduction by WCA and SSA with other algorithms. In addition, it is observed from Table 6, that annual cost is reduced from 37,800 \$ to 24,835.32 \$ by WCA and 37,800 \$ to 24,817.68 \$, by SSA. It is noticed that subsequent net saving is achieved by SSA of 12,982.32 \$, outperforming all other algorithms. The minimum bus voltage before capacitor allocation is 0.9092 p.u. However, after capacitor allocation the minimum bus voltage obtained is 0.9308 p.u. **Fig. 18** Illustrates the improved voltage profile obtained by WCA and SSA compared with the base case. It is noticed that the bus voltage at each obtained by both algorithms is almost identical. Hence in Fig 18 voltage profile obtained by both the algorithms got overlapped. It can be concluded that both algorithms have the capability of minimizing power loss and operating costs. SSA is more capable to handle complex combinatorial optimization problems of capacitor allocation.



**Fig. 16.** SLD of 69 – bus distribution network.

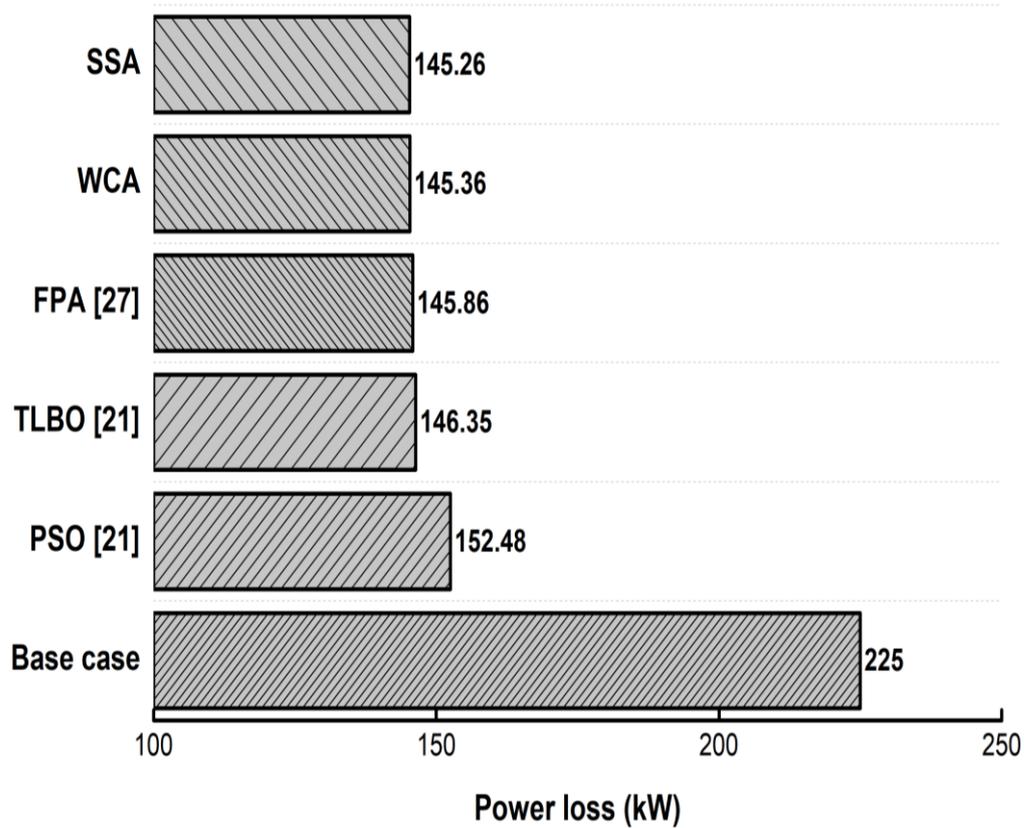
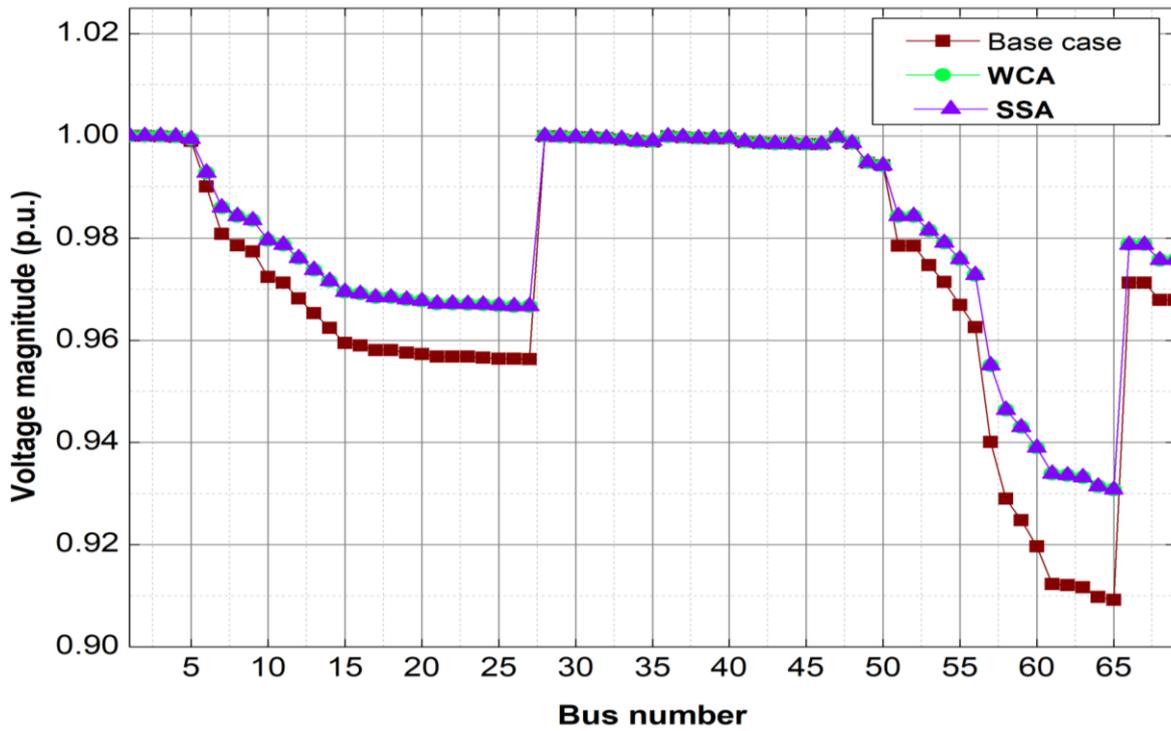


