Einstein's field equation of a quantum particle using wave function

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Abstract

Constructed an equation in context of General Relativity by employing the conjunction of wave function's derivatives and Schrodinger's Equation, confirming its validity by reducing the parameters to Planck's scale. The results we get from our fabricated equation is somehow providing the satisfactory solution in terms of its structure and dimension which corresponds to Einstein's field equation. The equation considers both mass and mass-less particles and determines their wavelengths and potential energy respectively. We have also discussed about the behavior of the equation when it is reduced to Planck's unit which leads to the accumulation of Newtonian values.

Keywords: quantum gravity; Einstein Field Equation; Planck's gravity; Wave Function; Gravitational Constant

1. Introduction:

General Relativity (GR) has passed every astrophysical test thrown at it, involving the situations that Einstein himself couldn't have imagined. GR is one of the remarkable theories of our time which elaborates phenomenon of gravity in a more detailed and beautiful way for almost all the astronomical object but, from the four fundamental forces of our universe, only gravity is lacking quantum description and if we use GR, it gets useless in describing gravity at quantum scale. To describe the phenomenon of gravity at quantum scale several theories were made such as quantization of general theory of relativity, loop quantum gravity, gauged supersymmetry and supersymmetric string theory, even though these theories are successful in describing gravity in quantum realm nonetheless on certain unfeasible conditions they either require extra dimensions or symmetries that is physically impossible in our 3+1 dimensional universe furthermore all those theories are not renormalizable. To quantize gravity, one can quantize space-time itself, Loop Quantum Gravity theory describes that a network of quantum loops of gravitational fields makes up space. However, in string theory, gravity is described by using a vibrating string whose attributes commensurate to hypothetical quantum mechanical particle graviton, the so called "source" of gravitational force. But here we will focus on the sub-result of loop quantum gravity which suggests that distance, area and volume has a minimum quantity i.e., derived from the Planck's distance and time also has a minimum quantity of Planck's time. Gravitation is merely one of the excitations of a string (or other stretched entity) that is living over a background metric space in string theory. According to what I understand, early developments of non-perturbative definition of theories, like M theory, also depend on the existence of such background metric space over which the theory is established for the development and its rendition. Thus, for physicists, the problem of quantum gravity is now reduced to an aspect of the problem of understanding what is the mysterious nonperturbative theory that has perturbative string theory as its perturbation expansion, and how to extract information on Planck scale physics from it [1]. In the end, we learn that a theory that can forecast events in the physical realm where both theories are relevant—in the regime of Planck scale phenomena, 10^{-33}cm—is necessary in order to have a conceptual basis capable of supporting both quantum mechanics and general relativity.

The result of the paper is confiscated through various attempt of hit and trial method which, in turn, served us a satisfactory equation that may potentially provide a notion about some uncommon connection between classical and quantum gravity. As a researcher, I think we should be open to any theories or methods which can help us solve the long-standing puzzle of gravity in the quantum realm. In the below section, I propose the analysis of wave function for a particle of mass 'm' in a certain region of space using Schrodinger's equation in contrast to Einstein's Field Equation (EFE). We formulate an equation using wave function Ψ of a particle relevant to the structure and dimensions of EFE and analyze the formulated equation for Planck scale space.

2. Modification of Schrödinger's Equation:

L. Smolin [5] stated that any correct quantum theory of gravity must have general relativity as the low energy limit and accepting this conjecture we attempt to design an equation for a quantum particle using its wave equation which will reduce to GR at low energy limit which we will accomplish by considering Planck's scale.

Now, let's considering, time independent Schrodinger's Equation (3-Dimensional, particle of mass m in a box of dimensions "$x \times y \times z$"):

$$\nabla^2\Psi\ (x,y,z) + \frac{8\pi^2 m(E-V)}{h^2}\Psi = 0$$

$$\nabla^2\Psi\ (x,y,z) = -\frac{2m(E-V)}{\hbar^2}\Psi \tag{1}$$

(Note that the terms in the brackets denote the derivatives taken with respect to the given variables.)

