The derivation of formula for neutron lifetime

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Abstract

We assume that neutron is simply composed of proton and electron. Then, in the process of electron escaping from the proton bondage, we discussed the interaction between electron and proton to derive the neutron lifetime formula. According to the different interaction between electron and proton, we get two formulas to calculate the neutron lifetime. The results are $878.58s$ and $877.77s$ respectively, which are in good agreement with the latest laboratory measurement.

Introduction

First, we introduce the first equation obtained to calculate the neutron lifetime, which is as follows:

$$t_n = \frac{\pi h}{\Delta mc^2} \frac{m_p}{m_e} \sqrt{\frac{ke^2}{Gm_p m_e}} \tag{1}$$

Where:

t_n is the lifetime of the neutron;

h is the Planck constant;

m_p is the mass of the proton;

m_e is the mass of the electron;

c is the speed of light in vacuum;

k is Coulomb's constant;

e is the elementary charge;

G is the universal gravitational constant;

π is the pi;

m_n is the mass of the neutron;

$\Delta m = m_n - m_p$;

Δm is the mass difference between the neutron and the proton, and which represents the mass lost in the decay of the neutron.

Here is how we derive the first neutron lifetime formula.

We assume that neutron is simply composed of proton and electron. Since both proton and electron have charge, their charge will interact with each other, resulting in Coulomb force F_e. Since proton and electron have mass, their mass will generate gravity F_n. Since both Coulomb and gravitational forces act on the same electron at the same time, the displacement produced by both forces is always the same. Assuming that the displacement of electrons under the action of Coulomb force is S, and that under the action of gravity is L, then $L = S$.

Assuming that the electron escapes from inside the proton with uniformly decelerating motion and zero final velocity, according to the displacement equation we get:

$$L = \frac{1}{2} a_n t_n^2$$

$$S = \frac{1}{2} a_e t_e^2 \tag{2}$$

a_n is the acceleration of electron caused by the gravity of proton; t_n is the action time of gravity; a_e is the acceleration of electron caused by the Coulomb force of proton; t_e is the action time of Coulomb force.

Since $L = S$, we get:

$$\frac{1}{2} a_n t_n^2 = \frac{1}{2} a_e t_e^2 \tag{3}$$

It is known from Newton's second law that $a_n = F_n / m_e$, and $a_e = F_e / m_e$, substituting them into equation (3), we get:

$$F_n t_n^2 = F_e t_e^2 \tag{4}$$

By transforming Equation (4), we get:

$$t_n = t_e \sqrt{\frac{F_e}{F_n}} \tag{5}$$

It is Knowed by the law of gravitation that $F_n = G m_p m_e / L^2$; from Coulomb's law we know that $F_e = k e^2 / S^2$, substituting them into equation (5) and combining $L = S$, we get:

$$t_n = t_e \sqrt{\frac{ke^2}{Gm_p m_e}} \tag{6}$$

t_e is the action time of Coulomb force and it is difficult to get it directly. But if t_e is proportional to a known time, then we find t_e as long as we find their proportional relationship. We assume that t_e has the following relation:

$$\frac{t_e}{T} = A\frac{m_p}{m_e} \tag{7}$$

If Δm annihilates into a photon, T is the period of the photon; of course, T can also be interpreted as the lifetime of Δm, Then divided by a 2π. A is the scale factor.

Since the period of a photon is the reciprocal of its frequency, according to the energy formula of a photon we get $T = h/\Delta mc^2$, substituting them into equation (7), we get:

$$t_e = \frac{Ah}{\Delta mc^2}\frac{m_p}{m_e} \tag{8}$$

Substituting equation (8) into equation (6) we get:

$$t_n = \frac{Ah}{\Delta mc^2}\frac{m_p}{m_e}\sqrt{\frac{ke^2}{Gm_p m_e}} \tag{9}$$

t_n is the action time of gravity. Here we give it another meaning, which is also the neutron lifetime. The latest experimental measurement value of neutron lifetime is $877.75s$[1]. Now substitute it into equation (9), and we get $A \approx 3.14$. It can be seen that it is very close to the value of π. We take $A = \pi$, and then substitute it into Equation (9), and we get:

$$t_n = \frac{\pi h}{\Delta mc^2}\frac{m_p}{m_e}\sqrt{\frac{ke^2}{Gm_p m_e}} \tag{10}$$

So far, we have derived the first formula to calculate the neutron lifetime. By calculating Equation (10), we get that the neutron lifetime is $878.5753s$.

