Investigation of the gravitational wave
of the binary star system J0651+2844

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The binary star system J0651 is expected to be be one of the brightest sources of gravitational waves in our galaxy. Despite its known frequency, the radiation could not be detected so far. A new method eliminates the strong phase modulation caused by the Earth's orbit and drastically reduces the bandwidth. Therefore, the GW may be identified in the records of numerous superconducting gravimeters. The determined parameters of GW agree well with the results of previous observations in the optical domain and the predictions of relativity.

1 Introduction

Our galaxy hosts the double star system J0651+2844 with orbital period of 12.75 minutes [1]. The emitted continuous gravitational wave (GW) excites the Earth to forced oscillations und should be clearly different from the natural resonances of the Earth: The frequency is very stable, has a much smaller half-width than the natural resonances of the Earth [2] and the average amplitude remains constant for years. As the Earth rotates around the sun and around its own axis, all GWs are phase modulated with the corresponding frequencies. A GW is also amplitude modulated because each antenna has some directivity. These signatures of different frequency are important criteria to identify GW signals in the records of gravimeters.

Although GWs were detected many years ago [3], no continuous signals have yet been received [4]. Is it due to insensitive antennas or unsuitable analysis programs? In order to clarify both questions, the longest scale existing on Earth – the Earth itself – was chosen as antenna. Gravimeters measure neither the diameter nor its change, but the acceleration at the ends of a diameter. The signals are evaluated using proven standard methods of communications engineering. Objectives of this paper are:

- Do gravimeters respond to GW? Are there any quality differences?
- How can the known techniques of telecommunication be combined to detect a GW in the existing records of gravimeters?
- Can we measure the GW emitted by J0651? No one has been able to do this yet.
- Are the results consistent with previous results [1]?

2 Instrumentation and noise

Because of the frequent disturbances caused by Earthquakes, the Power Spectral Density (PSD) of superconducting gravimeters in the range around 2 mHz is about 10^{-18} m^{2}s^{-3} Hz

[8] [9]. To reduce noise, signals always go through filters during their processing. The measurable noise amplitude after the filter is calculated according to the equation

$$A_{noise} = \sqrt{PSD \cdot BW} \tag{1}$$

where BW is the bandwidth. One cannot narrow the bandwidth of the signal processing arbitrarily in order to eliminate the disturbing noise. Because that increases the necessary time span T_{min} that the filter needs to settle. The relationship (2) was first formulated by Küpfmüller [10], is reminiscent of the Heisenberg uncertainty principle and of high importance for communications engineering.

$$T_{min} \cdot BW \geq 0.5 \tag{2}$$

When searching for GWs, this means: If you want to fall below the (arbitrary) target value $A_{noise} = 10^{-14}$ m/s^2, the bandwidth of the filters must not exceed 1 nHz (equation (1)). Because of (2), gravimeters must be operated for at least 15 years and the frequency of the GW must not vary by more than 0.5 nHz during the entire period to keep the signal within the filter range.

3 Receiving and preparing the data

Gravity data have been recorded for decades and IGETS Potsdam [12] stores correspondingly long data series. Previously used for Earthquake research, the gravimeters are mounted directly on the ground and therefore respond to distant, minute ground motions. Stronger Earthquakes overload the sensors and cause data gaps of several minutes. Before release, the gaps are filled by artificial tidal signals, then the signals pass through low-pass filters with a cutoff frequency of 8.3 mHz. When searching for GWs in the frequency range around 2 mHz, the influence of the variable air mass above the gravimeter may be taken into account.

GWs differ from Earth's natural resonances by characteristic modulations (for data sets spanning several years) that allow their identification:

- The frequency of a GW is well defined (narrow linewidth) and increases slowly. The natural frequencies of the Earth are damped oscillations and differ from GW by considerably larger half-widths. There are no sudden amplitude changes.
- The frequency of a GW is phase modulated (PM) with $f_{year} = 31.688$ nHz because the Earth orbits the sun. This modulation creates additional spectral lines at a distance of $n \cdot f_{year}$. The modulation index of the PM may be calculated from the amplitudes of these sidebands.
- Amplitude and direction of polarization of continuous GWs are constant. Because the gravimeters are directionally sensitive antennas that rotate with the Earth, the received signal may be amplitude modulated at 12 or 24 hour intervals.

