Helmholtzian Factorization vs Dirac Factoring
Detailed Fermion Mass-Color-Charge Architecture
Weak Interaction and Strong Force Field Equations

Through the Helmholtzian factorization of the Helmholtz/Klein-Gordon equation the mass is a sum of squares of a 4-tuple of the four-vector-doublet. The Dirac equation factoring shows that the electron satisfies via a mass-singlet. Thus, in combination, the heavy leptons and quarks are separated as: $(0,0,0,m_0)$ heavy leptons satisfying the Dirac factoring, and $(m_1,m_2,m_3,0)$ quarks satisfying the Helmholtzian factoring, as pythagorean quadruples.

Through the pythagorean quadruple primitives $(2,5,14,15) \Rightarrow \left(\mp\frac{1}{3},\mp\frac{5}{6},\mp\frac{7}{3},\frac{5}{2}\right),\left(\pm\frac{2}{3},\pm\frac{5}{3},\pm\frac{14}{3},5\right)$, charge and color are manifested, plus the mass magnitude $|m|^2$ is always the sum-of-squares of the mass-constituents via Klein-Gordon equation. Plus, by establishing the fermion architecturethe weak interaction is manifested (including the masses of the $W^{\pm}$ and Z^0 , via the fundamental dimensionless physical constant k).

And continuing the Helmholtzian factorization/Dirac equation factoring analysis, extended through the Helmholtzian factorization to all particle types and interactions (quarks, neutrinos, weaks, heavy leptons).

In general, the Helmholtzian factorization yields the Helmholtz partial differential equation in four-vectors [3].

$$(\Box + |m|^2) = D_B D_A$$

In [4] I showed that the heavy leptons and quarks are separated as:

$$m \equiv (m_1,m_2,m_3,m_0) = (m_1,m_2,m_3,0) + (0,0,0,m_0) = m_q + m_\ell$$

$$m_q \equiv (m_1,m_2,m_3,0) \ , \ m_\ell \equiv (0,0,0,m_0)$$

such that:

$(\Box + |m_\ell|^2)$ satisfies the Dirac factoring, and:

$(\Box + |m_q|^2)$ satisfies the Helmholtzian factorization, as a pythagorean quadruple [5][6], yielding the

$(\Box + |m_\nu|^2)$ satisfies the Helmholtzian factorization, with color characteristics along with Dirac characteristics; but more, as a pythagorean quintuple.

Dirac facoring uses the $2x2$ Pauli matrices: $\boldsymbol{\sigma}^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\boldsymbol{\sigma}^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\boldsymbol{\sigma}^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

along with the $2x2$ identity matrix: $\boldsymbol{\sigma}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{I}_2$

to factor the Klein-Gordon equation.

Whenever $\psi_D \equiv \begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix}$ is a 2^M-dimensional vector, via a matrix differential operator factorization, it may be written (in the Dirac representation):

$$\left[i\gamma_i \bullet \vec{\nabla} + \left(i\boldsymbol{\sigma}^0\mathbf{I}_2\frac{\partial}{\partial t} - \gamma_0 m\right)\right]\psi_D = \mathbf{0} \quad \text{or:} \quad \left[\gamma_i \bullet \vec{\nabla} + \gamma_0\frac{\partial}{\partial t} - \mathbf{I}_2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} m\right]\psi_D = \mathbf{0}$$

where:

$\gamma_1 \equiv \boldsymbol{\sigma}^1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\gamma_2 \equiv \boldsymbol{\sigma}^2\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\gamma_3 \equiv \boldsymbol{\sigma}^3\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\gamma_0 \equiv \mathbf{I}_2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
or:	(depends on the	representation)	
$\gamma_1 \equiv \boldsymbol{\sigma}^1\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\gamma_2 \equiv \boldsymbol{\sigma}^2\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\gamma_3 \equiv \boldsymbol{\sigma}^3\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\gamma_0 \equiv \mathbf{I}_2\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

