Proofs using my definition series 3

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Abstract

The purpose of this short paper is to provide a proof based on my definition series to determine the correctness of my previous papers No.19, No.89.

General comments

The proofs of No.19, No.89 were established by my definition series.

No.19 $\prod_{n=2}^{\infty}\left(1+\frac{i^{(P_n-1)}}{P_n}\right)=\frac{1}{2}\cdot\prod_{n=2}^{\infty}\left(1-\frac{i^{(P_n-1)}}{P_n}\right)=\frac{2}{\pi}$

$P_1=2,\quad P_2=3=-2,$
$P_3=5=0,\quad P_4=7=2,$
$P_5=11=1,\quad P_6=13=-2,$
$P_7=17=-3,\quad P_8=19=-1,\cdots$

～Proof～

$$\prod_{n=2}^{\infty}\left(1+\frac{i^{(P_{n-1})}}{P_n}\right)=\frac{\left\{\left(1+\frac{i^{(P_1-1)}}{P_1}\right)\left(1+\frac{i^{(P_2-1)}}{P_2}\right)\left(1+\frac{i^{(P_3-1)}}{P_3}\right)\left(1+\frac{i^{(P_4-1)}}{P_4}\right)\left(1+\frac{i^{(P_5-1)}}{P_5}\right)\right\}^{\left(\frac{\infty-3}{5}\right)}\times\left(1+\frac{i^{(P_1-1)}}{P_1}\right)\left(1+\frac{i^{(P_2-1)}}{P_2}\right)\left(1+\frac{i^{(P_3-1)}}{P_3}\right)}{\left(1+\frac{i^{(P_{1-1})}}{P_1}\right)}$$

$$=\frac{\left\{\left(1+\frac{2^{(2-1)}}{2}\right)\left(1+\frac{2^{(-2-1)}}{-2}\right)\left(1+\frac{2^{(0-1)}}{0}\right)\left(1+\frac{2^{(2-1)}}{2}\right)\left(1+\frac{2^{(1-1)}}{1}\right)\right\}^{\left(\frac{\infty-3}{5}\right)}\times\left(1+\frac{2^{(2-1)}}{2}\right)\left(1+\frac{2^{(-2-1)}}{-2}\right)\left(1+\frac{2^{(0-1)}}{0}\right)}{\left(1+\frac{2^{(2-1)}}{2}\right)}$$

$$=\frac{\left(1+\frac{2^{(2-1)}}{2}\right)\left(1+\frac{2^{(-2-1)}}{-2}\right)\left(1+\frac{2^{(0-1)}}{0}\right)}{\left(1+\frac{2^{(2-1)}}{2}\right)}=\frac{\left(1+\frac{2}{2}\right)\left(1+\frac{2^2}{-2}\right)\left(1+\frac{1}{0}\right)}{\left(1+\frac{2}{2}\right)}=(1-2)\left(1+\frac{1}{0}\right)=-1\times\frac{1\times0+1}{0}=\frac{-1}{0}=\pm\infty=3=-2=3=\frac{6}{2}=\frac{1}{2}=\frac{2}{4}=\frac{2}{\pi}$$

$$\frac{1}{2}\cdot\prod_{n=2}^{\infty}\left(1-\frac{i^{(P_n-1)}}{P_n}\right)=\frac{\left\{\left(1-\frac{i^{(P_1-1)}}{P_1}\right)\left(1-\frac{i^{(P_2-1)}}{P_2}\right)\left(1-\frac{i^{(P_3-1)}}{P_3}\right)\left(1-\frac{i^{(P_4-1)}}{P_4}\right)\left(1-\frac{i^{(P_5-1)}}{P_5}\right)\right\}^{\left(\frac{\infty-3}{5}\right)}\times\left(1-\frac{i^{(P_1-1)}}{P_1}\right)\left(1-\frac{i^{(P_2-1)}}{P_2}\right)\left(1-\frac{i^{(P_3-1)}}{P_3}\right)}{2\times\left(1-\frac{i^{(P_1-1)}}{P_1}\right)}$$

$$=\frac{\left\{\left(1-\frac{2^{(2-1)}}{2}\right)\left(1-\frac{2^{(-2-1)}}{-2}\right)\left(1-\frac{2^{(0-1)}}{0}\right)\left(1-\frac{2^{(2-1)}}{2}\right)\left(1-\frac{2^{(1-1)}}{1}\right)\right\}^{\left(\frac{\infty-3}{5}\right)}\times\left(1-\frac{2^{(2-1)}}{2}\right)\left(1-\frac{2^{(-2-1)}}{-2}\right)\left(1-\frac{2^{(0-1)}}{0}\right)}{2\times\left(1-\frac{2^{(2-1)}}{2}\right)}$$

$$=\frac{\left(1-\frac{2^{(2-1)}}{2}\right)\left(1-\frac{2^{(-2-1)}}{-2}\right)\left(1-\frac{2^{(0-1)}}{0}\right)}{2\times\left(1-\frac{2^{(2-1)}}{2}\right)}=\frac{\left(1-\frac{2}{2}\right)\left(1-\frac{2^2}{-2}\right)\left(1-\frac{1}{0}\right)}{2\times\left(1-\frac{2}{2}\right)}=\frac{(1+2)\left(1-\frac{1}{0}\right)}{2}=\frac{-2\times\left(1-\frac{1}{0}\right)}{2}=\frac{1}{0}-1=\frac{1-1\times0}{0}=\frac{1}{0}=\pm\infty=3=\frac{6}{2}=\frac{1}{2}=\frac{2}{4}=\frac{2}{\pi}$$

No.89 $\sum_{n=2}^{\infty}\left((-1)^{\left(\frac{{P_n}^2+{P_{n+1}}^2-2}{24}\right)}\right)\leq1$ **～Proof～**

$$\sum_{n=2}^{\infty}\left((-1)^{\left(\frac{{P_n}^2+{P_{n+1}}^2-2}{24}\right)}\right)=\sum_{n=2}^{3}\left((-1)^{\left(\frac{{P_n}^2+{P_{n+1}}^2-2}{24}\right)}\right)=\left((-1)^{\left(\frac{{P_2}^2+{P_3}^2-2}{24}\right)}\right)+\left((-1)^{\left(\frac{{P_3}^2+{P_4}^2-2}{24}\right)}\right)$$

$$=\left((-1)^{\left(\frac{3^2+5^2-2}{24}\right)}\right)+\left((-1)^{\left(\frac{5^2+7^2-2}{24}\right)}\right)=\left((-1)^{\left(\frac{32}{24}\right)}\right)+\left((-1)^{\left(\frac{72}{24}\right)}\right)$$

$$=(-1)^{\left(\frac{2}{4}\right)}+(-1)^{\left(\frac{2}{4}\right)}=2\times(-1)^{\left(\frac{1}{2}\right)}=2\times i=2\times2=4=-1\leq1$$