- The reception frequency of a GW is probably phase-modulated with $f_{day} = 11.6\ \mu$Hz because the Earth rotates around its own axis in 23.93447192 hours. After Earthquakes, one observes that the natural frequencies of the Earth split into several singlets f, $f \pm f_{day}$ and $f \pm 2 \cdot f_{day}$ [5]. There are various reasons for this: The Coriolis force as a result of the Earth's rotation produces splitting because deformations of the Earth's crust spread faster in the direction of the rotation than in the opposite direction. Then there is the elliptical shape of the Earth: the path across the poles is 67 km shorter than the equator. Whether this also applies to GW still needs to be investigated.

Thus, the criteria to be tested are fixed: frequency spacing of the spectral lines, amplitude and phase ratios, frequency drift. The database consists of the records of eight gravimeters (PE, OS, BF1+2, MB, ST, CO and CB), which recorded particularly low-noise data between the years 2009 and 2018. To ensure high frequency resolution, 120 consecutive records from each gravimeter (one month each) are concatenated into a single data chain and all tide signals below 100 μHz are removed by a high-pass filter. In a last step the center frequency of the search range is reduced from 2613.68 μHz to 150 μHz.

4 The search for GW

Eleven years ago the binary system J065133.338+284423.37 was discovered and the orbital period determined [1]. This star system consists of two white dwarfs and generates gravitational waves, which have never been detected, although the frequency is known. The goal of this investigation is: Is it possible to find spectral lines in the vicinity of 2613.67 μHz that fulfill the above list of requirements and are therefore probably produced by a GW?

No single spectrum of a gravimeter shows anomalies with sufficient S/N to indicate a GW. Therefore, the available long-term records are phase-coherently added to improve the S/N. The procedure is based on the following consideration: Seven gravimeters are located – closely adjacent – in Central Europe and one on the opposite side of the Earth in Canberra/Australia. The shortest distance L is about 12×10^6 m long and passes almost through the center of the Earth. When a GW passes by, L oscillates in the same rhythm and sinusoidally with the maximum amplitude ΔL. Since L is much smaller than the assumed wavelength of the GW ($\lambda \approx 10^{11}$ m), the strain h is calculated with the approach

$$h = \Delta L/L = h_0 \cdot sin(\omega t) \tag{3}$$

Previous estimates give values $h_0 \approx 10^{-18}$ for the binary system J0651 in our Galaxy [16]. Thus the vertical acceleration $\ddot{L}$ of the Earth's surface is

$$\ddot{L} = L \cdot \omega^2 \cdot h_0 = 12 \times 10^6\ m \cdot (2\pi \cdot 2613 \times 10^{-6}\ \mathrm{Hz})^2 \cdot 10^{-18} \approx 3.2 \times 10^{-15}\ \frac{m}{s^2} \tag{4}$$

This value almost reaches the estimated sensitivity of superconducting gravimeters (see section 2) and with a sufficiently good analysis program one should be able to detect the GW of J0651.

In the investigation [6] it is shown that the S/N improves considerably if one removes all phase modulations [7]. Therefore, one looks for spectral lines in the vicinity of 2613 μHz whose amplitude increase when one compensates the PM with f_{year}. Any GW from outside our solar system must show this PM. Now we focus on the solution near $f_{GW} = 2613.682$ μHz.

5 Influence of the PM at 2613 μHz

We always receive modulated GWs, never constant frequencies. In order to prove that a signal is a GW, as many signatures as possible must be checked and confirmed (see Section 3). Any kind of modulation generates sidebands (spectral lines next to f_{GW}) which affect the reception of the GW:

- In PM, the sidebands take their energy from the central frequency. Even if only a single frequency phase modulates the signal, several spectral lines appear on either side of f_{GW}. If one compensates the PM, these sidebands disappear and the amplitude of the central frequency increases.

- The bandwidth of the signal processing must be at least wide enough to contain all essential sidebands of the PM. This increases the noise level (see equation (1)). If all modulations are compensated, bandwidth and noise may be reduced drastically.

