The lower estimate of the chromatic number of the planeis more than 6.

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Abstract: The preprint provides a consideration of limiting the lower limit of the chromatic number to 7.

The covering of the Euclidean plane with equal circles is considered in order to determine the value of the chromatic number.

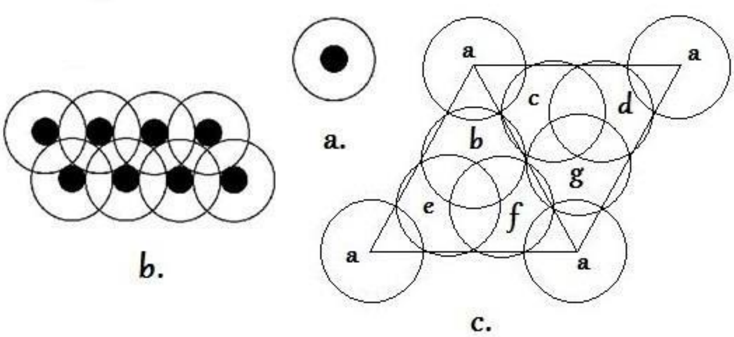
1. Construction of a minimal monochromatic lattice.

By convention, there is a parameter g that limits the dimension of the area of the same color. The minimum shape of an area of one color is less than the parameter g there is a circle with a radius $2r = g$.

In the case of packing a set of given circles (of equal sizes), the parameter g determines the area forbidden for color around the colored circle, that is, a circular area with $R = 3r$ is formed, the center of which coincides with the center of the colored circle.

As is known, the densest packing of equal circles on the plane is formed with the symmetry of a hexagonal lattice, which can also be represented as consisting of symmetrical rhombic cells.

In the case of a dense packing of circles of the same color on the plane, the cells are rhombuses with sides and a small semi-axis equal to 4r. The monochrome circles at the nodes of this lattice form a minimal monochromatic lattice (the fulfillment of the parameter g for one color on the plane).



2. Calculation of the chromatic number of the plane.

Let the area of a one-color circle be 1, radius $r = \sqrt{1/\pi}$. The area of the rhombicregion of the lattice

$$S_{rh} = 2 * l^2 \frac{\sqrt{3}}{4} \quad l = 4\sqrt{1/\pi} \quad S_{rh} = 8 * \frac{\sqrt{3}}{\pi}$$

Accordingly the maximum area density of monochrome circles for amonochromatic lattice is $d_m = \frac{1}{S_{rh}}$

The chromatic number is the minimum number of minimal monochromatic lattices completely covering the plane.

Considering the coverage density of one monochromatic lattice, the number of lattices for a complete coverage of the plane is 4.41… lattices without intersections ($\chi > 4$).

In a monochromatic lattice, the area of each cell accounts for the area of one shaded circle. Thus, the chromatic number of the plane is equal to the minimum numberof densely packed colored circles completely covering the specified rhombic cell of the monochromatic lattice.

As is known, the minimum packing density of equal circles completely covering the plane is $D \approx 1{,}209$ for a hexagonal packing of circles. The minimally dense total coverage of a monochromatic lattice cell is 4,41… $*$ 1,209… = 5,33…, i.e. 6 circles, thus ($\chi > 5$).

However,thelatticeoftheindicatedhexagonalpackingofcirclesandthemonochromaticlatticehave non-multiple dimensions of rhombic cells, so their circles do not coincide, which leads to acompaction of the packing of circles with a superposition of lattices. For a superposition of a hexagonal and one monochromatic lattice, the cell coverage of the monochromatic lattice is $(5.33 \ldots + 1)$. Deformation (compression) of the hexagonal lattice to a monochromatic lattice is possible to reduce the density of the plane covering with circles less than (5,33..+1). The circle of the monochromatic cell becomes the circle of the hexagonal lattice. The side of the rhombus of the hexagonal lattice is $\sqrt{3}$, when combined along two faces, the rows of both lattices on opposite faces differ by an amount of $(4 - 2\sqrt{3})$, taking into account the overlap of the circles of the hexagonal lattice, a remainder of overlapping circles with a width of $(\frac{\sqrt{3}}{2} - (4 - 2\sqrt{3}))$ along the two faces of the monochromatic lattice arises. Compression of the rhombus of the hexagonal lattice to the rhombus of the monochromatic lattice by this value leads to an increase in the density of the coating circles by 1.1718 times, that is, from 5.33 ... to 6.245..., thus $\chi > 6$. (an example of packing 7 circles in cell image c).