For:

$$\begin{pmatrix} \phi_D^C \\ -\phi_D^D \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right)\phi_D^A - i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^B \\ i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^A - \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right)\phi_D^B \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right) \end{pmatrix}\begin{pmatrix} \phi_D^C \\ -\phi_D^D \end{pmatrix} = \mathbf{0}$$

$$= \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right)\phi_D^C + i\boldsymbol{\sigma}\bullet\vec{\nabla}(-\phi_D^D) \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^C - \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right)(-\phi_D^D) \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t} + m\right)\phi_D^C - i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^D \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^C + \mathbf{I}_2\left(-i\frac{\partial}{\partial t} + m\right)\phi_D^D \end{pmatrix}$$

$$= \begin{pmatrix} -i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^D + \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right)\phi_D^C \\ \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right)\phi_D^D - i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^C \end{pmatrix} = \begin{pmatrix} i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^D - \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right)\phi_D^C \\ \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right)\phi_D^D - i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^C \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right)\phi_D^D - i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^C \\ i\boldsymbol{\sigma}\bullet\vec{\nabla}\phi_D^D - \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right)\phi_D^C \end{pmatrix} = \begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & -\mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^D \\ \phi_D^C \end{pmatrix}$$

these are the same in different components, which become the single equation above (given the appropriate $\boldsymbol{\gamma}_i$ representations)

and:

$$\begin{pmatrix} \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) & -i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right) & i\boldsymbol{\sigma}\bullet\vec{\nabla} \\ -i\boldsymbol{\sigma}\bullet\vec{\nabla} & \mathbf{I}_2\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

$$\Rightarrow \begin{pmatrix} \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right)\left(-i\frac{\partial}{\partial t}+m\right) - \boldsymbol{\sigma}^2\vec{\nabla}\bullet\vec{\nabla} & 0 \\ 0 & \boldsymbol{\sigma}^2\bullet\vec{\nabla}\bullet\vec{\nabla} + \mathbf{I}_2\left(i\frac{\partial}{\partial t}+m\right)\left(-i\frac{\partial}{\partial t}+m\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

$$\Rightarrow \begin{pmatrix} \mathbf{I}_2\left(\frac{\partial^2}{\partial t^2}+m^2\right) - \mathbf{I}_2\vec{\nabla}\bullet\vec{\nabla} & 0 \\ 0 & -\mathbf{I}_2\bullet\vec{\nabla}\bullet\vec{\nabla} + \mathbf{I}_2\left(\frac{\partial^2}{\partial t^2}+m^2\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

$$\Rightarrow \begin{pmatrix} \mathbf{I}_2\left(\frac{\partial^2}{\partial t^2}+m^2\right) - \mathbf{I}_2\sum_{\nu=1}^{3}\frac{\partial^2}{\partial^2 x_\nu} & 0 \\ 0 & -\mathbf{I}_2\sum_{\nu=1}^{3}\frac{\partial^2}{\partial^2 x_\nu} + \mathbf{I}_2\left(\frac{\partial^2}{\partial t^2}+m^2\right) \end{pmatrix}\begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} = \mathbf{0}$$

$$\Rightarrow \left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\begin{pmatrix} \begin{pmatrix} \phi_D^A \\ \phi_D^B \end{pmatrix} \\ \begin{pmatrix} \phi_D^C \\ \phi_D^D \end{pmatrix} \end{pmatrix} = \left(\nabla^2 - \frac{\partial^2}{\partial t^2} - m^2\right)\begin{pmatrix} \begin{pmatrix} \begin{pmatrix} \psi_D^{A1} \\ \psi_D^{A2} \end{pmatrix} \\ \begin{pmatrix} \psi_D^{B1} \\ \psi_D^{B2} \end{pmatrix} \end{pmatrix} \\ \begin{pmatrix} \begin{pmatrix} \psi_D^{D1} \\ \psi_D^{D2} \end{pmatrix} \\ \begin{pmatrix} \psi_D^{C1} \\ \psi_D^{C2} \end{pmatrix} \end{pmatrix} \end{pmatrix} = \mathbf{0}$$

The Helmholtzian operator factorization is:

$$\mathbf{J} \equiv D_B D_A \mathbf{f} = ((\Box - |m|^2))\mathbf{f}$$

where:

$$D_B \equiv \begin{pmatrix} D_0 & D_3^{\Leftrightarrow} & -D_2^{\Leftrightarrow} & -D_1 \\ -D_3^{\Leftrightarrow} & D_0 & D_1^{\Leftrightarrow} & -D_2 \\ D_2^{\Leftrightarrow} & -D_1^{\Leftrightarrow} & D_0 & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0^{\Updownarrow} \end{pmatrix} \quad \& \quad D_A \equiv \begin{pmatrix} D_0^{\Updownarrow} & -D_3^{\Leftrightarrow} & D_2^{\Leftrightarrow} & -D_1 \\ D_3^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_1^{\Leftrightarrow} & -D_2 \\ -D_2^{\Leftrightarrow} & D_1^{\Leftrightarrow} & D_0^{\Updownarrow} & -D_3 \\ -D_1^{\Updownarrow} & -D_2^{\Updownarrow} & -D_3^{\Updownarrow} & D_0 \end{pmatrix}$$