Fig. 17. Comparison of SSA with different algorithms based on loss reduction for 69 – bus system

Table 7

Comparison of results obtained by the proposed algorithm with other techniques (69 – bus)

Parameter	Base case	PSO [21]	TLBO [21]	FPA [27]	WCA	SSA
Power loss (kW)	225	152.48	146.35	145.86	<b>145.36</b>	<b>145.26</b>
Loss reduction (%)	-	32.23	34.95	35.17	<b>35.39</b>	<b>35.44</b>
Annual cost (\$)	37,800	NA	25,033.2	24,972.78	<b>24,835.32</b>	<b>24,817.68</b>
Net saving (\$)	-	NA	12766.8	12827.22	<b>12964.68</b>	<b>12982.32</b>
Capacitor size (kVAR) @ bus location	-	241 @ 46 365 @ 47 1015 @ 50	600 @ 12 1050 @ 61 150 @ 64	450 @ 11 150 @ 22 1350 @ 61	<b>300 @ 17</b> <b>1200 @ 60</b> <b>300 @ 65</b>	<b>300 @ 10</b> <b>300 @ 17</b> <b>1200 @ 60</b>
Net kVAR injected	-	1621	1800	1950	<b>1800</b>	<b>1800</b>
Min. Voltage (p.u.)	0.9092	NA	0.9313	0.9496	<b>0.9307</b>	<b>0.9308</b>
Max. Voltage (p.u.)	NA	NA	NA	NA	<b>1.0000</b>	<b>1.0000</b>



**Fig. 18.** Improved voltage profile obtained after OSCB allocation using SSA for 69-bus system

#### 6.1.5. 85-bus system

The SLD of the 85-bus distribution network is shown in **Fig. 19**. The network line voltage rating is 11 kV. The bus and line data of the network is taken from **Table A5**. The total load demand of real and reactive on the network is 2570.28 kW and 2621.936 kVAR, respectively. The power loss of the network at the base case is 316.097 kW. WCA gives the optimal bus locations as 7, 34, 66 with optimal capacitor bank sizes of 1050, 600, 600 kVAR, respectively. Whereas SSA gives the optimal bus locations as 8, 33, 67 and optimal capacitor bank sizes as 1050, 750, and 450 kVAR, respectively. The results obtained by both algorithms are presented in **Table 8**. It is noticed that after capacitor allocation the power loss reduction obtained by WCA and SSA are 149.37 kW and 148.91 kW, respectively. The percentage power loss reduction obtained by WCA and SSA compared to the base case is 52.74% and 52.89%, respectively. **Fig. 20**, Illustrates the comparison of obtained power loss reduction by WCA and SSA with other algorithms. In addition, it is observed from Table 6, that annual cost is reduced from 53,104.3 \$ to 25,597.59 \$ by WCA and 53,104.3 \$ to 25,571.88 \$, by SSA. It is noticed that subsequent net saving is achieved by SSA of 27,532.42 \$, outperforming all other algorithms. The minimum bus voltage before capacitor allocation is 0.8713 p.u. However, after capacitor allocation the minimum bus voltage obtained is 0.9222 p.u. **Fig. 21** Illustrates the improved voltage profile obtained by WCA and SSA compared with the base case. It can be concluded that both algorithms have the capability of minimizing power loss and operating costs. SSA is more capable to handle complex combinatorial optimization problems of capacitor allocation.

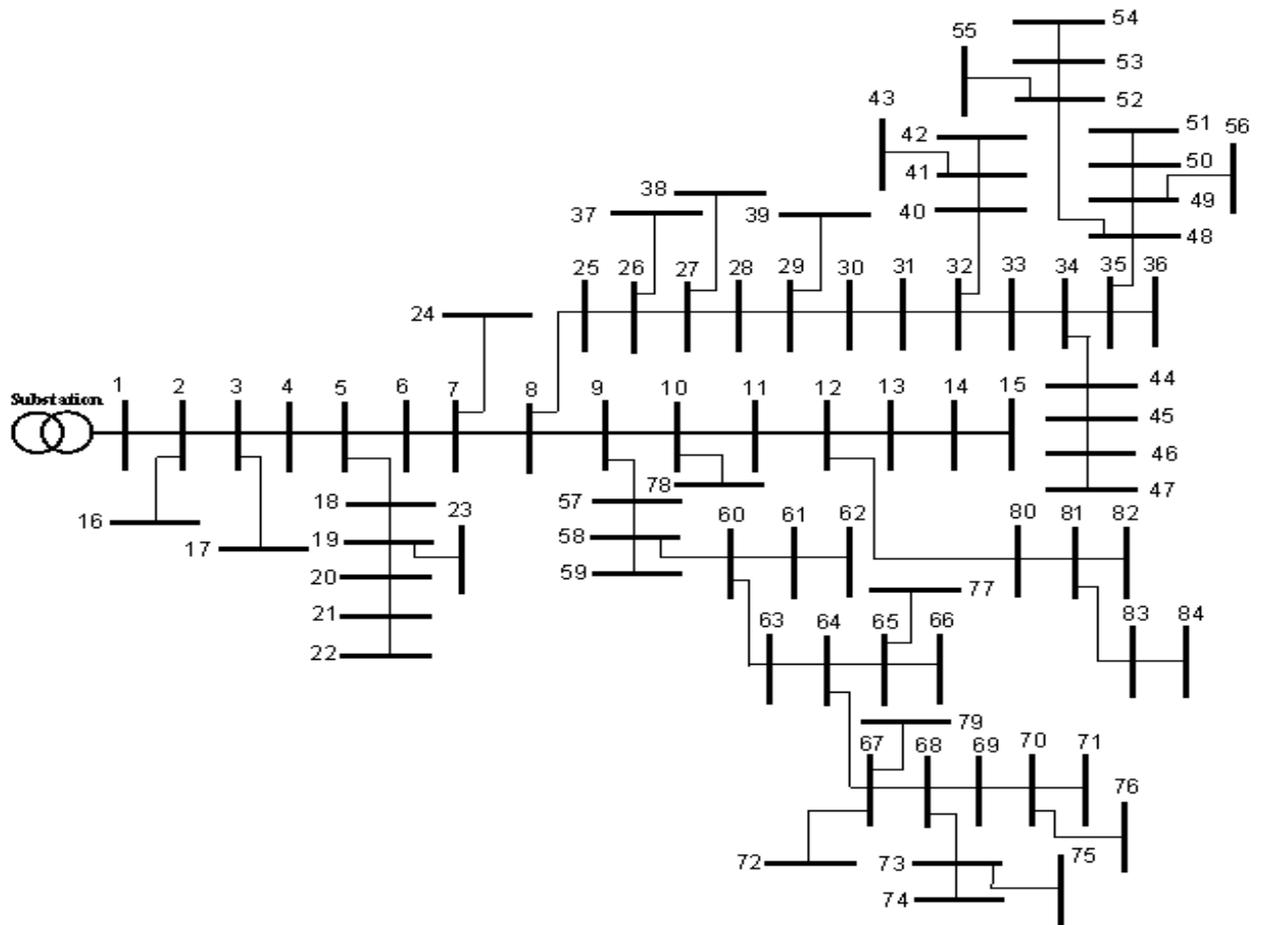


Fig. 19. SLD of 85 – bus distribution network.