Using, wave-function $\Psi = Ae^{\left(\frac{i}{\hbar}(Px-Et)\right)}$, and taking its double-partial derivative with respect to time coordinate [2].

$$\frac{\partial^2\Psi\ (t)}{\partial t^2} = \frac{(-i.E)^2}{\hbar^2}\Psi$$

Now, expanding one energy term in the equation in terms of kinetic energy ($\frac{1}{2}mv^2,\ where\ "v"\ is\ the\ speed\ of\ the\ particle$) and potential energy ($V$) and leaving another term intact, we will get.

$$\frac{\partial^2\Psi\ (t)}{\partial t^2} = -\frac{(\frac{1}{2}mv^2+V)\times E}{\hbar^2}\ \Psi = -\frac{2}{\hbar^2}\left(\frac{1}{4}v^2\left(m+\frac{2V}{v^2}\right)E\right)\Psi$$

$$\rightarrow \frac{1}{v^2}\frac{\partial^2\Psi\ (t)}{\partial t^2} = -\frac{2}{\hbar^2}\left(\frac{1}{4}\left(m+\frac{2V}{v^2}\right)E\right)\Psi \tag{2}$$

Both (1) and (2) are dimensionally satisfying each other now, adding both of the equation,

$$\nabla^2\Psi\ (x,y,z) + \frac{1}{v^2}\frac{\partial^2\Psi\ (t)}{\partial t^2} = -\frac{2}{\hbar^2}\left(m(E-V)+\frac{1}{4}\left(m+\frac{2V}{v^2}\right)E\right)\Psi \tag{3}$$

Considering the right side of the equation first,

$$= -\frac{2}{\hbar^2}\left(\left(\frac{Em}{4}+\frac{EV}{2v^2}\right)+Em-Vm\right)\Psi = -\frac{2}{\hbar^2}\left(\frac{5Em}{4}+V\left(\frac{E}{2v^2}-m\right)\right)\Psi$$

$$= -\frac{2}{\hbar^2}\left(\frac{5Em}{4}+\frac{V}{4}\left(\frac{2E}{v^2}-m-3m\right)\right)\Psi \qquad \because \frac{2E}{v^2}-m=\frac{2V}{v^2}$$

$$\Longrightarrow -\frac{2}{\hbar^2}\left(\frac{5Em}{4}+\frac{V}{4}\left(\frac{2V}{v^2}\right)-\frac{3Vm}{4}\right)\Psi = -\frac{2}{\hbar^2}\left(\frac{5Em}{4}+\frac{V^2}{2v^2}-\frac{3Vm}{4}\right)\Psi$$

$$= \left(-\frac{2}{\hbar^2}\left(\frac{5Em}{4}+\frac{V^2}{2v^2}-\frac{3Vm}{4}\right)\right)\Psi \tag{4}$$

Now, considering left side of the equation,

$$= \nabla^2\Psi\ (x,y,z) + \frac{1}{v^2}\frac{\partial^2\Psi\ (t)}{\partial t^2} \tag{5}$$

According to wave equation we can write second term of the above equation as:

$$\frac{1}{v^2}\frac{\partial^2\Psi}{\partial t^2} = \frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2} = \nabla^2\Psi(x,y,z)$$

3. Result & Discussion:

So, the equation (4) can be written as: $= 2.\nabla^2\Psi$

$$2.\nabla^2\Psi = \left(-\frac{2}{\hbar^2}\left(\frac{5Em}{4}+\frac{V^2}{2v^2}-\frac{3Vm}{4}\right)\right)\Psi \Longrightarrow \nabla^2\Psi = \left(-\frac{m}{\hbar^2}.\frac{1}{4}\left(5E+\frac{2V^2}{mv^2}-3V\right)\right)\Psi \quad (6)$$

Multiplying and dividing the volume "η" of the box in right side of the equation:

$$\nabla^2\Psi = \left(-\frac{m}{4\hbar^2}.\eta\left(\frac{5E}{\eta}+\frac{V^2}{\left(\frac{1}{2}mv^2\right)\eta}-\frac{3V}{\eta}\right)\right)\Psi$$

Where, E/η is the energy density which we can denote as T_{00} using Einstein's stress-energy Tensor and $\frac{V}{\eta}$ is already a pressure term and be denoted as T_{xx} where, $x \neq 0$.[3], [4]

$$\nabla^2\Psi = \left(-\frac{m}{4\hbar^2}.\eta\left(5T_{00}+\frac{V}{\left(\frac{1}{2}mv^2\right)}T_{xx}-3T_{xx}\right)\right)\Psi$$

$$\frac{\nabla^2\Psi}{\Psi} = \left(-\frac{m}{4\hbar^2}.\eta\left(5T_{00}+\left(\frac{V}{\left(\frac{1}{2}mv^2\right)}-3\right)T_{xx}\right)\right) \quad (7)$$

where, $\hbar = \frac{h}{2\pi}$,

$\frac{\nabla^2\Psi}{\Psi}$ is reduced to $-k^2$ i.e. the negative square of wave number.