and:

$$D_i^+ \equiv (\partial_i + m_i) \quad , \quad D_i^- \equiv (\partial_i - m_i)$$

$$D_i \equiv \begin{pmatrix} D_i^+ & 0 \\ 0 & D_i^- \end{pmatrix} \;,\; D_i^{\Updownarrow} \equiv \begin{pmatrix} D_j^- & 0 \\ 0 & D_i^+ \end{pmatrix} \;,\; D_i^{\Leftrightarrow} \equiv \begin{pmatrix} 0 & D_i^- \\ D_i^+ & 0 \end{pmatrix} \;,\; D_i^{\Leftrightarrow\Updownarrow} \equiv \begin{pmatrix} 0 & D_i^+ \\ D_j^- & 0 \end{pmatrix}$$

is, as in the case of the Dirac factoring, commutative.

So, in the case of the heavy leptons, is:

$$(\Box + |m_0|^2) = D_B D_A$$

where:

$$
\left\{
\begin{array}{l}
D_B = \begin{pmatrix}
\begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix} & \begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix} & -\begin{pmatrix} 0 & D_2^- \\ D_2^- & 0 \end{pmatrix} & -\begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix} \\
-\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix} & \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix} & \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix} & -\begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix} \\
-\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix} & -\begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix} & \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix} & -\begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix} \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix} & -\begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix} & -\begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix} & \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}
\end{pmatrix} \\
D_A = \begin{pmatrix}
\begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix} & -\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix} & \begin{pmatrix} 0 & D_2^- \\ D_2^- & 0 \end{pmatrix} & -\begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix} \\
\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix} & \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix} & -\begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix} & -\begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix} \\
-\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix} & \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix} & \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix} & -\begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix} \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix} & -\begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix} & -\begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix} & \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}
\end{pmatrix}
\end{array}
\right.
$$

$$
\Rightarrow \left\{
\begin{array}{l}
D_B = \begin{pmatrix}
(\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & -(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
-(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)
\end{pmatrix} \\
D_A = \begin{pmatrix}
(\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
-(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)
\end{pmatrix}
\end{array}
\right.
$$

So, in the case: $D_B D_A \mathbf{f} = (\Box + |m_0|^2)\mathbf{f} = \mathbf{0}$

$\Rightarrow D_A \mathbf{f} = \mathbf{F}$

$$
\Rightarrow \left\{
\begin{array}{l}
D_B = \begin{pmatrix}
(\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & -(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
-(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)
\end{pmatrix} \\
D_A = \begin{pmatrix}
(\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
-(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)
\end{pmatrix}
\end{array}
\right.
$$

$$
\Rightarrow \left\{
\begin{array}{l}
D_B\mathbf{F} = \begin{pmatrix}
(\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & -(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
-(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)
\end{pmatrix}
\begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} = \begin{pmatrix}
(\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1) \\
(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2) \\
-(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2) & (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1) & (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0) & -(\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3) \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1) & -(\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2) & -(\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3) & (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)
\end{pmatrix}
\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix}
\end{array}
\right.
$$