Theoretically, a PM signal occupies infinite bandwidth. In practice, one restricts to the strongest sideband frequencies in the vicinity of f_{GW}. This range is called the Carson bandwidth and calculated using the modulation index η. The source J0651 is close to the ecliptic and the maximum frequency shift due to the Doppler effect may reach the value

$$\Delta f_{year} = f_{GW} \cdot \left(\sqrt{\frac{c + v_{Earth}}{c - v_{Earth}}} - 1 \right) \approx 261 \; n\text{Hz}. \tag{5}$$

Then the modulation index is

$$\eta = \frac{\Delta f}{f_{modulation}} = \frac{\Delta f_{year}}{f_{year}} = 8.25 \tag{6}$$

and the signal must be processed with bandwidth

$$BW_{Carson} = 2 \cdot f_{year}(\eta + 1) = 586 \text{ nHz}. \tag{7}$$

This large area contains about 19 spectral lines of the sought-after GW plus an unnecessary amount of noise. In order to amplify and to determine the central frequency f_{GW}, we need to eliminate the PM and reduce the bandwidth. For $\eta = 8.25$ it follows from formulas (12) and (13) in [7]:

- If one compensates the PM, the associated sidebands disappear and the amplitude of the central spectral line at f_{GW} increases by the factor 2.8.
- Therefore, one may reduce the bandwidth of the signal processing from 586 nHz (equation (7)) to 1.5 nHz (equation (2)), which decreases the amplitude of the noise by a factor of 20.

Both effects multiply and improve the S/N by a factor of 3100 and thus the probability of detecting a GW.

6 The compensation of the phase modulation

In [15] it was accidentally discovered that the S/N of a GW increases when the records of European and Australian gravimeters are *subtracted* in phase. This empirical finding is confirmed in the frequency range around 2 mHz. Therefore, the records of eight low-noise gravimeters are combined: The data combination PE+OS+BF1+BF2+MB+ST+CO – 7*CB passes through an IQ bandpass filter which reduces the center frequency 2613 μHz to 150 μHz. The bandwidth of 100 μHz is sufficient to pass the essential sideband frequencies of the suspected PM. The subsequent decimation of the file length by a factor of 24 accelerates the data processing. The usual approach for a phase modulated oscillation with constant frequency is

$$y = sin(2\pi t \cdot f_{GW} + \phi_{modulation}) \tag{8}$$

The simple equation (8) contains two parameters f_{GW} and ϕ which must be adapted to the problem: The frequency f_{GW} of J0651 is not constant, but increases proportionally to the time. The phase $\phi_{modulation}$ indicates the position of the gravimeters on the surface of the Earth on their way around the sun. The non-circular orbit is approximated by the following approach:

$$\phi_y = \eta_y \cdot sin(p_y + t \cdot \omega_y) + \eta_{2y} \cdot sin(p_{2y} + t \cdot 2\omega_y) + \eta_{3y} \cdot sin(p_{3y} + t \cdot 3\omega_y) \tag{9}$$

Thereby applies $\omega_y = 2\pi \cdot f_{year}$;
η_{2y} and p_{2y} are amplitude and phase of the double frequency,
η_{3y} and p_{3y} are amplitude and phase of the triple frequency.

To model the PM in the daily rhythm, two summands are sufficient:

$$\phi_d = \eta_d \cdot sin(p_d + t \cdot \omega_d) + \eta_{2d} \cdot sin(p_{2d} + t \cdot 2\omega_d) \quad (10)$$

Thereby applies $\omega_d = 2\pi \cdot f_{day}$;
η_{2d} and p_{2d} are amplitude and phase in a 12-hour rhythm.

Translated into mathematical language, the output from the auxiliary oscillator is:

$$y = sin(2\pi t(f_{GW} + t \cdot k_{drift}) + \phi_y + \phi_d) \quad (11)$$

The equation (11) contains 17 parameters to describe the receivable GW. The initial values for J0651 are:
$f_{GW} \approx 2613.683\ \mu$Hz (frequency of the GW; depends on the year)
$k_{drift} \approx 30 \times 10^{-18}\ s^{-2}$, twice the value from [1]
Technical remark: Both phase modulation and amplitude modulation with a fixed frequency f_{mod} produce spectral lines at a distance f_{mod} above and below the carrier frequency (sidebands). These differ in their phases. The compensation method MSH [6] used in this section reacts *only* to the sidebands caused by PM. Once these are removed, the remaining sidebands may be analyzed using AM demodulation methods (see section 10).
A time-consuming iteration optimizes the 17 parameters in equation (11) and improves the S/N of the GW so much that the remaining spectral line at f_{GW} becomes unmistakable. The iteration ends when the amplitude of f_{GW} stops increasing. Then, the PM and the frequency drift of the auxiliary oscillator (in the MSH process) match the values of the original GW signal. Figure 1 shows the result.

