Kouider Number with base a

Kouider Mohammed Ridha

Department of Mathematics, Applied Mathematics Laboratory, University of Mohamed Khider, Biskra, Algeria
Email: mohakouider@gmail.com
Second email: ridha.kouider@univ-biskra.dz

Abstract

Kouider function with basis a is a new numerical function which presented by Kouider (2021,[1]). In this paper we interesting in study this function for all natural number. In order to discover his properties. Therefore we present a new number called kouider number with bases a which we go developed in this study.

Keywords: *Kouider function with basis a.*

.

1. Introduction

Kouider [2] introduced a new function called the Kouider function with basis a. He is defined as follows.

Let $[x]$ be integer part of real number x .The Kouider function with basis a is numeric function define for all $x>0$ to $\mathbb{R}$ where $s=[\ln x/\ln a]$ by:

$$K_a(x)=ax-a^{s+1}+1 \qquad (1)$$

Where $a\in\mathbb{R}_+^*-\{1\}$. He also defined the derivative function of his function by

$$K_a^{'}(x)=\ln a\left(K_a(x)-1\right) \qquad (2)$$

Where $a\in\mathbb{R}_+^*-\{1\}$.

We will use $K_a(n)$ to represent the Kouider number or Kouider sequences as follows:

$$K_a(n)=a\times n-a^{s+1}+1 \qquad (3)$$

where a is a non-zero natural number that is different to the number 1. And $n\in\mathbb{N}^*$ with s represents the integer part of the number $\ln(n)/\ln(a)$.

The following table present $K_a(n)$ the kouider number for $a=\{2;3;4;5;6;7;8;9;10\}$ with $n=\{1;2;3;\cdots\cdots;12\}$. We can form this table by using the property shown in the relation (3) . This is in order to generate Kouider numbers and know their values and behavior.

n / a	1	2	3	4	5	6	7	8	9	10	11	12
2	1	1	3	1	3	5	7	1	3	5	7	9
3	1	4	1	4	7	10	13	16	1	4	7	10
4	1	5	9	1	5	9	13	17	21	25	29	33
5	1	6	11	16	1	6	11	16	21	26	31	36
6	1	7	13	19	25	1	7	13	19	25	31	37
7	1	8	15	22	29	36	1	8	15	22	29	36
8	1	9	17	25	33	41	49	1	9	17	25	33
9	1	10	19	28	37	46	55	64	1	10	19	28
10	1	11	21	31	41	51	61	71	81	1	11	21
11	1	12	23	34	45	56	67	78	89	100	1	12
12	1	13	25	37	49	61	73	85	97	109	121	1

Table(1): Kouider numbers for $a=\{2;\cdots\cdots;12\}$ with $n=\{1;\cdots\cdots;12\}$.

It is clear that the number s defined in the relationship (3) is a natural number. The following table represents the values of the natural number s attached to each Kouider number. with base a

n / a	1	2	3	4	5	6	7	8	9	10	11	12
2	0	1	1	2	2	2	2	3	3	3	3	3
3	0	0	1	1	1	1	1	1	2	2	2	2
4	0	0	0	1	1	1	1	1	1	1	1	1
5	0	0	0	0	1	1	1	1	1	1	1	1
6	0	0	0	0	0	1	1	1	1	1	1	1
7	0	0	0	0	0	0	1	1	1	1	1	1
8	0	0	0	0	0	0	0	1	1	1	1	1
9	0	0	0	0	0	0	0	0	1	1	1	1
10	0	0	0	0	0	0	0	0	0	1	1	1
11	0	0	0	0	0	0	0	0	0	0	1	1
12	0	0	0	0	0	0	0	0	0	0	0	1

Table(2): Values of s related to $K_a(n)$ for $a=\{2;\cdots\cdots;12\}$ and $n=\{1;\cdots\cdots;12\}$.

In the following, we will present what we have noticed about the behavior of the Kouider numbers shown in the table (1).

We can rewrite (3) the basic formula, which defines Kouider's numbers, as follows:

$$\begin{cases} K_a(n)=a\times m+1 \\ m=(n-a^s) \end{cases} \qquad (4)$$

where $a\in\mathbb{N}^*-\{1\}$. And $n\in\mathbb{N}^*$ with $s=[\ln n/\ln a]$.

Form (4), we get that $m=0$ if $n=a^s$. Then the it's clear that the Kouider number $K_a(n)=1$.

