Higher Order Belief Divergence

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Abstract

In this paper, a novel belief divergence, higher order belief Jensen-Shannon divergence is proposed to measure the discrepancy between BPAs in Dempster–Shafer evidence theory.

Keywords: Dempster–Shafer evidence theory, Belief divergence, Higher order, Fractal, Multi-source information fusion, Pattern classification.

1. The proposed method

Definition 1 (Higher order basic probability assignment). Let a frame of discernment be $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ with its powerset 2^Θ, and let m be a BPA on Θ. A novel BPA, higher order basic probability assignment (HOBPA) m^h is defined as

$$\begin{aligned} m^h(A_i) &= \sum_{A_i \subseteq A_j} h^{||A_j|-|A_i||} \frac{m(A_j)}{(h+1)^{|A_j|} - h^{|A_j|}} \\ &= \sum_{A_i \subseteq A_j} \frac{m^{h-1}(A_j)}{(h+1)^{|A_j|} - h^{|A_j|}}, \end{aligned} \tag{1}$$

where h is the order coefficient, and m^0 denotes the zeroth order of m, i.e., m itself; A_i represents any hypothesis in 2^Θ expect the empty set $\emptyset$; A_j is hypotheses in 2^Θ that make A_i be the subsets of A_j; $|A_i|$ represents the cardinality of A_i and $|A_j|$ represents the cardinality of A_j.

Definition 2 (HOBJS divergence) Let a frame of discernment be $\Theta = \{\theta_1, \theta_2, \cdots, \theta_N\}$ with powerset 2^Θ, and let m_1 and m_2 be two BPAs on Θ. Higher order belief Jensen-Shannon (HOBJS) divergence between m_1 and m_2 is defined as

$$\begin{aligned} D^h_{BJS}(m_1||m_2) = &\frac{1}{2} \sum_{A_i \subseteq \Theta} m^h_1(A_i) \log \frac{2m^h_1(A_i)}{m^h_1(A_i) + m^h_2(A_i)} \\ &+ \frac{1}{2} \sum_{A_i \subseteq \Theta} m^h_2(A_i) \log \frac{2m^h_2(A_i)}{m^h_1(A_i) + m^h_2(A_i)}, \end{aligned} \tag{2}$$

where h is the order coefficient, and A_i represents any hypothesis in 2^Θ except the empty set $\emptyset$.

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