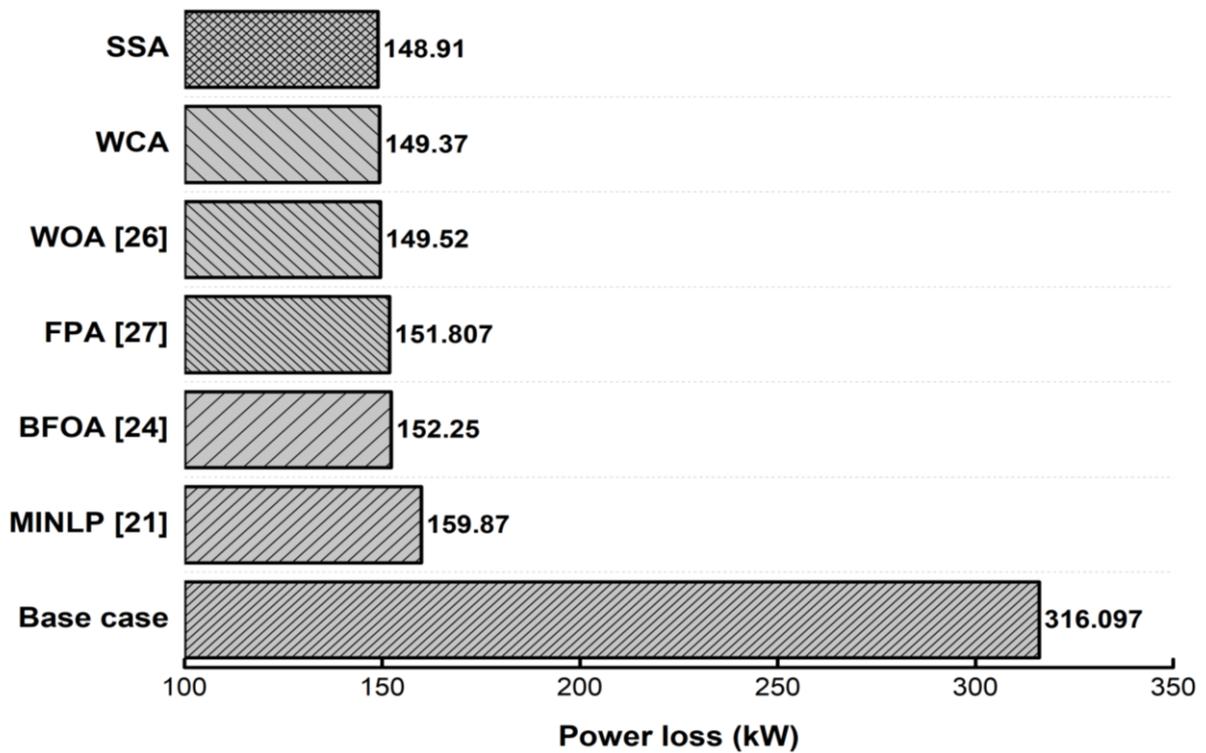
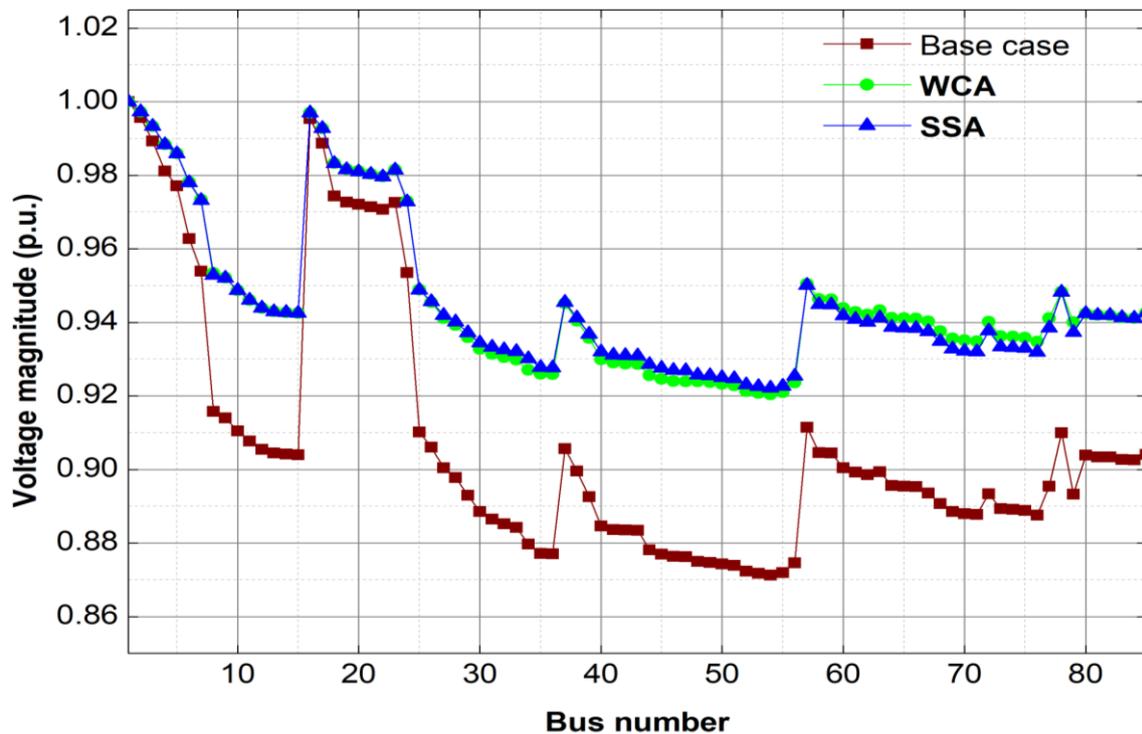


Fig. 20. Comparison of SSA with different algorithms based on loss reduction for 85 – bus system