Propertie1.1: Let $a\in\mathbb{N}-\{0;1\}$ and $n\geq 1$ we have

1) $K_a(n+1)=K_a(n)+a$ for $a^s\leq n<a^{s+1}$ (5)
2) $K_a(n+1)=1$ if $s=1$.
3) $K_a(1)=1$ (6)

Proof: ¨Proof of (5). By the formula (4), we have

$$K_a(n+1)=a\times(n+1-a^s)+1=a\times(n-a^s)+1+a$$

Then under (4) we find

$$K_a(n+1)=K_a(n)+a.$$

Proof of (6). We have for $n=1$ we get $s=0$ then we found $m=0$ and under (4) we find $K_a(1)=1$.

From the table(1), we can notice that some Kouider numbers are repeated between a^s and a^{s+1} , two successive exponents of base a . For example, note the following Kouider numbers with base 2 $K_2(n)$ or 3 for $n=\{1;\cdots\cdots;24\}$:

1, 1,3,1,3,5,7,1,3,5,7,9,11,13,15,1,3,5,7,9,11,13,15,17,…..

2^0 2^1 2^2 2^3 2^4

For numbers $K_2(n)$ we note that the number of $K_2(n)$ numbers between 2^s and 2^{s+1} is 2^s for $2^s \le n \le 2^{s+1}-1$ or is $2^{s+1}-2^s$. Also, the number of recurring numbers of $K_2(n)$ for $2^s \le n \le 2^{s+1}-1$ is $2^{s-1} = 2^s - 2^{s-1}$. Once we read $K_2(n)$ numbers, we can say that they are odd numbers

In this context, is it possible to find a value for the number $s = [\ln n / \ln 2]$ that includes all the numbers $K_2(n)$ written in the form (4).?.

In general from Table (1), it can be said that the number of $K_a(n)$ numbers between a^s and a^{s+1} for $a^s \le n \le a^{s+1}-1$ is $a^{s+1} - a^s = a^s(a-1)$. Also, the number of recurring numbers of $K_a(n)$ for $a^s \le n \le a^{s+1}-1$ is $a^s - a^{s-1} = a^s(1-a^{-1})$. They all represent a numerical sequence defined for every two natural numbers $n \ge 1$ and $a \ge 2$ by:

$$U_a(n) = a \times (n - a^s) + 1 \qquad (4)$$

where $s = [\ln n / \ln a]$ with $a^s \le n \le a^{s+1}-1$.

The following table shows what the numbers $K_a(n)-1$ will be:

n / a	1	2	3	4	5	6	7	8	9	10	11	12
2	**0**	**0**	**2**	**0**	**2**	**4**	**6**	**0**	**2**	**4**	**6**	**8**
3	**0**	**3**	**0**	**3**	**6**	**9**	**12**	**15**	**0**	**3**	**6**	**9**
4	**0**	**4**	**8**	**0**	**4**	**8**	**12**	**16**	**20**	**24**	**28**	**32**
5	**0**	**5**	**10**	**15**	**0**	**5**	**10**	**15**	**20**	**25**	**30**	**35**
6	**0**	**6**	**12**	**18**	**24**	**0**	**6**	**12**	**18**	**24**	**30**	**36**
7	**0**	**7**	**14**	**21**	**28**	**35**	**0**	**7**	**14**	**21**	**28**	**35**
8	**0**	**8**	**16**	**24**	**32**	**40**	**48**	**0**	**8**	**16**	**24**	**32**
9	**0**	**9**	**18**	**27**	**36**	**45**	**54**	**63**	**0**	**9**	**18**	**27**
10	**0**	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**0**	**10**	**20**
11	**0**	**11**	**22**	**33**	**44**	**55**	**66**	**77**	**88**	**99**	**0**	**12**
12	**0**	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**0**

Table(3): Numbers $K_a(n)-1$ for $a = \{2;\cdots\cdots;12\}$ with $n = \{1;\cdots\cdots;12\}$.

We note that the values of the numbers $K_a(n)-1$ are multiples of base a with $K_a(a^s)-1=0$ between a^s and a^{s+1} for $a^s \le n \le a^{s+1}-1$. They all represent a numerical sequence defined by $U_a(n)-1$ as on (7) for each natural number $n \ge 1$ with $a \ge 2$.