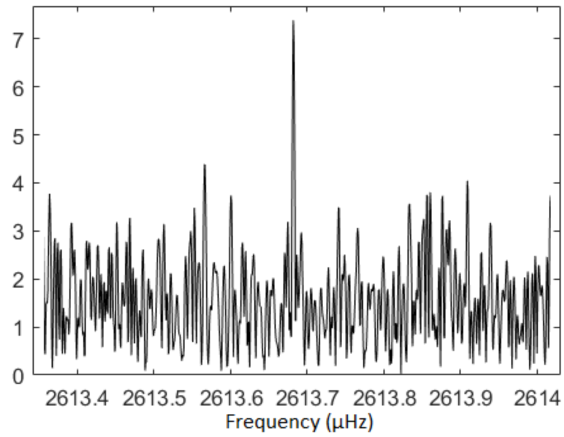

Figure 1): *Spectrum of the environment of f_{GW} after compensation of all phase modulations and the drift. Before compensation of PM, the total energy of a GW is distributed on many spectral lines and therefore disappears in the noise. After compensation, the energy is concentrated on a single line.*

The central spectral line in figure 1 has the extraordinarily narrow half-width 2.4 nHz. The corresponding Q-factor is about 10^6, far exceeding the range measured for Earth natural resonances. The frequency drift of f_{GW} is $(23.3 \pm 0.4) \times 10^{-18}$ s^{-2}. General relativity predicts orbital decay $(28 \pm 6) \times 10^{-18}$ s^{-2} due to gravitational wave radiation. Hermes [1] observed $(33.5 \pm 9.6) \times 10^{-18}$ s^{-2} from model-dependent light curve fits.

7 PM in annual rhythm

With the approach (9) one determines the slow PM (equation (9)). Due to the Doppler effect, the periodic change in the distance between the source of the GW and the Earth causes a periodic change in frequency, which is shown in Figure 2. For a perfectly circular orbit, one expects a sinusoidal shape. Since the mean maximum value of figure 2 is approximately equal to the result of equation (6), the source of the GW must be close to the plane of the ecliptic. The actual ecliptic latitude of J0651 is 5.8^0. The ecliptic longitude of the source of the GW can only be estimated inaccurately, because the maxima in figure 2 cannot be localized unambiguously.

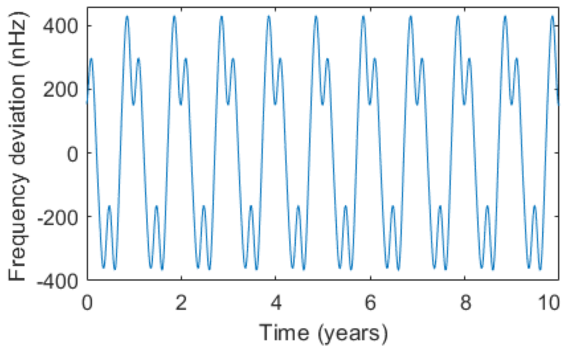

Figure 2): *The frequency deviation around f_{GW} caused by the earth's orbit should change sinusoidally. The striking deviation remains during the whole analysis period of ten years and therefore cannot be caused by the poor S/N of the GW.*

8 PM in daily rhythm

With the approach (10) one checks whether the GW is phase modulated with the period 24 hours (or shorter). A comparison of the measured parameters $\eta_d = 1.1$ and $\eta_{2d} = 2.6$ show that the PM in the rhythm of 12 hours is predominant. The small additional PM in the 24-hour rhythm is due to the asymmetric position of the gravimeters on the Earth's surface. The maximum frequency deviation $\Delta f_{day} = \eta_{2d} \cdot f_{day} = 82$ nHz (equation 6) is remarkable: The low peripheral speed of the Earth at the equator of 460 m/s causes a maximum frequency deviation of ± 1.3 nHz in the 24-hour rhythm. The measured frequency deviation Δf_{day} is 60 times larger and changes in a 12-hour rhythm. This very strong PM is not generated by the Doppler effect.

In anticipation of section 10, it must be emphasized that the MSH method *only* detects and eliminates sidebands generated by phase modulation. MSH ignores sidebands generated by other types of modulation such as amplitude modulation.