**Table 8**Comparison of results obtained by the proposed algorithm with other techniques (**85 – bus**)

Parameter	Base case	MINLP [21]	BFOA [24]	FPA [27]	WOA [26]	WCA	SSA
Power loss (kW)	316.097	159.87	152.25	151.807	149.52	<b>149.37</b>	<b>148.91</b>
Loss reduction (%)	-	49.42	51.83	51.97	52.69	<b>52.74</b>	<b>52.89</b>
Annual cost (\$)	53,104.3	27637	27,027.07	25971.576	NA	<b>25,597.59</b>	<b>25,571.88</b>
Net saving (\$)	-	25467.3	26077.23	27132.724	NA	<b>27506.71</b>	<b>27532.42</b>
Capacitor size (kVAR) @ bus location	-	300 @ 7 700 @ 8 900 @ 29 500 @ 58	840 @ 9 660 @ 34	1200 @ 8 600 @ 36 600 @ 72	490 @ 25 709 @ 34 566 @ 67 417 @ 80	<b>1050 @ 7</b> <b>600 @ 34</b> <b>600 @ 66</b>	<b>1050 @ 8</b> <b>750 @ 33</b> <b>450 @ 67</b>
Net kVAR injected	-	2400	2150	2400	2182	<b>2250</b>	<b>2250</b>
Min. Voltage (p.u.)	0.8713	0.9171	0.9180	0.9235	0.9214	<b>0.9204</b>	<b>0.9222</b>
Max. Voltage (p.u.)	0.9957	NA	NA	NA	NA	<b>0.9973</b>	<b>0.9973</b>

**Fig. 21.** Improved voltage profile obtained after OSCB allocation using SSA for 85-bus system

## 6.2. Convergence property

The power loss convergence characteristics of WCA and SSA for 9 – bus system is illustrated in **Fig. 22**. The annual cost minimization convergence characteristics of WCA and SSA for 9 – bus system is illustrated in **Fig. 23**. It can be noticed that both algorithms have smooth convergence characteristics for minimization of power loss and annual cost.

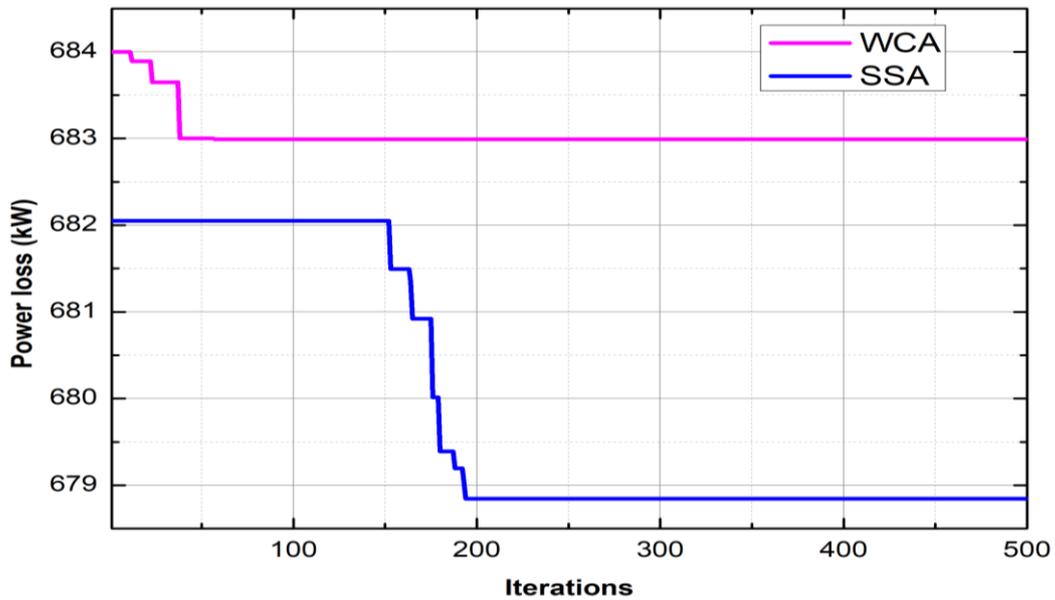


Fig. 22. Convergence characteristics of WCA and SSA for 9 – bus system (power loss)

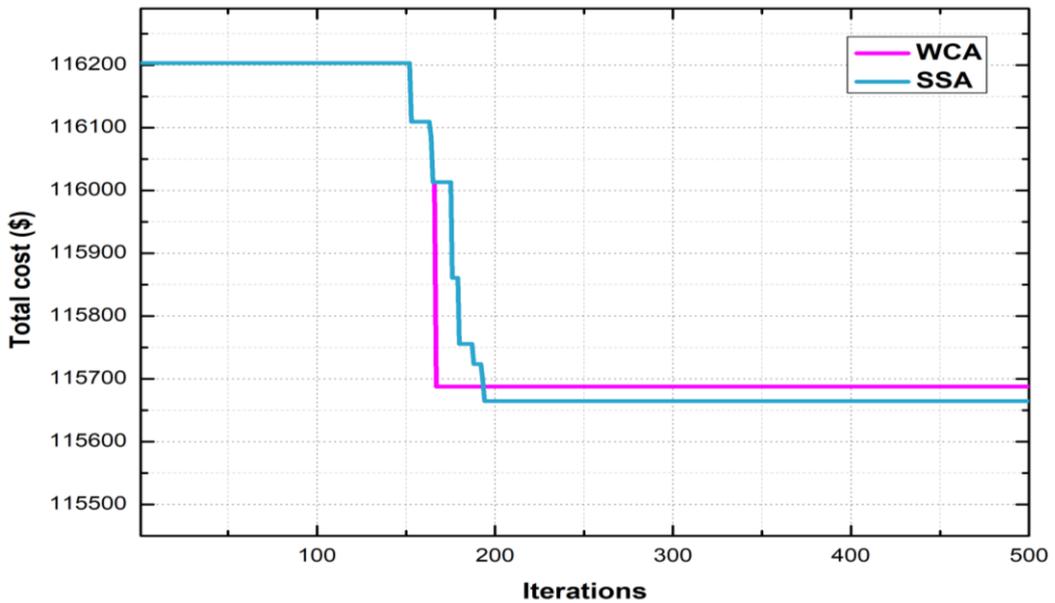


Fig. 23. Convergence characteristics of WCA and SSA for 9 – bus system (annual cost minimization)

### 6.3. Computational time

In the present study for each test system, both algorithms have run 50 times. Since WCA and SSA are stochastic optimization techniques hence for evaluating the performance of the best, average, and worst power loss, corresponding standard deviation (SD), variance, and average computational time MATLAB built-in functions have been used and obtained simulation results are presented in **Table 9**. The methodology for finding best, average, worst, standard deviation, and variance of given solution set can be evaluated using different techniques presented.