$\frac{\nabla^2\Psi}{\Psi} = -k^2 = -1/\lambda^2$, ($\lambda$ is the wavelength of the particle)

Manipulated Tensor terms seems to be the diagonal of the energy-momentum tensor. Considering this, we can rewrite equation (6) in a more generalized way for particle of mass m in a region of volume η.

$$\frac{1}{\lambda^2} = \frac{m\eta}{4\hbar^2}\sum_{\mu=0}^{1} T_{\mu\mu} \quad (8)$$

Where $T_{\mu\mu} = \begin{bmatrix} 5\ E/\eta & 0 \\ 0 & \left(\frac{V}{\frac{1}{2}mv^2}-3\right)\frac{V}{\eta} \end{bmatrix}$

Right side of equation (8) seems similar to the structure of Einstein's Field Equation i.e. $8\pi GT_{\mu\nu}/c^4$. If dimensionally we analyse the above equation with respect to right hand side of Einstein Field equation, then $G/c^4 \equiv \frac{m\eta}{\hbar^2}$. And, if we reduce the mass of particle and volume of the region (where particle exist) to Planck's scale (i.e., Planck's mass m_p & Planck's volume l_p^3) in $\frac{m\eta}{\hbar^2}$, it yields exactly G/c^4. The eruption of gravity (or GR) at Planck's scale may signifies the existence of a certain particle which generates gravity, a particle similar to graviton yet but have the composition of Planck's mass in a Planck's volume.

It somehow vaguely relates both classical and quantum mechanics with each other. We took another step toward understanding quantum gravity which may bring the problem of quantum gravity at rest and can potentially become the source of new science.

Also, we apply equation (7) on a particle with no mass like light, we get:

$$\frac{1}{\lambda^2} = \frac{1}{32\pi}\left(\frac{8\pi m\eta}{\hbar^2}\left(T_{00}+\left(\frac{V}{\left(\frac{1}{2}mv^2\right)}-3\right)\frac{V}{\eta}\right)\right) \rightarrow \frac{1}{\lambda^2} = \frac{1}{4}\left(\left(\frac{m\eta}{\hbar^2}T_{00}+\frac{m\eta}{\hbar^2}.\left(\frac{V}{\left(\frac{1}{2}mv^2\right)}\right)\frac{V}{\eta}-\frac{m\eta}{\hbar^2}.3\ \frac{V}{\eta}\right)\right)$$

$\rightarrow \frac{1}{\lambda^2} = \frac{1}{4}\left(\left(\frac{m\eta}{\hbar^2}.T_{00}+\frac{1}{\hbar^2}.\left(\frac{V^2}{\left(\frac{1}{2}c^2\right)}\right)-\frac{m\eta}{\hbar^2}.3\ \frac{V}{\eta}\right)\right)$ putting $m = 0$ for rest mass of light, we get:

$$\rightarrow \frac{1}{\lambda^2} = \frac{1}{4}\frac{1}{\hbar^2}\left(\frac{V^2}{\left(\frac{1}{2}c^2\right)}\right) \rightarrow V = \frac{\pi}{\sqrt{2}}\left(\frac{hc}{\lambda}\right)$$

We are getting potential energy of light which seems to be $\frac{\pi}{\sqrt{2}}$ times the total energy of light wave. For now, it's not known that what the potential energy of light or any massless particle depicts here but we are hoping to find out in future that's why I am writing here for further future reference.

4. Conclusion:

The paper extended the interpretation of the wave function beyond estimating the probability of a particle's position nevertheless using the wave function and Schrödinger's Equation we are able to deduce an equation similar to the structure of Einstein's Field Equation which may be able to explain the source of gravity at quantum realm, a particle with a Planck's scale property can produce gravity Furthermore, we may have unexpectedly found the potential energy of the massless particle. The whole paper is subjected to finding general relativity in the quantum realm and is seemingly successful in doing so, this is just another step in understanding quantum gravity and in my view, the equation that we have found will play a crucial role in uncovering more quantum physics from this universe. Yet, theoretically, this theory has a long way to be proved still we think it will be helpful if everyone in the scientific community could judge and use it for future reference.

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