$$
\Rightarrow \left\{
\begin{array}{l}
D_B\mathbf{F} = \begin{pmatrix}
(\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)F^1 + (\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3)F^2 - (\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2)F^3 - (\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1)F^0 \\
-(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3)F^1 + (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)F^2 + (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1)F^3 - (\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2)F^0 \\
(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2)F^1 - (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1)F^2 + (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)F^3 - (\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3)F^0 \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1)F^1 - (\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2)F^2 - (\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3)F^3 + (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)F^0
\end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} = \begin{pmatrix}
(\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)f^1 - (\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3)f^2 + (\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2)f^3 - (\mathbf{I}_2\partial_1 + \boldsymbol{\sigma}^3 m_1)f^0 \\
(\boldsymbol{\sigma}^1\partial_3 - \boldsymbol{\sigma}^3 m_3)f^1 + (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)f^2 - (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1)f^3 - (\mathbf{I}_2\partial_2 + \boldsymbol{\sigma}^3 m_2)f^0 \\
-(\boldsymbol{\sigma}^1\partial_2 - \boldsymbol{\sigma}^3 m_2)f^1 + (\boldsymbol{\sigma}^1\partial_1 - \boldsymbol{\sigma}^3 m_1)f^2 + (\mathbf{I}_2\partial_0 - \boldsymbol{\sigma}^3 m_0)f^3 - (\mathbf{I}_2\partial_3 + \boldsymbol{\sigma}^3 m_3)f^0 \\
-(\mathbf{I}_2\partial_1 - \boldsymbol{\sigma}^3 m_1)f^1 - (\mathbf{I}_2\partial_2 - \boldsymbol{\sigma}^3 m_2)f^2 - (\mathbf{I}_2\partial_3 - \boldsymbol{\sigma}^3 m_3)f^3 + (\mathbf{I}_2\partial_0 + \boldsymbol{\sigma}^3 m_0)f^0
\end{pmatrix} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix}
\end{array}
\right.
$$

$$\Rightarrow \begin{cases} D_B\mathbf{F} = \begin{pmatrix} \mathbf{I}_2(\partial_0 F^1 - \partial_1 F^0) - \boldsymbol{\sigma}^1(\partial_2 F^3 - \partial_3 F^2) + \boldsymbol{\sigma}^3[(m_0 F^1 - m_1 F^0) + (m_2 F^3 - m_3 F^2)] \\ \mathbf{I}_2(\partial_0 F^2 - \partial_2 F^0) - \boldsymbol{\sigma}^1(\partial_3 F^1 - \partial_1 F^3) + \boldsymbol{\sigma}^3[(m_0 F^2 - m_2 F^0) + (m_3 F^1 - m_1 F^3)] \\ \mathbf{I}_2(\partial_0 F^3 - \partial_3 F^0) - \boldsymbol{\sigma}^1(\partial_1 F^2 - \partial_2 F^1) + \boldsymbol{\sigma}^3[(m_0 F^3 - m_3 F^0) + (m_1 F^2 - m_2 F^1)] \\ -\mathbf{I}_2(\partial_1 F^1 + \partial_2 F^2 + \partial_3 F^3 - \partial_0 F^0) + \boldsymbol{\sigma}^3(m_1 F^1 + m_2 F^2 + m_3 F^3 - m_0 F^0) \end{pmatrix} = \mathbf{0} \\ D_A\mathbf{f} = \begin{pmatrix} \mathbf{I}_2(\partial_0 f^1 - \partial_1 f^0) + \boldsymbol{\sigma}^1(\partial_2 f^3 - \partial_3 f^2) - \boldsymbol{\sigma}^3[(m_0 f^1 + m_1 f^0) + (m_2 f^3 - m_3 f^2)] \\ \mathbf{I}_2(\partial_0 f^2 - \partial_2 f^0) + \boldsymbol{\sigma}^1(\partial_3 f^1 - \partial_1 f^3) - \boldsymbol{\sigma}^3[(m_0 f^2 + m_2 f^0) + (m_3 f^1 - m_1 f^3)] \\ \mathbf{I}_2(\partial_0 f^3 - \partial_3 f^0) + \boldsymbol{\sigma}^1(\partial_1 f^2 - \partial_2 f^1) - \boldsymbol{\sigma}^3[(m_0 f^3 + m_3 f^0) + (m_1 f^2 - m_2 f^1)] \\ -\mathbf{I}_2(\partial_1 f^1 + \partial_2 f^2 + \partial_3 f^3 - \partial_0 f^0) + \boldsymbol{\sigma}^3(m_1 f^1 + m_2 f^2 + m_3 f^3 + m_0 f^0) \end{pmatrix} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} \end{cases} \quad (1)$$

This is the general case, also the equations for the neutrinos.