Propertie1.2: Let $a \ge 2$ and $a^s \le n < a^{s+1}$ we have

$$\sum_{i=a^s}^{a^{s+1}-1} 1 = a^s(a+1) \qquad (7)$$

Proof: ¨Proof of (7). Let us know the following sequence $V_a(n) = 1$ for $a^s \le n \le a^{s+1}-1$. We mentioned earlier that the number of boundaries between a^s and a^{s+1} is $a^s(a+1)$. Then,

$$\sum_{i=a^s}^{a^{s+1}-1} 1 = \underbrace{1+1+\cdots+1}_{a^s(a+1)} = a^s(a+1) \times 1 = a^s(a+1).$$

Referring to Table 3, specifically for the Kouider numbers with base 2, you will find that all of them represent even numbers. That is why you will find that the union of the Kouider numbers with base 2 written as $K_2(n)-1$ and $K_2(n)$ will give us the set of natural numbers, this means $\{K_2(n)-1\} \cup \{K_2(n)\} = \mathbb{N}$¨ for all $a^s \le n \le a^{s+1}-1$. The question is, is there a natural number s for which the union of the two sets $\{K_2(n)-1\}$ and $\{K_2(n)\}$ gives us all the natural numbers $\mathbb{N}$?. For example, for $s=3$ we will find that for $2^3 \le n \le 2^4-1$:

$\{K_2(n)\} = \{1,3,5,7,9,11,13,15\}$ and

$\{K_2(n)-1\} = \{0,2,4,6,8,10,12,14\}$

then

$\{K_2(n)-1\} \cup \{K_2(n)\} = \{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15\}$.

Propertie1.3: Let $a=2$ and $a^s \le n < a^{s+1}$ we have

$$\sum_{i=2^s}^{2^{s+1}-1} i = 3 \times 2^{s-1}(2^s - 1) \qquad (8)$$

Proof: ¨Proof of (8). We mentioned that

$$\sum_{i=2^s}^{2^{s+1}-1} i = 2^s + \cdots + 2^{s+1} - 1$$

The previous sum can also be written in

$$\sum_{i=2^s}^{2^{s+1}-1} i = (2^{s+1}-1) + (2^{s+1}-2) + \cdots + 2^s$$

Then

$$\sum_{i=2^s}^{2^{s+1}-1} i = (2^{s+1}-1) + (2^{s+1}-2) + (2^{s+1}-3) + \cdots + (2^{s+1}-2^s)$$

$$= \underbrace{2^{s+1} + 2^{s+1} + \cdots + 2^{s+1}}_{2^s} - (1+2+3+\cdots+2^s)$$

$$= 2^{s+1} \times 2^s - 2^{s-1}(1+2^s) = 2^{2s+1} - 2^{2s-1} - 2^{s-1}$$

After simple calculations we will find that

$$\sum_{i=2^s}^{2^{s+1}-1} i = \frac{3}{2} \times 2^s (2^s - 1)$$

Properties1.4: For all natural number $n \ge 1$ and $s = [\ln n / \ln 2]$. We have :

1) $K_2(n) = 2n - M_{s+1}$

where $M_{s+1} = 2^{s+1} - 1$ is Mersenne numbers

2) For $2^s \le n \le 2^{s+1}-1$ we get

$$\sum_{i=2^s}^{2^{s+1}-1} i = \frac{3}{2} \times M_s \times (M_s + 1) \qquad (9)$$

3) under (7) for $a=2$ we have

$$\sum_{i=2^s}^{2^{s+1}-1} 1 = 2^s = M_s + 1 \qquad (10)$$

where $M_s = 2^s - 1$ is Mersenne numbers for $s = [\ln n / \ln 2]$

Proof: The validity of the two properties (9) and (10) can be proven using Mersenne numbers and some simple mathematical calculations in the two relations (7) for $a=2$ and (8) respectively.

Propertie1.5: Let $2^s \le n < 2^{s+1}$ and $s = [\ln n / \ln 2]$ we have

$$\sum_{i=2^s}^{2^{s+1}-1} K_2(i) = M_s{}^2 - 1 \qquad (11)$$

where M_s is Mersenne numbers

Proof: ¨Proof of (11). For $n \ge 1$ and $s = [\ln n / \ln 2]$. We have

$$K_2(n) = 2n - 2^{s+1} + 1$$

We point out that we will prove that property (11) is true for $2^s \le n < 2^{s+1}$. As we mentioned before, for $2^s \le n < 2^{s+1}$. The number $s = [\ln n / \ln 2]$ will be constant. Then

$$\sum_{i=2^s}^{2^{s+1}-1} K_2(i) = \sum_{i=2^s}^{2^{s+1}-1} (2i - 2^{s+1} + 1) = 2 \sum_{i=2^s}^{2^{s+1}-1} i - (2^{s+1}-1) \sum_{i=2^s}^{2^{s+1}} 1.$$

Using (7) for $a=2$ and (8) we get

$$\sum_{i=2^s}^{2^{s+1}-1} K_2(i) = 3 \times 2^s (2^s - 1) - (2^{s+1}-1) 2^s$$

For M_s is Mersenne numbers, we have

$$\sum_{i=2^s}^{2^{s+1}-1} K_2(i) = (M_s+1)(M_s-1) = M_s^2 - 1$$

With $s = [\ln n / \ln 2]$ is constant.