**Table 9**

The best, average, and worst power loss with SD and variance obtained by WCA and SSA for 50 trial runs

Algorithm	Test system	Power loss (kW)					Computational time (s)
		Worst	Average	Best	SD	Variance	
WCA	9 – bus system	678.845	678.845	678.845	0.0000	0.00000	3.15
	33 – bus system	135.01	133.11	132.35	0.8737	0.76335	8.05
	34 – bus system	161.18	160.83	160.58	0.2487	0.06185	8.07
	69 – bus system	147.11	146.01	145.26	0.7528	0.56670	16.32
	85 – bus system	150.72	149.19	148.91	0.5733	0.32867	23.45
SSA	9 – bus system	682.99	682.99	682.99	0.0000	0.00000	3.15
	33 – bus system	135.34	133.44	132.66	0.8635	0.74563	8.05
	34 – bus system	163.25	161.96	160.80	0.2487	0.06185	8.07
	69 – bus system	147.26	146.53	145.36	0.3528	0.12446	16.32
	85 – bus system	150.21	149.69	149.37	0.3733	0.13935	23.45

## 7. Conclusion and future scope

In this paper, two novel metaheuristic algorithms have been implemented for solving capacitor allocation optimization problems in the distribution network. The first algorithm is inspired by the water cycle process of nature in the real world where streams and rivers flow to the sea known as the water cycle algorithm (WCA). The second algorithm is inspired by salp swarming behavior in oceans for navigating and foraging is known as the salp swarm algorithm (SSA). Both the algorithms are tested on standard 9, 33, 34, 69, and 85 – bus distribution networks. The efficiency of WCA and SSA is assessed in terms of power loss minimization and net saving maximization. The power loss reduction achieved by the present algorithms is compared with the existing techniques. The comparison confirms that both the algorithms are suitable and capable of solving optimal capacitor allocation problems in distribution networks.

The better results of the present optimization algorithms show their capabilities of local (intensification) and global search (diversification). To crisscross the feasibility of present algorithms, both algorithms are effectively implemented for solving capacitor allocation problems in small, medium, and large distribution networks. However, the application of the present algorithm is not restricted only to solving capacitor allocation problems, it can be utilized in related areas of research for future work. The effective implementation of WCA and SSA for solving various optimization problems, hybridization of WCA and SSA can be implemented and tested with benchmark functions and various well-established optimization algorithms, and the effective implementation of novel optimization algorithms for solving optimal capacitor allocation problems in the distribution network.

## References

- [1] K. S. Sambaiah and T. Jayabarathi, "Loss minimization techniques for optimal operation and planning of distribution systems: A review of different methodologies," *Int. Trans. Electr. Energy Syst.*, vol. 30, no. 2, p. 12230, 2020.
- [2] Y. G. Bae, "Analytical Method of Capacitor Allocation on Distribution Primary Feeders," in *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-97, no. 4, pp. 1232-1238, July 1978, doi: 10.1109/TPAS.1978.354605.
- [3] J. J. Grainger and S. H. Lee, "Optimum Size and Location of Shunt Capacitors for Reduction of Losses on Distribution Feeders," in *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-100, no. 3, pp. 1105-1118, March 1981, doi: 10.1109/TPAS.1981.316577.
- [4] M. M. A. Salama, E. A. A. Mansour, A. Y. Chikhani and R. Hackam, "Control of Reactive Power in Distribution Systems with an End-Load and Varying Load Condition," in *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-104, no. 4, pp. 941-947, July 1985, doi: 10.1109/TPAS.1985.319095.
- [5] Baran, M. and Wu, F.F., "Optimal sizing of capacitors placed on a radial distribution system". *IEEE Transactions on Power Delivery*, vol. 4, no. 1, pp.735-743, 1989.
- [6] Chiang H, Wang J. Optimal capacitor placements in distribution systems. II. Solution algorithms and numerical results. *IEEE Trans Power Deliv* 1990;5:643–9. doi:10.1109/61.53065.
- [7] Yann-Chang Huang, Hong-Tzer Yang, Ching-Lien Huang. Solving the capacitor placement problem in a radial distribution system using Tabu Search approach. *IEEE Trans Power Syst* 2002;11:1868–73. doi:10.1109/59.544656.
- [8] Levitin, G., Kalyuzhny, A., Shenkman, A., Chertkov, M. Optimal capacitor allocation in distribution systems using a genetic algorithm and a fast energy loss computation technique. *IEEE Trans Power Deliv* 2000;5(2):623-628.
- [9] Das D. Novel method for solving radial distribution networks. *IEE Proc - Gener Transm Distrib* 2002;141:291. doi:10.1049/ip-gtd:19949966.
- [10] Swarup KS. Genetic algorithm for optimal capacitor allocation in radial distribution systems. *6th Int Conf Evol Comput* 2005;2005:152–9. doi:10.1109/INDCON.2005.1590245.
- [11] Prasad P V, Sivanagaraju S, Sreenivasulu N. A Fuzzy-Genetic Algorithm for Optimal Capacitor. *ARPN J Eng Appl Sci* 2007;2:28–32.
- [12] H. M. Khodr, F. G. Olsina, P. M. D. O. De Jesus, and J. M. Yusta, "Maximum savings approach for location and sizing of capacitors in distribution systems," *Electr. Power Syst. Res.*, vol. 78, no. 7, pp. 1192–1203, 2008.
- [13] Hamouda A, Lakehal N, Zehar K. Heuristic method for reactive energy management in distribution feeders. *Energy Convers Manag* 2010;51:518–23. doi:10.1016/j.enconman.2009.10.016.
- [14] Elmaouhab A, Boudour M, Gueddouche R. New Evolutionary Technique for Optimization Shunt Capacitors in Distribution Networks. *J Electr Eng* 2011;62:163–7. doi:10.2478/v10187-011-0027-x.
- [15] S. A. Taher, A. Karimian, and M. Hasani, "A new method for optimal location and sizing of capacitors in distorted distribution networks using PSO algorithm," *Simul. Model. Pract. Theory*, vol. 19, no. 2, pp. 662–672, 2011.
- [16] R. S. Rao, S. V. L. Narasimham, and M. Ramalingaraju, "Optimal capacitor placement in a radial distribution system using Plant Growth Simulation Algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 33, no. 5, pp. 1133–1139, 2011.