The equation (10) can also be simplified, thus giving:

$$t_n = \frac{\pi\lambda_e}{\Delta m}\sqrt{\frac{r_e m_p}{G}} \tag{11}$$

Or:

$$t_n = \frac{2\pi^2 r_e}{\Delta m}\sqrt{\frac{a_0 m_p}{G}} \tag{12}$$

In the above two equations, the λ_e is the Compton wavelength of the electron. r_e is the classical radius of the electron. a_0 is the Bohr radius.

The lifetime distance of the Δm

L is the gravitational action distance between proton and electron, here it is also the lifetime distance of the Δm. The equation (2) combining $F_n = Gm_p m_e/L^2$, we get:

$$L = \sqrt[3]{\frac{Gm_p t_n^2}{2}} \tag{13}$$

This formula is the lifetime distance of Δm. Substituting the calculation results of neutron lifetime of Equation (10) into Equation (13), we obtain $L_1 = 3.50572 \times 10^{-11} m$, and $L_1/a_0 = 0.66248$, which is about the length of $2/3$ Bohr radius. Its significance is that Δm begins to decay when it moves to about $2/3$ Bohr radius from the proton.

The second principle for calculating the neutron lifetime is as follows. Assuming that the spin direction of Δm is reversed, the magnetic field of Δm and the magnetic field of proton are mutually exclusive, forcing Δm to stay away from proton, and the distance between Δm and proton increases continuously, which leads to the decay of neutron. The process is similar to the hyperfine splitting of the ground state of hydrogen atom, here Δm have a similar spin-flip jump.

The second method to deduce neutron lifetime is the interaction between gravitational and magnetic forces.

Based on the motion of the electron in the magnetic field of the proton, the magnetic field force between the electron and the proton can be obtained F_B:

$$F_B = \frac{ke^2 g_p g_e}{\pi L^2} \tag{14}$$

g_p is the spin g-factor of the proton; g_e is the spin g-factor for electron.

Δm generates displacement under the action of magnetic field force, and the magnetic field force works on Δm, so, we get:

$$L = \frac{ke^2 g_p g_e}{2(\Delta m - m_e)c^2} \frac{m_p}{m_e} \frac{\Delta m}{m_e} \frac{1}{b^n} \tag{15}$$

$b = m_n/m_p = 1.001378419$, or $b = g_e/2 = 1.00115965218$, $n = 0, \pm 1, \pm 2 \cdots\cdots$

b is a parameter that varies with the measurement accuracy.

(15) equation combined with (2) equation, and $a_n = Gm_p \Delta m/m_e L^2$, the second

equation for calculating the neutron lifetime can be obtained as follows:

$$t_n = \frac{ke^2 g_p g_e}{2(\Delta m - m_e)c^2} \frac{m_p}{m_e} \frac{\Delta m}{m_e} \frac{1}{b_1^{n_1}} \sqrt{\frac{ke^2 g_p g_e}{G m_p \Delta m b_2^{n_2}}} \tag{16}$$

b_1, n_1, b_2, n_2 The range of values is the same as b and n, this allows them to take different values. When the $b_1 = 1.001378419$, the $n_1 = 2$, the $n_2 = 0$, the result of equation (16) is calculated as:

$$t_n = 877.77s \tag{17}$$

Comparing the latest neutron lifetime measurements $877.75s$, it can be found that the results calculated by this equation are in good agreement. Of course, different values are taken for the (16) equation b_1, n_1, b_2, n_2 , We will get many different results, which are mainly adjusted for different neutron lifetime measurements. When n_1 and n_2 are equal to 0, the lifetime of neutron is $t_n = 880.19s$.

The lifetime distance of Δm calculated by Equation (16) is: $L_2/a_0 \approx 0.9023$. The different lifetime distances of Δm should be related to the different decay modes of neutrons.

In this paper, the formula of neutron lifetime is derived by some assumptions, although there are a lot of unreasonable places, but it can also show that the neutron lifetime can be derived from the theory.

Finally, this paper presents again a very simple empirical formula related to the neutron lifetime:

$$t_n \propto \frac{h}{3\left(m_p c\right)^2} \tag{18}$$

The neutron lifetime calculated by this empirical formula is $t_n = 878.411s$.

Reference

[1] arXiv:2106.10375v2