We can say that the sum (11) is the same as the following sum

$$1+3+5+\cdots+2m+1 = M_s^2 - 1 \qquad (13)$$

Propertie1.6: Let $m \in \mathbb{N}^*$. We have

$$\sum_{i=1}^{m} i = 2M_s^2 - M_s - 3 \qquad (14)$$

where M_s is Mersenne numbers with $s = [\ln n / \ln 2]$ for $2^s \leq n < 2^{s+1}$

Proof: Proof of (14). For $m \geq 1$. We have

$$\sum_{i=1}^{m} i = 1+2+3+4+5+\cdots+m$$

We said earlier that the Kouider numbers with base 2, which are written in one of the two forms $K_2(n)-1$ and $K_2(n)$, are numbers whose union gives the set of natural numbers for $2^s \leq n < 2^{s+1}$ and $s = [\ln n / \ln 2]$. Where s is a constant number large enough to include most of the natural numbers!!!.

So, the previous sum can be written in the following form

$$\sum_{i=1}^{m} i = \sum_{i=2^s}^{2^{s+1}-1} (2K_2(i)-1) = 2\sum_{i=2^s}^{2^{s+1}-1} K_2(i) - \sum_{i=2^s}^{2^{s+1}-1} 1$$

Using (10) and (11) we get

$$\sum_{i=1}^{m} i = \sum_{i=2^s}^{2^{s+1}-1} (2K_2(i)-1) = 2(M_s^2-1) - (M_s+1)$$

Then

$$\sum_{i=1}^{m} i = 2M_s^2 - M_s - 3.$$

Next, by the two properties (1.6) and (1.5), we conclude that

$$2+4+6+\cdots+2m = \sum_{i=1}^{m} i - \sum_{i=2^s}^{2^{s+1}-1} K_2(i)$$

Using the two relations (13) and (14), we find that

$$2+4+6+\cdots+2m = M_s^2 - M_s - 2 \qquad (15)$$

Furthermore we present a new fast algorithm to calculate $K_a(n)$. The algorithm is used to compute The Kouider Numbers with basis a by an program R .

```
Kouider.R<-function(n,a)
{t=log(n)/log(a)
s=trunc(t)
K=a*n-a^(s+1)+1
print(K)
print("Kouider number is successful")
}
```

Example 1.1:

Let us calculate $K_2(n)$ for $n = 10^6$ and $n = 10^{12}$. Using the algorithm given earlier, we find the following results

```
> Kouider.R(10^6,2)
[1] 951425
[1] "Kouider number is successful"
""""""""""""""""""""""""""""""""""""""""""""""""""""""

> Kouider.R(10^12,2)
[1] 900488372225
[1] "Kouider number is successful"
""""""""""""""""""""""""""""""""""""""""""""""""""""""
```

2. Kouider Triangle

Referring to table (1), we will find the form of a triangle of Kouider numbers $K_a(n)$ with base a, which is equal to 1, $K_a(n) = 1$. And we marked it in red. We will find that it is formed in one case, which is $K_a(n) = 1$ for $a = n$ with $s = 0$ this is what we will find in table (2). Besides that under (6) we have $K_a(1) = 1$.

Although the number $s = [\ln n / \ln a]$ does not exist for $a = n = 1$. However, it is possible to calculate the number $K_1(1)$ using the relationship (4) as

$$K_1(1) = 1 \times (1 - 1^s) + 1 = 1 \qquad (16)$$

Thus, we can define the Kouider triangle as shown in the following table

n / a	1	2	3	4	5	6	7	8	9	10	11	12
1	1											
2	1	1										
3	1	4	1									
4	1	5	9	1								
5	1	6	11	16	1							
6	1	7	13	19	25	1						
7	1	8	15	22	29	36	1					
8	1	9	17	25	33	41	49	1				
9	1	10	19	28	37	46	55	64	1			
10	1	11	21	31	41	51	61	71	81	1		
11	1	12	23	34	45	56	67	78	89	100	1	
12	1	13	25	37	49	61	73	85	97	109	121	1

Table(3): Kouider triangle for $a = \{1;\cdots\cdots;12\}$ with $n \leq a$.