- [17] M. Ramalinga Raju, K. V. S. Ramachandra Murthy, and K. Ravindra, "Direct search algorithm for capacitive compensation in radial distribution systems," *Int. J. Electr. Power Energy Syst.*, vol. 42, no. 1, pp. 24–30, 2012.
- [18] S. P. Singh and A. R. Rao, "Optimal allocation of capacitors in distribution systems using particle swarm optimization," *Int. J. Electr. Power Energy Syst.*, vol. 43, no. 1, pp. 1267–1275, 2012.
- [19] El-Fergany AA. Optimal capacitor allocations using evolutionary algorithms. *IET Gener Transm Distrib* 2013;7:593–601. doi:10.1049/iet-gtd.2012.0661.
- [20] A. Kavousi-Fard and T. Niknam, "Considering uncertainty in the multi-objective stochastic capacitor allocation problem using a novel self adaptive modification approach," *Electr. Power Syst. Res.*, vol. 103, pp. 16–27, 2013.
- [21] Sultana, S. and Roy, P.K., "Optimal capacitor placement in radial distribution systems using teaching learning based optimization". *International Journal of Electrical Power & Energy Systems*, vol. 54, pp.387-398, 2014.
- [22] C. S. Lee, H. V. H. Ayala, and L. D. S. Coelho, "Capacitor placement of distribution systems using particle swarm optimization approaches," *Int. J. Electr. Power Energy Syst.*, vol. 64, pp. 839–851, 2015.
- [23] S. K. Injeti, V. K. Thunuguntla, and M. Shareef, "Optimal allocation of capacitor banks in radial distribution systems for minimization of real power loss and maximization of network savings using bio-inspired optimization algorithms," *Int. J. Electr. Power Energy Syst.*, vol. 69, pp. 441–455, 2015.
- [24] K. R. Devabalaji, K. Ravi, and D. P. Kothari, "Optimal location and sizing of capacitor placement in radial distribution system using Bacterial Foraging Optimization Algorithm," *Int. J. Electr. Power Energy Syst.*, vol. 71, no. October, pp. 383–390, 2015.
- [25] N. Gnanasekaran, S. Chandramohan, P. S. Kumar, and A. Mohamed Imran, "Optimal placement of capacitors in radial distribution system using shark smell optimization algorithm," *Ain Shams Eng. J.*, vol. 7, no. 2, pp. 907–916, 2016.
- [26] D. B. Prakash and C. Lakshminarayana, "Optimal siting of capacitors in radial distribution network using Whale Optimization Algorithm," *Alexandria Eng. J.*, vol. 56, no. 4, pp. 499–509, 2017.
- [27] V. Tamilselvan, T. Jayabarathi, T. Raghunathan, and X. S. Yang, "Optimal capacitor placement in radial distribution systems using flower pollination algorithm," *Alexandria Eng. J.*, 2018.
- [28] Kola Sampangi, Sambaiah, and Jayabarathi Thangavelu. "Optimal capacitor allocation in distribution networks for minimization of power loss and overall cost using water cycle algorithm and grey wolf optimizer." *International Transactions on Electrical Energy Systems*, vol. 30, no. 5, 2020.
- [29] A. A. Z. Diab and H. Rezk, "Optimal Sizing and Placement of Capacitors in Radial Distribution Systems Based on Grey Wolf, Dragonfly and Moth–Flame Optimization Algorithms," *Iran. J. Sci. Technol. - Trans. Electr. Eng.*, vol. 43, no. 1, pp. 77–96, 2019.
- [30] M. W. Saddique, S. S. Haroon, S. Amin, A. R. Bhatti, I. A. Sajjad, and R. Liaqat, "Optimal Placement and Sizing of Shunt Capacitors in Radial Distribution System Using Polar Bear Optimization Algorithm," *Arab. J. Sci. Eng.*, vol. 46, no. 2, pp. 873–899, 2021.
- [31] A. Selim, S. Kamel, and F. Jurado, "Capacitors Allocation in Distribution Systems Using a Hybrid Formulation Based on Analytical and Two Metaheuristic Optimization Techniques," *Comput. Electr. Eng.*, vol. 85, 2020.
- [32] G. G. Soma, "Optimal Sizing and Placement of Capacitor Banks in Distribution Networks Using a Genetic Algorithm," *Electricity*, vol. 2, no. 2, pp. 187–204, 2021.
- [33] M. Asadi et al., "Optimal Placement and Sizing of Capacitor Banks in Harmonic Polluted Distribution Network," in 2021 IEEE Texas Power and Energy Conference, TPEC 2021, 2021.

- [34] T. P. M. Mtonga, K. K. Kaberere, and G. K. Irungu, "Optimal Shunt Capacitors' Placement and Sizing in Radial Distribution Systems Using Multiverse Optimizer," *IEEE Can. J. Electr. Comput. Eng.*, vol. 44, no. 1, pp. 10–21, 2021.
- [35] A. Naderipour et al., "Spotted hyena optimizer algorithm for capacitor allocation in radial distribution system with distributed generation and microgrid operation considering different load types," *Sci. Rep.*, vol. 11, no. 1, pp. 1–15, 2021.
- [36] M. Babanezhad, S. Arabi Nowdeh, A. Y. Abdelaziz, K. M. AboRas, and H. Kotb, "Reactive power based capacitors allocation in distribution network using mathematical remora optimization algorithm considering operation cost and loading conditions," *Alexandria Eng. J.*, vol. 61, no. 12, pp. 10511–10526, 2022.
- [37] Teng, J.H.: "A network-topology-based three-phase load flow for distribution systems", *Proc. Nat. Sci. Council, Republic of China*, vol. 24, no. 4, pp. 259–264, 2000.
- [38] Teng, J.H., "A direct approach for distribution system load flow solutions". *IEEE Transactions on power delivery*, vol. 18, no. 3, pp.882-887, 2003.
- [39] H. Eskandar, A. Sadollah, A. Bahreininejad, and M. Hamdi, "Water cycle algorithm - A novel metaheuristic optimization method for solving constrained engineering optimization problems," *Comput. Struct.*, vol. 110–111, pp. 151–166, 2012.
- [40] A. A. Heidari, R. Ali Abbaspour, and A. Rezaee Jordehi, "An efficient chaotic water cycle algorithm for optimization tasks," *Neural Comput. Appl.*, vol. 28, no. 1, pp. 57–85, 2017.
- [41] A. A. Heidari, R. Ali Abbaspour, and A. Rezaee Jordehi, "Gaussian bare-bones water cycle algorithm for optimal reactive power dispatch in electrical power systems," *Appl. Soft Comput. J.*, vol. 57, pp. 657–671, 2017.
- [42] S. Mirjalili, A. H. Gandomi, S. Z. Mirjalili, S. Saremi, H. Faris, and S. M. Mirjalili, "Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems," *Adv. Eng. Softw.*, vol. 0, pp. 1–29, 2017.

