On Legendre's conjecture

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Abstract:- In this paper, we interesting in most conjecture problem relies with the prime number which is Legendre's conjecture. *We also introduced polynomials that check this conjecture* with algebraic proof. *Also, we reinforced the conjecture with some rules.*

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1. Introduction

In 1912, during the International Conference on Mathematics, Edmund Landau posed four basic problems about prime numbers, among them the **Legendre's conjecture**. Which states:

Is there always at least one prime number between consecutive square number n^2 and $(n+1)^2$?

After algebraic operations, we found that the numbers that fulfil Legendre's conjecture are written in the following form

$$n^2+n-1=(n+1)^2-(n+2) \qquad (1)$$

for $n\in\mathbb{N}$ where $n\geq 2$ with $n\neq 5k+2$ for $k\geq 1$ is a natural number. And of course, we found that the numbers written in the form (1) are prime numbers if $n\neq 5k+2$ where $k\geq 1$.

It is easy to prove that the numbers written in the form (1) satisfy the relationship

$$n^2<n^2+n-1<(n+1)^2$$

by proof the correctness of the following:

$n^2-(n^2+n-1)<0$ and $(n+1)^2-(n^2+n-1)>0$

for $n\geq 2$.

In the second step we will prove that

$PGCD(n^2;n^2+n-1)=1$ it is true for every natural number $n\in\mathbb{N}^*$ We have $n^2+n-1=n^2\times 1+n-1$, then

$$PGCD(n^2;n^2+n-1)=PGCD(n^2;n-1)$$

Also, we get $1\times n^2+(n-1)(n+1)=1$. That is, there are two integers such that $\alpha=1$ and $\beta=(n+1)$. According to Puzo's theorem we have n^2 and $n-1$ are relatively prime. Then n^2 and n^2+n-1 are relatively prime also.

On the other hand, we find for $PGCD(n;n-1)=1$ that n and $n-1$ are relatively prime for every natural number $n\in\mathbb{N}^*$. Because, according to Puzo's theory we have $n-(n-1)=1$. We conclude under Dirichlet's theorem that for $PGCD(n;n-1)=1$ there is infinitely many primes given by

$$p(n)=n^2+n-1 \qquad (2)$$

Let Δ be the discriminant of (2), $\Delta=5\equiv 1[4]$. It can be said that the polynomial (2) overlaps the definition of Rabinowitsch polynomial for instant see [4].

Next, we consider the natural number $n\geq 1$ where $n=5k+r$ with $r=\{0,1,2,3,4\}$ and $k\in\mathbb{N}$. And we will write the polynomial (2) in terms of k for the following values of $n=\{5k,5k+1,5k+2,5k+3,5k+4\}$ respectively. Then

First, for $n=5k$ we find

$$n^2+n-1=25k^2+5k-1=5k(5k+1)-1=5m-1$$

And for $n=5k+1$ we find

$$n^2+n-1=25k^2+15k+1=5k(5k+3)+1=5m+1$$

Next, for $n=5k+2$ we find

$$n^2+n-1=25k^2+25k+5=5(5k^2+5k+1)=5m$$

And for $n=5k+3$ we find

$$n^2+n-1=25k^2+35k+11=5(5k^2+7k+2)+1=5m+1$$

Finally, for $n=5k+4$ we find

$$n^2+n-1=25k^2+45k+19=5(5k^2+9k+4)-1=5m-1$$

According to the results obtained previously, we conclude that the polynomial given in (2) became an odd number for $n=\{5k,5k+1,5k+3,5k+4\}$ It is written in one of the two forms $5m+1$ or $5m-1$.

Since for $n=5k+2$ the polynomial (2) is a multiple of 5 and is written in the form $5m$ with $m\in\mathbb{N}$.Of course, except for number 5 for $k=0$.

Also, we give more than one polynomial that give a prime number. Which define as the following:

$25n^2+5n-1$, $25n^2+15n+1$ and $25n^2+35n+11$ with $5n^2+5n+1$. Finally, $25n^2+45n+19$.

There more, we reduced for the numerical results of the polynomial (2) shown in table (1), this relation as following: for consecutive $p_j(n)$ and $p_{j+1}(n)$ we have

$$p_{j+1}(n)=p_j(n-1)+2n \qquad (3)$$

Since , if $n=5k+2$ for $k\geq 1$. We get

$$p_{j+1}(n)=p_j(n-2)+2(2n-1) \qquad (4).$$

And that through the two relations (3) and (4) we can write an algorithm for this formula for $2\leq n<22$. See the following table.

Table 1: Values of the polynomial n^2+n-1 for $n=\{1;\cdots\cdots;34\}$.

n	n^2	$p(n)=n^2+n-1$	$(n+1)^2$	Is Prime
1	1	1	4	
2	4	5	9	5
3	9	11	16	
4	16	19	25	
5	25	29	36	
6	36	41	49	
7	49	55	64	5×11
8	64	71	81	
9	81	89	100	
10	100	109	121	
11	121	131	144	
12	144	155	169	5×31
13	169	181	196	
14	196	209	225	
15	225	239	256	
16	256	271	289	
17	289	305	324	5×61
18	324	341	361	
19	361	379	400	
20	400	419	441	
21	441	461	484	
22	484	505	529	5×101
23	529	551	576	
24	576	599	625	
25	625	649	676	11×59
26	676	701	729	
27	729	755	784	5×151
28	784	811	841	
29	841	869	900	11×79
30	900	929	961	
31	961	991	1024	
32	1024	1055	1089	5×211
33	1089	1121	1156	
34	1156	1189	1225	29×41
35	1225	1259	1296	
36	1296	1331	1369	$11\times11\times11$
37	1369	1405	1444	5×281
38	1444	1481	1521	
35	1521	1559	1600	
40	1600	1639	1681	11×149

2. CONCLUSIONS

In this paper we have introduced a Legendre's conjecture, which poses the following question:

Is there at least one prime number between n^2 and $(n+1)^2$?.

After algebraic operations shown in table (1) we found that the defined polynomial is as (2). But for $n=5k+2$ we found that the Legendre's conjecture is not true because our polynomial gives us a multiple of 5 and therefore the number is not prime number.

For this we will put a new conjecture, it is as follows:

For the natural number $n\geq1$ where $n=5k+r$ with $r=\{0,1,3,4\}$ and $k\in\mathbb{N}$. There infinite prime numbers written in the form $p(n)=n^2+n-1$ and checked

$$n^2<p(n)<(n+1)^2.$$

It's clear from table (1) this conjecture is often true for $2\leq n<22$ without $n=5k+2$ for $k\in\mathbb{N}^*$. And after $n=22$ we note that, in addition to the numbers are written as (2) for $n=5k+2$ with $k\in\mathbb{N}^*$ from table (1), we find other numbers are not prime. But it can be expressed by multiplying prime numbers

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