9 Checking the amplitude constancy of the GW

A spectrum contains no information whether and how the amplitude of a signal changes over time. A phase-sensitive integrator acts like an extremely narrow-band filter and answers this question. One possible method is based on the addition theorem of trigonometric functions and measures only a single frequency f_{check}. The SG data are digitized and consist of discrete values. Between successive readings z_n and z_{n+1} of the signal, a certain time interval passes (the sampling time T_s). With each step, the phase angle increases by the value $\alpha = 2\pi T_s \cdot f_{check}$. If one wants to know whether the received noise z_n, z_{n+1}, z_{n+2}.... contains a signal of frequency f_{check}, one sums the amplitudes by alternately calculating the two formulas:

$$x_{n+1} = z_n + cos(\alpha)x_n + sin(\alpha)y_n \tag{12}$$

$$y_{n+1} = cos(\alpha)y_n - sin(\alpha)x_n \tag{13}$$

The sequence of values x_n and y_n depends on the choice of parameters:

- Without an injected signal and with the initial values $x_1 = 1$, $y_1 = 0$, the formulas calculate a table of values for $x = sin(2\pi tf)$ and $y = cos(2\pi tf)$ with constant amplitude.
- Setting $x_1 = y_1 = 0$ and injecting a monochromatic signal z_n whose frequency matches f_{check}, the formulas calculate an oscillation whose amplitude increases in proportion to time.
- If the injected frequency differs from f_{check} or if the injected signal z_n varies in phase or amplitude, the output signal of the integrator is small and not linear.
- If noise is injected, the formulas calculate a low-bandwidth frequency mixture in the vicinity of f_{check} , whose amplitude fluctuates irregularly.

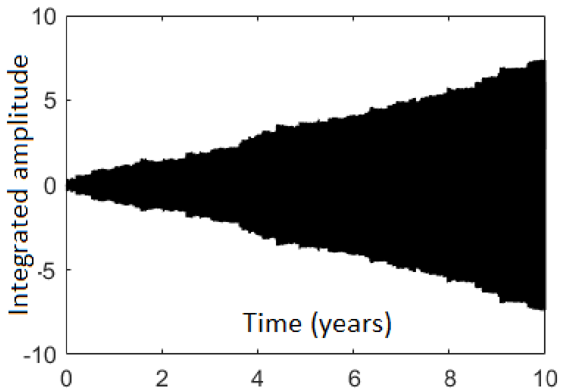

Figure 3): *After the signal of the GW is freed from all PM and the frequency drift, the frequency is constant. The slightest phase fluctuations would produce noticeable deviations from linearity. Apparently, Earthquakes cannot affect the signal even though the gravimeters are mounted directly on the ground.*

Choosing $f_{check} = 2613$ μHz and injecting the signal recorded by the gravimeters, we obtain the result shown in Figure 3. Since the integrated amplitude increases in proportion to time, the injected signal mixture contains a signal whose frequency and amplitude are constant throughout the entire ten-year period – as expected from a continuous GW.

10 Amplitude modulation in half-day rhythm

The MSH procedure determines and compensates PM. Subsequently, there are no more sidebands that can be assigned to a PM. This does not exclude that further modulations are present. If the GW is additionally amplitude modulated (AM), sidebands with corresponding frequencies still exist. If the signal is modulated with a *single* frequency f_{AM}, the spectrum shows three spectral lines with the mutual spacing f_{AM}. With a period of 12 hours, the wide gap between f_{GW} and the 23 μHz distant sidebands is filled with noise. With low S/N, the weak spectral lines can hardly be found in the noise of the many other GWs. To isolate the neccessary AM sidebands, one needs a matched filter (BW = 3 nHz) and a precise value of f_{AM}. This enables the precise measurement of the modulation frequency.

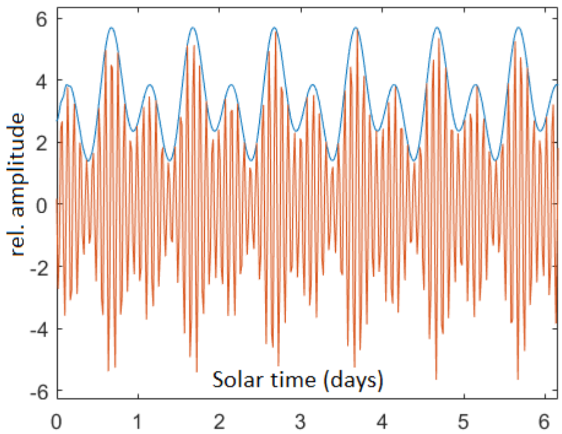

Figure 4): *Highly stretched section of the amplitude modulation of the GW. Red: Signal of the GW after frequency shift to f_{ZF} and after the matched filter. Blue: Envelope for better visualization. This fine structure remains constant during the ten-year total duration. The modulation index A_{max}/A_{min} is remarkably large. The time counting of the t-axis starts on 2009-01-01 at 0 o'clock.*

