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A Proof of Mersenne Prime Conjecture

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Abstract:

The Mersenne Prime Conjecture was proposed in 1644, which refers to whether there are infinitely many Mersenne primes in a positive integer of the form 2^n - 1. The distribution of prime numbers is a deterministic random distribution, so problems related to prime numbers can be studied, analyzed and proved by probability statistics.

This paper proves the conjecture by judging the convergence of the series. At the same time, after the 51st Mersenne prime was proved in 2018, a conservative prediction is made, that is, there are at least 52 Mersenne primes in the Mersenne numbers less than $10^{215000000}$; a more accurate prediction is that there are at least 52 Mersenne numbers in the Mersenne numbers less than $10^{103000000}$ prime numbers.

Key words:

Mersenne Prime Conjecture, Probability and Statistics, Prime Number Theorem, Series, Deterministic Random Distribution

Mersenne numbers are positive integers of the form 2^n - 1, where n is a natural number. In 1644, Mersenne gave the prime numbers contained in the Mersenne numbers less than 257 (although there were errors and omissions), which were called Mersenne primes, which aroused lasting research interest. But whether there are infinitely many Mersenne primes, there is no clear conclusion [1].

This paper uses the probability and statistics method to prove the conjecture, and now the method is introduced as follows:

1. Proof Method of Mersenne Prime Conjecture

Since prime numbers do not have any mathematical model to represent them with complete accuracy, prime numbers are randomly distributed on the number line [2][3]. The distribution of prime numbers is a deterministic random distribution, so problems related to prime numbers can be studied, analyzed and proved by means of probability and statistics methods [4][5][6].

Let n>1 be any positive integer. According to the prime number theorem, the probability that 2^n-1 is a prime number is conservatively estimated to be about 1/ln(2^n-1), so the number (cumulative probability) of Mersenne primes less than 2^n is conservatively estimated to be about

$$\sum_{x=2}^{n} \frac{1}{ln(2^x - 1)}$$

Since ln(2^x- 1) approaches ln(2^x) as x increases, the above formula can be simplified to

$$\frac{1}{\ln(2)} \sum_{x=2}^{n} \frac{1}{x}$$

And because when n approaches infinity, the harmonic series in the above formula diverges, and the function value, that is, the number of Mersenne primes, tends to infinity. This proves the Mersenne prime conjecture, that there are infinitely many Mersenne primes.

Because the positive integer mantissa of form 2^n - 1 can only be 1, 3, 5 and 7; and because when the mantissa is 1, 3, 7 and 9, the probability of a positive integer m being a prime number is 10/4*1/ln(m) , so when the mantissas are 1, 3, 5, and 7, the probability that a positive integer m is prime is 10/4*1/ln(m)*3/4. After simplification the more precise probability that a positive integer of form 2^n - 1 is prime is about 15/8*/ln(2^n-1). Therefore, when n>1 is any positive integer, a more accurate estimate of the number (cumulative probability) of Mersenne primes less than 2^n is about

$$\frac{15}{8\ln(2)} \sum_{x=2}^{n} \frac{1}{x}$$

2. Experimental verification

See Table 1 for the predicted and actual numbers of Mersenne primes in different positive integer intervals.

Table 1 Distribution of Mersenne prime numbers in positive integer intervals

Positive integer intervals [1， n]	Estimation of number of Mersenne prime	Exact estimation of number of Mersenne primes	Actual number of Mersenne primes [1]
10^{5}	3.4	6.4	5
10^{25}	5.8	10.8	9
10^{125}	8.1	15.2	12
10^{625}	10.4	19.5	15
10^{3125}	12.7	23.9	22
10^{15625}	15.1	28.2	27
10^{78125}	17.4	32.6	31
10^{360625}	19.6	36.7	33
$10^{1953125}$	22.0	41.3	37
$10^{9765625}$	24.3	45.6	43
$10^{48828125}$	26.7	50.0	51
$10^{103000000}$	27.7	52.0	—
$10^{215000000}$	28.8	54.0	—

3. Conclusion

In the positive integers of the form $2^n - 1$, that is, Mersenne numbers, there are infinitely many prime numbers; the number of Mersenne primes in Mersenne numbers less than 2^n is approximately

$$\frac{15}{8\ln(2)}\sum_{x=2}^{n}\frac{1}{x}$$

Conjecture: Conservatively predict that there are at least 52 Mersenne primes in Mersenne numbers less than $10^{215000000}$; a more accurate prediction is that there are at least 52 Mersenne primes in Mersenne numbers less than $10^{103000000}$.

4. References

[1] https://www.mersenne.org/primes/

[2] Baoyang Liu and Yawei Liu. On the distribution of twin primes. Mathematics Learning and Research, 2011, (15) 87-88

[3] G. H.Hardy and E. M.Wright. An Introduction to the Theory of Numbers, 6th ed., Oxford University Press, 2008

[4] Zhi Li and Hua Li. A Proof of Goldbach Conjecture. viXra:2111.0093

[5] Zhi Li and Hua Li.Proofs of Twin Prime Number Conjecture and First Hardy-Littlewood Conjecture. viXra:2111.0098

[6] Zhi Li and Hua Li. Proof of the n^2+1 conjecture. https://zhuanlan.zhihu.com/p/555825398