## Appendices

**Table A1**

Line and bus data of 9 – bus test system

line no.	from bus, $m$	to bus, $m + 1$	$R_{m,m+1}$ ( $\Omega$ )	$X_{m,m+1}$ ( $\Omega$ )	$P_L$ (kW)	$Q_L$ (kVAR)
1	1	2	0.1233	0.4127	1840	460
2	2	3	0.0140	0.6051	980	340
3	3	4	0.7463	1.2050	1790	446
4	4	5	0.6984	0.6084	1598	1840
5	5	6	1.9831	1.7276	1610	600
6	6	7	0.9053	0.7886	780	110
7	7	8	2.0552	1.1640	1150	60
8	8	9	4.7953	2.7160	980	130
9	9	10	5.3434	3.0264	1640	200

**Table A2**

Line and bus data of 33 – bus test system

line no.	from bus, $m$	to bus, $m + 1$	$R_{m,m+1}$ ( $\Omega$ )	$X_{m,m+1}$ ( $\Omega$ )	$P_L$ (kW)	$Q_L$ (kVAR)
1	1	2	0.0922	0.0477	100	60
2	2	3	0.493	0.2511	90	40
3	3	4	0.366	0.1864	120	80
4	4	5	0.3811	0.1941	60	30
5	5	6	0.819	0.707	60	20
6	6	7	0.1872	0.6188	200	100

7	7	8	0.7114	0.2351	200	100
8	8	9	1.03	0.74	60	20
9	9	10	1.04	0.74	60	20
10	10	11	0.1966	0.065	45	30
11	11	12	0.3744	0.1238	60	35
12	12	13	1.468	1.155	60	35
13	13	14	0.5416	0.7129	120	80
14	14	15	0.591	0.526	60	10
15	15	16	0.7463	0.545	60	20
16	16	17	1.289	1.721	60	20
17	17	18	0.732	0.574	90	40
18	2	19	0.164	0.1565	90	40
19	19	20	1.5042	1.3554	90	40
20	20	21	0.4095	0.4784	90	40
21	21	22	0.7089	0.9373	90	40
22	3	23	0.4512	0.3083	90	50
23	23	24	0.898	0.7091	420	200
24	24	25	0.896	0.7011	420	200
25	6	26	0.203	0.1034	60	25
26	26	27	0.2842	0.1447	60	25
27	27	28	1.059	0.9337	60	20
28	28	29	0.8042	0.7006	120	70
29	29	30	0.5075	0.2585	200	600
30	30	31	0.9744	0.963	150	70
31	31	32	0.3105	0.3619	210	100
32	32	33	0.341	0.5302	60	40

**Table A3**

Line and bus data of 34 – bus test system

line no.	from bus, $m$	to bus, $m + 1$	$R_{m,m+1}$ ( $\Omega$ )	$X_{m,m+1}$ ( $\Omega$ )	$P_L$ (kW)	$Q_L$ (kVAR)
1	1	2	0.117	0.048	230	142.5
2	2	3	0.10725	0.044	0	0
3	3	4	0.16445	0.04565	230	142.5
4	4	5	0.15	0.0415	230	142.5
5	5	6	0.15	0.0415	0	0
6	6	7	0.3144	0.054	0	0
7	7	8	0.2096	0.036	230	142.5
8	8	9	0.3144	0.054	230	142.5
9	9	10	0.2096	0.036	0	0
10	10	11	0.131	0.0225	230	142.5
11	11	12	0.1048	0.018	137	84
12	3	13	0.1572	0.027	72	45
13	13	14	0.2096	0.036	72	45
14	14	15	0.1048	0.018	72	45
15	15	16	0.0524	0.009	13.5	7.5
16	6	17	0.1794	0.0498	230	142.5
17	17	18	0.16445	0.04565	230	142.5
18	18	19	0.2079	0.0473	230	142.5
19	19	20	0.189	0.043	230	142.5
20	20	21	0.189	0.043	230	142.5
21	21	22	0.262	0.045	230	142.5
22	22	23	0.262	0.045	230	142.5

23	23	24	0.3144	0.054	230	142.5
24	24	25	0.2096	0.036	230	142.5
25	25	26	0.131	0.0225	230	142.5
26	26	27	0.1048	0.018	137	85
27	7	28	0.1572	0.027	75	48
28	28	29	0.1572	0.027	75	48
29	29	30	0.1572	0.027	75	48
30	10	31	0.1572	0.027	57	34.5
31	31	32	0.2096	0.036	57	34.5
32	32	33	0.1572	0.027	57	34.5
33	33	34	0.1048	0.018	57	34.5

**Table A4**

Line and bus data of 69 – bus test system

line no.	from bus, $m$	to bus, $m + 1$	$R_{m,m+1}$ ( $\Omega$ )	$X_{m,m+1}$ ( $\Omega$ )	$P_L$ (kW)	$Q_L$ (kVAR)
1	1	2	0.0005	0.0012	0	0
2	2	3	0.0005	0.0012	0	0
3	3	4	0.0015	0.0036	0	0
4	4	5	0.0251	0.0294	0	0
5	5	6	0.366	0.1864	2.6	2.2
6	6	7	0.3811	0.1941	40.4	30
7	7	8	0.0922	0.047	75	54
8	8	9	0.0493	0.0251	30	22
9	9	10	0.819	0.2707	28	19
10	10	11	0.1872	0.0619	145	104
11	11	12	0.7114	0.2351	145	104
12	12	13	1.03	0.34	8	5
13	13	14	1.044	0.345	8	5.5
14	14	15	1.058	0.3496	0	0
15	15	16	0.1966	0.065	45.5	30
16	16	17	0.3744	0.1238	60	35
17	17	18	0.0047	0.0016	60	35
18	18	19	0.3276	0.1083	0	0
19	19	20	0.2106	0.069	1	0.6
20	20	21	0.3416	0.1129	114	81
21	21	22	0.014	0.0046	5	3.5
22	22	23	0.1591	0.0526	0	0
23	23	24	0.3463	0.1145	28	20
24	24	25	0.7488	0.2745	0	0
25	25	26	0.3089	0.1021	14	10
26	26	27	0.1732	0.0572	14	10
27	3	28	0.0044	0.0108	26	18.6
28	28	29	0.064	0.1565	26	18.6
29	29	30	0.3978	0.1315	0	0
30	30	31	0.0702	0.0232	0	0
31	31	32	0.351	0.116	0	0
32	32	33	0.839	0.2816	14	10
33	33	34	1.708	0.5646	19.5	14
34	34	35	1.474	0.4673	6	4
35	3	36	0.0044	0.0108	26	18.55
36	36	37	0.064	0.1565	26	18.55
37	37	38	0.1053	0.123	0	0
38	38	39	0.0304	0.0355	24	17
39	39	40	0.0018	0.0021	24	17
40	40	41	0.7283	0.8509	1.2	1
41	41	42	0.31	0.3623	0	0