Possible AM periods of 12 and 24 hours follow from the rotation period of the Earth; other rhythms were not studied. Figure 4 shows the results:

- Rough structure: the mean amplitude is very well constant during the whole time span of ten years (±3%). That causes the linear increase of the integrated amplitude in figure 3. The small deviations are a consequence of the not perfectly rectangular filter shapes.
- Fine structure: The GW of J0651 is amplitude modulated with two time constants: 11.9673 ± 0.00005 hours (modulation index = 25%) and 23.93459 ± 0.00001 hours (modulation index = 28%).
- 18.8 ± 0.05 hours after midnight the amplitude of the GW reaches its maximum value (measured in the solar time frame starting on 2009-01-01 at 0 o'clock).

- A weaker maximum follows 10.6 hours later. The phase shift between high and low maximum remains constant during the ten-year measurement period. These details are mean values over the entire measurement period of 3652 days.
- The oscillation period of both AM deviates only 5×10^{-6} from the sidereal day length 23.93447192 hours.

The striking deviation from synodic daylength 24 hours suggests a GW source outside the solar system. A possible explanation: If one interprets the straight line between Europe and Canberra/Australia as a linear GW antenna, it rotates by 180^o per 11.9678 hours. As with all linear antennas, the reception amplitude depends on the angle between the orientation of this antenna and the propagation direction of the GW. Without knowledge of the directional pattern of this antenna, the direction to the source of the GW cannot be determined.

11 Summary and Discussion

In all records of the eight superconducting gravimeters (at different locations), one finds a spectral line at $f_{GW} = 2613.682\ \mu$Hz whose frequency and mean amplitude are strikingly constant throughout the ten-year measurement period. All modulations of this signal indicate an excitation by a GW:

- The signal is phase modulated with f_{year} and the frequency deviation Δf_{year} corresponds to the Doppler shift calculated from the motion of the Earth in orbit.
- The signal is also phase modulated at a much higher frequency, which matches very well with the half sidereal day duration. Therefore, it is very unlikely that the signal is generated in the Sun or in near-Earth space.
- The frequency deviation of this PM in a 12-hour rhythm far exceeds the limit due to the low circumferential velocity at the equator. In addition, this PM would have to have a 24-hour rhythm. What value would a measurement have if it didn't deliver at least one surprising result?
- The signal is amplitude modulated at the rhythm of half the sidereal day duration. Rotating dipole antennas provide such signals.
- The spectral line f_{GW} does not correspond to any of the many natural resonances of the Earth, differs by a much smaller half-width and is not split into several modes. Therefore it can be excluded that f_{GW} has anything to do with the natural resonances of the Earth.
- Integration of the signal with a phase-sensitive method confirms an extremely small variation of the phase during the ten-year measurement period. This rules out excitation by non-synchronized Earthquakes.

- In January 2009, J0651 generated a GW of frequency 2613.6835(2) μHz.
- The frequency drift of f_{GW} is $(23.3 \pm 0.4) \times 10^{-18}$ s^{-2}. General relativity predicts orbital decay $(28 \pm 6) \times 10^{-18}$ s^{-2} due to gravitational wave radiation.

There is no physical reason that any resonator (Weber bar) is necessary to receive GWs. As with electromagnetic waves, a resonant antenna improves reception, but is not a prerequisite. 50 years of unsuccessful attempts to receive GW with *Weber bars* of various designs allow only one conclusion: it is not the design of the antenna that determines success, but the principle of data evaluation.

The results open a new frontier for experimentalists to explore and theorists to explain.

12 Data availability

The recordings of all gravimeters may be downloaded free of charge from GFZ Potsdam [12]. A detailed description of the IGETS data base, the IGETS products and the registration procedure are described in [13]. The raw data are formatted as ASCII files and cover one month each. They may be requested as MATLAB files from the author.

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