Note:

least of the mass constituents isn't quite right, plus since charge is defined by m_0 for heavy leptons, the correct mathematical definition for charge is:

charge $\equiv m_0 + nearest0(m_1, m_2, m_3)$

where:

$$nearest0(m_1, m_2, \ldots, m_n) \equiv \begin{cases} \min(m_1, m_2, \ldots, m_n) \ , \ \ if \ \min(m_1, m_2, \ldots, m_n) \geq 0 \\ \max(m_1, m_2, \ldots, m_n) \ , \ \ if \ \max(m_1, m_2, \ldots, m_n) < 0 \end{cases}$$

thus, for quarks: charge $= nearest0(m_1, m_2, m_3)$

for heavy leptons: charge $= m_0$

for light leptons: $m_0 = -nearest0(m_1, m_2, m_3)$

$\Rightarrow$ charge $= 0$

and, so, for neutrinos: $\exists \lambda \ni |m_\nu|^2 = \left[\left(\pm\frac{1}{3}\right)^2 + \left(\pm\frac{5}{6}\right)^2 + \left(\pm\frac{7}{3}\right)^2 + \left(\mp\frac{1}{3}\right)^2 \right]\lambda^2$

$$= \left[2(1)^2 + \left(\frac{5}{2}\right)^2 + (7)^2 \right]\left(\frac{\lambda}{3}\right)^2$$

$$= \left[(2)^2 + (5)^2 + (14)^2 + (1)^2 \right]\left(\frac{\lambda}{6}\right)^2$$

$$= \left[(15)^2 + 1 \right]\left(\frac{\lambda}{6}\right)^2 = [225 + 1]\left(\frac{\lambda}{6}\right)^2$$

$$= [226]\left(\frac{\lambda}{6}\right)^2 = \left(\frac{113}{3}\right)\lambda^2$$

Note also that neutrino and meson oscillation (B-:Bbar, Kaon, etc.) is manifested via their (m_1, m_2, m_3) triplets. A similar baryon oscillation may be observed, as well.

For quarks: $m_0 = 0$:

$$\Rightarrow \begin{cases} D_B\mathbf{F} = \begin{pmatrix} \mathbf{I}_2(\partial_0 F^1 - \partial_1 F^0) - \boldsymbol{\sigma}^1(\partial_2 F^3 - \partial_3 F^2) + \boldsymbol{\sigma}^3[-m_1 F^0 + (m_2 F^3 - m_3 F^2)] \\ \mathbf{I}_2(\partial_0 F^2 - \partial_2 F^0) - \boldsymbol{\sigma}^1(\partial_3 F^1 - \partial_1 F^3) + \boldsymbol{\sigma}^3[-m_2 F^0 + (m_3 F^1 - m_1 F^3)] \\ \mathbf{I}_2(\partial_0 F^3 - \partial_3 F^0) - \boldsymbol{\sigma}^1(\partial_1 F^2 - \partial_2 F^1) + \boldsymbol{\sigma}^3[-m_3 F^0 + (m_1 F^2 - m_2 F^1)] \\ -\mathbf{I}_2(\partial_1 F^1 + \partial_2 F^2 + \partial_3 F^3 - \partial_0 F^0) + \boldsymbol{\sigma}^3(m_1 F^1 + m_2 F^2 + m_3 F^3) \end{pmatrix} = \mathbf{0} \\ D_A\mathbf{f} = \begin{pmatrix} \mathbf{I}_2(\partial_0 f^1 - \partial_1 f^0) + \boldsymbol{\sigma}^1(\partial_2 f^3 - \partial_3 f^2) - \boldsymbol{\sigma}^3[m_1 f^0 + (m_2 f^3 - m_3 f^2)] \\ \mathbf{I}_2(\partial_0 f^2 - \partial_2 f^0) + \boldsymbol{\sigma}^1(\partial_3 f^1 - \partial_1 f^3) - \boldsymbol{\sigma}^3[m_2 f^0 + (m_3 f^1 - m_1 f^3)] \\ \mathbf{I}_2(\partial_0 f^3 - \partial_3 f^0) + \boldsymbol{\sigma}^1(\partial_1 f^2 - \partial_2 f^1) - \boldsymbol{\sigma}^3[m_3 f^0 + (m_1 f^2 - m_2 f^1)] \\ -\mathbf{I}_2(\partial_1 f^1 + \partial_2 f^2 + \partial_3 f^3 - \partial_0 f^0) + \boldsymbol{\sigma}^3(m_1 f^1 + m_2 f^2 + m_3 f^3) \end{pmatrix} = \begin{pmatrix} F^1 \\ F^2 \\ F^3 \\ F^0 \end{pmatrix} \end{cases} \quad (2)$$

