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Proof of Fermat's Number Conjecture

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Abstract:

The natural numbers defined by the sequence $F(n) = 2^{2^n} + 1$, n = 0,1,2,..., are called Fermat numbers. Fermat's conjecture states that there are only finitely many prime numbers in Fermat numbers. It was proposed in 1640 and has not been proved for more than 380 years.

The prime number distribution is a deterministic random distribution, so problems related to prime numbers can be studied, analyzed and proved by probability statistics.

In this paper, the probability and statistics method is used to prove the conjecture by judging whether the series converges. Our new conjecture is that there are only five known Fermat primes, namely 3, 5, 17, 257, and 65537.

Key words:

Fermat number conjecture, probability and statistics, prime number theorem, series, deterministic random distribution

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This paper uses the probability and statistics method to prove the conjecture, and now the method is introduced as follows:

1. Proof method of Fermat number conjecture

Since there is no mathematical model for prime numbers that can be completely and accurately represented [1][2], the distribution of prime numbers is a deterministic random distribution, so the problems related to prime numbers can be studied, analyzed and proved by using probability and statistics methods [3][4][5].

Let n = 0,1,2,..., according to the prime number theorem, the probability of $2^{2^n}+1$being a prime number is about $1/\ln(2^{2^n}+1)$, so the number of prime numbers (cumulative probability) when n is any positive integer is about

$$\sum_{x=0}^{n} \frac{1}{ln(2^{2^x}+1)}$$

Because when x increases, $ln(2^{2^x}+1)$ approaches the value of $ln(2^{2^x})$, so the above formula can be simplified to

$$\frac{1}{\ln(2)} \sum_{x=0}^{n} \frac{1}{2^x}$$

And because the series $\sum \frac{1}{x^p}$ converges when p>1; for a fixed p, when x is sufficiently large, $2^x>x^p$, so $1/2^x<1/x^p$; when n tends to infinity, the above function converges. This proves the Fermat number conjecture, that is, there are only finitely many prime numbers in Fermat numbers.

2. Conclusion and conjecture

There are only finitely many prime numbers in the Fermat number sequence F(n) = $2^{2^n}+1$, n = 0,1,2,... . Our new conjecture is that there are only five known Fermat primes, namely 3, 5, 17, 257, and 65537.

3. References

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