It is worth noting that the Kouider numbers within the values of Kouider's triangle all share a single value for s, which is $s = 0$.

Below we will present some properties related to the Kouider numbers for $s = 0$

Propertie2.1: Let $a \leq n$ and $n \in \mathbb{N}^*$. Under (4) we have

$$\begin{cases} K_a(n) = a \times (n-1) + 1 & if \quad a < n \\ K_a(n) = 1 & if \quad a = n \end{cases} \qquad (17)$$

Proof: Proof of (17). For $a \leq n$ then,

$$\frac{\ln a}{\ln n} \leq 1$$

For $s = [\ln n / \ln a]$, implies

$$\begin{cases} s = 1 & if \quad a < n \\ s = 0 & if \quad a = n \end{cases}$$

Then , Under (4) we have

$$\begin{cases} K_a(n) = a \times (n-1) + 1 & if \quad a < n & for \quad s = 0 \\ K_a(n) = 1 & if \quad a = n & for \quad s = 1 \end{cases}$$

In general, the relationship (17) is always achieved for $a \leq n$ and $n \in \mathbb{N}^*$. That is, any Kouider number found inside Kouider's triangle achieves the relationship (17). See table (4)

Properties2.2: Author relations from table (4)

1) $K_a(a^s) = 1 \quad with \quad a = n \quad for \quad s = 1$

2) $K_a(1) = 1 \quad with \quad a < n \quad for \quad s = 0$

3) And with $s = 0$ for $n \geq 1$ where $a \neq n$, we have

$$\begin{cases} K_3(2) = 4 & for \quad a = 3 \\ K_a(n+1) = K_a(n) + K_{a-1}(2) & for \quad 4 \leq a \leq n-1 \end{cases} \qquad (18)$$

4) And for each base a and $n \in \mathbb{N}^*$ where $a < n$ with

$a \geq 3$ for $s=0$, we find that

$$\begin{cases} K_3(2)=4 \\ K_a(n+1)=K_a(1)+nK_{a-1}(2) \end{cases} \quad (19)$$

4) For $2 \leq a \leq n$ we have

$$\begin{cases} K_2(2)=1 \quad for \quad a=2 \quad with \quad s=1 \\ K_a(2)=K_a(1)+a \quad for \quad a=3,\ldots,n \quad with \quad s=0 \end{cases} \quad (20)$$

4) Also with $s=0$, and for each base $a=\{3,\ldots,n-1\}$ where $n \in \mathbb{N}^*$ we have

$$K_a(n+1)=K_a(n)+a \quad (21)$$

5) Under (17) with $s=0$, and for each base $a=\{3,\ldots,n-1\}$ where $n \in \mathbb{N}^*$ we have

$$\begin{cases} K_a(n+1)=an+1 \quad for \quad s=0 \\ K_a(n+1)=1 \quad for \quad s=1 \end{cases} \quad (22)$$

Proof:

1) Proof of (18). For $a<n$ with $s=0$ and by using the formula(17). We have,

$K_a(n+1)=a(n+1-1)+1$

$=a(n-1)+1+a$

$=a(n-1)+1+a-1+1$

$=a(n-1)+1+(a-1)(2-1)+1,$

Then $K_a(n+1)=K_a(n)+K_{a-1}(2)$ with $a \leq n-1$ where $n \geq 1$. Since $K_a(1)=1$ and $K_1(1)=K_2(2)=1$. Then for $3 \leq a \leq n-1$, we will find this numerical sequence

$$\begin{cases} K_3(2)=4 \quad for \quad a=3 \\ K_a(n+1)=K_a(n)+K_{a-1}(2) \quad for \quad 4 \leq a \leq n-1 \end{cases}$$

2) Proof of (19). In order to prove the validity of the relationship , we will apply the property (18) with the values of the natural number $a=\{4,5,6\}$.