42	42	43	0.041	0.0478	6	4.3
43	43	44	0.0092	0.0116	0	0
44	44	45	0.1089	0.1373	39.22	26.3
45	45	46	0.0009	0.0012	39.22	26.3
46	4	47	0.0034	0.0084	0	0
47	47	48	0.0851	0.2083	79	56.4
48	48	49	0.2898	0.7091	384.7	274.5
49	49	50	0.0822	0.2011	384.7	274.5
50	8	51	0.0928	0.0473	40.5	28.3
51	51	52	0.3319	0.1114	3.6	2.7
52	9	53	0.174	0.0886	4.35	3.5
53	53	54	0.203	0.1034	26.4	19
54	54	55	0.2842	0.1447	24	17.2
55	55	56	0.2813	0.1433	0	0
56	56	57	1.59	0.5337	0	0
57	57	58	0.7837	0.263	0	0
58	58	59	0.3042	0.1006	100	72
59	59	60	0.3861	0.1172	0	0
60	60	61	0.5075	0.2585	1244	888
61	61	62	0.0974	0.0496	32	23
62	62	63	0.145	0.0738	0	0
63	63	64	0.7105	0.3619	227	162
64	64	65	1.041	0.5302	59	42
65	11	66	0.2012	0.0611	18	13
66	66	67	0.0047	0.0014	18	13
67	12	68	0.7394	0.2444	28	20
68	68	69	0.0047	0.0016	28	20

**Table A5**

Line and bus data of 85 – bus test system

line no.	from bus, $m$	to bus, $m + 1$	$R_{m,m+1}$ ( $\Omega$ )	$X_{m,m+1}$ ( $\Omega$ )	$P_L$ (kW)	$Q_L$ (kVAR)
1	1	2	0.108	0.075	0	0
2	2	3	0.163	0.112	0	0
3	3	4	0.217	0.149	56	57.58
4	4	5	0.108	0.074	0	0
5	5	6	0.435	0.298	35.28	36.28
6	6	7	0.272	0.186	0	0
7	7	8	1.197	0.82	35.28	36.28
8	8	9	0.108	0.074	0	0
9	9	10	0.598	0.41	0	0
10	10	11	0.544	0.373	56	57.58
11	11	12	0.544	0.373	0	0
12	12	13	0.598	0.41	0	0
13	13	14	0.272	0.186	35.28	36.28
14	14	15	0.326	0.223	35.28	36.28
15	2	16	0.728	0.302	35.28	36.28
16	3	17	0.455	0.189	112	115.17
17	5	18	0.82	0.34	56	57.58
18	18	19	0.637	0.264	56	57.58
19	19	20	0.455	0.189	35.28	36.28
20	20	21	0.819	0.34	35.28	36.28
21	21	22	1.548	0.642	35.28	36.28
22	19	23	0.182	0.075	56	57.58
23	7	24	0.91	0.378	35.28	36.28
24	8	25	0.455	0.189	35.28	36.28

25	25	26	0.364	0.151	56	57.58
26	26	27	0.546	0.226	56	57.58
27	27	28	0.273	0.113	35.28	36.28
28	28	29	0.546	0.226	0	0
29	29	30	0.546	0.226	35.28	36.28
30	30	31	0.273	0.113	14	14.39
31	31	32	0.182	0.075	0	0
32	32	33	0.182	0.075	35.28	36.28
33	33	34	0.819	0.34	0	0
34	34	35	0.637	0.264	0	0
35	35	36	0.182	0.075	56	57.58
36	26	37	0.364	0.151	56	57.58
37	27	38	1.002	0.416	56	57.58
38	29	39	0.546	0.226	35.28	36.28
39	32	40	0.455	0.189	35.28	36.28
40	40	41	1.002	0.416	0	0
41	41	42	0.273	0.113	35.28	36.28
42	41	43	0.455	0.189	35.28	36.28
43	34	44	1.002	0.416	35.28	36.28
44	44	45	0.911	0.378	35.28	36.28
45	45	46	0.911	0.378	35.28	36.28
46	46	47	0.546	0.226	14	14.39
47	35	48	0.637	0.264	0	0
48	48	49	0.182	0.075	0	0
49	49	50	0.364	0.151	36.28	37.31
50	50	51	0.455	0.189	56	57.58
51	48	52	1.366	0.567	0	0
52	52	53	0.455	0.189	35.28	36.28
53	53	54	0.546	0.226	56	57.58
54	52	55	0.546	0.226	56	57.58
55	49	56	0.546	0.226	14	14.39
56	9	57	0.273	0.113	56	57.58
57	57	58	0.819	0.34	0	0
58	58	59	0.182	0.075	56	57.58
59	58	60	0.546	0.226	0	0
60	60	61	0.728	0.302	56	57.58
61	61	62	1.002	0.415	56	57.58
62	60	63	0.182	0.075	14	14.39
63	63	64	0.728	0.302	0	0
64	64	65	0.182	0.075	0	0
65	65	66	0.182	0.075	56	57.58
66	64	67	0.455	0.189	0	0
67	67	68	0.91	0.378	0	0
68	68	69	1.092	0.453	56	57.58
69	69	70	0.455	0.189	0	0
70	70	71	0.546	0.226	35.28	36.28
71	67	72	0.182	0.075	56	57.58
72	68	73	1.184	0.491	0	0
73	73	74	0.273	0.113	56	57.58
74	73	75	1.002	0.416	35.28	36.28
75	70	76	0.546	0.226	56	57.58
76	65	77	0.091	0.037	14	14.39
77	10	78	0.637	0.264	56	57.58
78	67	79	0.546	0.226	35.28	36.28
79	12	80	0.728	0.302	56	57.58
80	80	81	0.364	0.151	0	0
81	81	82	0.091	0.037	56	57.58
82	81	83	1.092	0.453	35.28	36.28
83	83	84	1.002	0.416	14	14.39
84	13	85	0.819	0.34	35.28	36.28