Correct setting of mapping function in one-to-one correspondence

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Abstract: In order to strictly discuss the one-to-one correspondence between the elements of sets, the number of elements that can participate in the correspondence was first discussed, and then the necessary conditions for the formation of injection and bijection were discussed. According to these conditions, it wss found that some mapping functions do not satisfy these conditions. For example, it is impossible to obtain a mapping function satisfying these conditions between any infinite set and its any proper subset.

Keywords: one-to-one correspondence; number of elements; necessary conditions for injection and bijection; infinite set; proper subset

1 Introduction

In literature [1], the author gave a method to check whether the one-to-one correspondence is correct, which irrefutably verified that it is impossible for any infinite set to have a one-to-one correspondence with any of its proper subsets. In this paper, the author will give the reason for Cantor's error based on the definition of bijection.

2 One-to-one correspondence and the number of elements

Since the one-to-one correspondence occurs between elements of sets, how many elements can participate in the correspondence is a problem that must be solved first. Otherwise, how can any discussion of one-to-one correspondence have any strictly mathematical meaning if even how many elements are in the one-to-one correspondence is a vague question?

The easiest way to know the number of elements is to count, so the number of elements can also be defined as the result of the count. The count can be counted directly, or the count object can be divided into small blocks, and then counted separately, and finally the count results of each small block are added to obtain the final count result. For example, if A and B intersect as Φ, and C=AUB, then the number of elements of C is the sum of the number of elements of A and the number of elements of B.

For finite sets, the above definitions and counting rules are clear and unambiguous. For infinite sets, there is no reliable reason to think that the above definitions and rules no longer hold, the only difference is that for infinite sets, since the number of elements is infinite, we cannot use natural numbers to represent the number of elements, but must use other notations, such as the ∞ notation used for mathematical analysis, represent the number of elements. For example, if A and B are both infinite sets, then $\infty_C=\infty_A+\infty_B$.

Some people may think that infinite sets and finite sets are fundamentally different, so they cannot be discussed in the above method. However, he should give the reasons.

The only reason he may give is that infinite sets can correspond one-to-one with a proper subset and various theories established on this basis, such as the addition rules of cardinal numbers, are different from finite sets, so infinite sets and finite sets have fundamentally different.

In literature [1], this was overturned, so these reasons no longer hold.

Besides, even if the addition rules of cardinal numbers still holds, it can't be used here, because here we are talking about the number of elements, not the cardinal numbers.

3 Definitions and conditions of injection, surjection and bijection

Definitions:

Injection: If for any two different elements x_1, x_2 in X, $x_1 \neq x_2$, the image $f(x_1) \neq (x_2)$ obtained by the mapping function y=f(x).

Surjection: Any element in Y is an image of an element in X.

Bijection: It is a mapping that is both injection and surjection.

Although the definition is very simple, it may not be applied correctly without careful analysis of it.

According to the definition of injection, it is not difficult to get:

1) The domain of definition of the injective function y=f(x) must include every element of X (hereinafter referred to as the mapping party).

2) The number of elements contained in the value domain of the injective function y=f(x) is strictly equal to the number of elements contained in the definition domain, i.e. the number of elements of X.

It is not difficult to see that if any of the above two conditions are not satisfied, the injection cannot be established. For example, since any element x_1 in X can be paired with another element x_2 in X to form a pair of elements x_1, x_2, if condition 1) is not satisfied, there is no guarantee that each of any two elements will be mapped to Y. For another example, if condition 2) is not satisfied, it is impossible for every x to have one and only one y, and injectivity does not hold.

Therefore, the above conditions are the necessary conditions for injection.

Similarly, the necessary conditions for the formation of bijection are:

1) There is a mapping function that can form an injection.

2) The number of elements contained in the value domain of the injective function is the same as the number of elements contained in the mapped party Y.

4 Application of the Conditions

It can be seen that, to discuss whether the bijection can be formed, we must first find the injective function.

Not all mapping functions are injective functions. For example, someone (Xue Wentian) proposed that the mapping of N→N can of course be bijective, but if let y=g(x)=x+1, that is, 1→2, 2→3, 3→4.... is an injection, not bijection (element 1 has no preimage). That is, the injective function between sets N and N is not bijection.

However, the elements belonging to the definition domain of the mapper are 1, 2, 3, 4....... It contains exactly every element of N, which meets the condition 1), but the elements belonging to its value domain are only 2, 3, 4... only includes each element of N-{1}, one element less than the mapping party, does not meet the condition 2), so the mapping: y=g(x)=x+1 is not an injective function!

According to the condition 1) of bijection, since it is not a injective function, how to discuss whether it is a bijection?

Then, is the mapping of N→N: 2→1, 3→2, 4→3.... injection ?

It is not difficult to see that the definition domain of the mapping party does not contain element 1, which does not meet the injection condition 1), so it is not injection.

It can be seen that it is wrong to think that a bijection can be changed to a injection only by shifting.

For another example, for the set $N_1=\{0\}\cup N$ and its proper subset N, any mapping of f:N1->N does not meet the condition of injection 2): The number of elements of the mapping party is greater than that of mapped party, so it is not injection. Although the mapping of $f:N\to N_1$ can be injection, it does not meet the bijective condition 2): The number of elements of the mapping party is less than that of mapped party and cannot be bijection, so there is no bijection between N_1 and N.

Not knowing the necessary conditions for the formation of injection and bijection, and setting the mapping function arbitrarily, are the reasons of Cantor's error.

[1][266] **viXra:2209.0016** **[pdf]** *submitted on 2022-09-03 00:48:50*

A Method to Check the Reliability of One-to-One Correspondence

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