Then for $a=4$ we get

$K_3(2)=4$

$K_4(1)=K_4(1)+0 \times K_3(2)$

$K_4(2)=K_4(1)+K_3(2)=K_4(1)+K_3(2)$

$K_4(3)=K_4(2)+K_3(2)=K_4(1)+2K_3(2)$

And for $a=5$,

$K_5(1)=K_5(1)+0 \times K_4(2)$

$K_5(2)=K_5(1)+K_4(2)$

$K_5(3)=K_5(2)+K_4(2)=K_5(1)+2K_4(2)$

$K_5(4)=K_5(3)+K_4(2)=K_5(1)+3K_4(2)$

Next with $a=6$ we have,

$K_6(1)=K_6(1)+0 \times K_5(2)$

$K_6(2)=K_6(1)+K_5(2)$

$K_6(3)=K_6(2)+K_5(2)=K_6(1)+2K_5(2)$

$K_6(4)=K_6(3)+K_5(2)=K_6(1)+3K_5(2)$

$K_6(5)=K_6(4)+K_5(2)=K_6(1)+4K_5(2)$

So in general we can find that $a \geq 3$ for $s=0$

$$\begin{cases} K_3(2)=4 \\ K_a(n+1)=K_a(1)+nK_{a-1}(2) \end{cases}$$

3) Proof of (19). To prove the relationship , we will use the same techniques we used to demonstrate the property(19). Then we have

$K_2(2)=K_2(1)=1$

$K_3(2)=4=K_3(1)+3$

$K_4(2)=5=K_4(1)+4$

$K_5(2)=6=K_5(1)+5$

$K_6(2)=7=K_6(1)+6$

$\vdots$

$\vdots$

$K_a(2)=1+a=K_a(1)+a$

Then we conclude

$$\begin{cases} K_2(2)=1 \quad for \quad a=n=2 \\ K_a(2)=1+a \quad for \quad a=3,\ldots,n \end{cases} \quad (23)$$

4) Proof of (21). We will use the relationship (17) with $s=0$. We have

$K_a(n+1)=a \times (n+1-1)+1=a \times (n-1)+1+a$

$=K_a(n)+a$

Then

$K_a(n+1)=K_a(n)+a$

4) Proof of (22). To prove the validity of (22) we use the relationship (17). And of course, after simple operations, we can prove the relationship (22).

Propertie2.3: With $s=0$, we have

$$\begin{cases} K_3(2)=4 \\ K_a(n+1)=1+nK_{a-1}(2) \end{cases} \quad (24)$$

Proof: Proof of (24). Merely, under (19) and $K_a(1)=1$ we get

$$\begin{cases} K_3(2)=4 \\ K_a(n+1)=1+nK_{a-1}(2) \end{cases}$$

In another way we have,

$1+nK_{a-1}(2)=1+n\left((a-1)(2-1)+1\right)$

$=1+an$

$=a(n+1-1)+1$

$=K_a(n+1).$

Properties2.2: With $s=0$, and under(17) if $n \geq 1$, we have

1) $K_{a+1}(n)=K_a(n)+(n-1) \quad (25)$

2) $K_{a+1}(n+1)-K_a(n+1)=n \quad (26)$

3) With $s=0$,for $a=n$ we have

$$K_n(n-1)=(n-1)^2 \quad (27)$$

Proof:

1) Proof of (25). Merely, under (17) we get

$K_{a+1}(n)=(a+1)\times(n-1)+1=a \times (n-1)+(n-1)+1$

$=a \times (n-1)+1+(n-1)$

Then

$$K_{a+1}(n)=K_a(n)+(n-1)$$

2) Proof of (26). Using the relationship (17),

$K_{a+1}(n+1)=(a+1)\times(n+1-1)+1=(a+1)\times n+1$

$=an+n+1=K_a(n+1)+n$

$K_{a+1}(n+1)-K_a(n+1)=n$

Then

$$K_{a+1}(n+1)=K_a(n+1)+n$$

2) Proof of (27). The relationship (27) can be checked by using the relationship (17). We have indicated this relationship in green in the table(4). You will notice them clearly, which comes immediately before the number 1.

Conclusion

In this paper we have introduced a new form for the natural numbers according to a new formula. We have called these numbers the Kouider numbers with base a, depending on the Kouider function with base a. It is worth noting that these numbers did not come from imagination, but rather as a result of a realistic story that took place in the first century BC. To view it, you can read the reference [1].

We provided some properties of the new numerical sequence, and we reached very surprising results related to it. We also found a new distribution of the natural numbers according to the formula, which may help in studying the prime numbers, which we will address in our next work.

Reference

[1] Kouider, Mohammed Ridha, "The Josephus Numbers" (August 7, 2019) . http://dx.doi.org/10.2139/ssrn.3433635

[2] Kouider, Mohammed Ridha, "Kouider Function have Basis a", (February 13, 2021). http://dx.doi.org/10.2139/ssrn.3785337