Note

$$D_B\mathbf{F} + D_A\mathbf{f} = \begin{pmatrix} \mathbf{I}_2(\partial_0(F^j + f^j) - \partial_j(F^0 + f^0)) - \boldsymbol{\sigma}^1(\nabla \times (\mathbf{F} + \mathbf{f}))_j + \\ + \boldsymbol{\sigma}^3\left[-m_j(F^0 + f^0) + (\mathbf{m} \times (\mathbf{F} - \mathbf{f}))_j \right] \\ -\mathbf{I}_2\left(\sum_{j=1}^{3} \partial_j(F^j + f^j) - \partial_0(F^0 + f^0) \right) + \boldsymbol{\sigma}^3\left(\sum_{j=1}^{3} m_j(F^j + f^j) \right) \end{pmatrix}$$

And: $m = (m_1, m_2, m_3) \leftrightarrow (-m_1, -m_2, -m_3) = -m$; $\left(m_i \leftrightarrow -m_i \right)$:

$$\Leftrightarrow D_B\mathbf{F} + D_A\mathbf{f} = \begin{pmatrix} \mathbf{I}_2(\partial_0(F^j + f^j) - \partial_j(F^0 + f^0)) - \boldsymbol{\sigma}^1(\nabla \times (\mathbf{F} + \mathbf{f}))_j + \\ + \boldsymbol{\sigma}^3\left[m_j(F^0 - f^0) + (\mathbf{m} \times (\mathbf{F} + \mathbf{f}))_j \right] \\ -\mathbf{I}_2\left(\sum_{j=1}^{3} \partial_j(F^j + f^j) - \partial_0(F^0 + f^0) \right) + \boldsymbol{\sigma}^3\left(\sum_{j=1}^{3} m_j(F^j - f^j) \right) \end{pmatrix}$$

(these equations underpin the quark mass-color strong force between mesons & baryons)

As you can see, the quark field components add/subtract component-wise by mass-color constituents per meson/baryon type.

So, we have gone from a vague symmetry group description, to a concrete description of field equations and architecture of the quarks, mesons, baryons and strong force:

(which determine the symmetry groups rather than the other way around)

$m_e = m_e$	$m_\mu = 5km_e$	$m_\tau = \left[\left(\frac{2}{1450}\right)(5k)^2 \right]^2 m_e$
$m_u = 5m_e$	$m_c = 60km_e$	$m_b = \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (5k)m_e$
$m_d = 10m_e$	$m_s = \left(\frac{23}{25}\right) \cdot (5k)m_e$	$m_t = 10\left[\left(\frac{3}{1004}\right)(6k)^2 \right]^2 m_e$

$m_u = \left[\left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2\right]m_e$	$m_c = \left[\left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2\right]12km_e$	$m_b = \left[\left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2\right]\frac{\sqrt{23}}{5} \cdot km_e$
$m_d = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)4m_e$	$m_s = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left(\frac{23}{30 \cdot 5}\right) \cdot 4km_e$	$m_t = \left(-\frac{1}{3},-\frac{5}{6},-\frac{7}{3}\right)\left[\left(\frac{3}{1004}\right)(6k)^2\right]^2 \frac{4}{3}m_e$

Note: $\left(+\frac{1}{3},+\frac{5}{6},+\frac{7}{3}\right)2 = \left(+\frac{2}{3},+\frac{5}{3},+\frac{14}{3}\right) = (2,5,14)\frac{1}{3} \rightarrow 2^2+5^2+14^2 = 15^2$

are pythagorean quadruples.
Thus, the pythagorean quadruple primitive multiple mass-constituents=**color**, and mass-constituents sum-of-squares=mass-magnitude $|m|^2$; and minimum mass-constituent=**charge**.
(Note, also, that since heavy leptons have a single mass constituent, the mass magnitude is a multiple of 1)

I brought you:

$$k = \frac{m_s}{m_u}\left(\frac{25}{23}\right) = \frac{1}{6}\frac{m_c}{m_d} = \frac{1}{5}\frac{m_\mu}{m_e} = \sqrt{\frac{m_b}{m_u}\sqrt{\frac{25}{23}}} = \frac{1}{6}\sqrt{\frac{1004}{3}\sqrt{\frac{m_t}{m_d}}} = \frac{1}{5}\sqrt{\frac{1450}{2}\sqrt{\frac{m_\tau}{m_e}}}$$

$$\Rightarrow \frac{m_t}{10 \cdot \left[\left(\frac{3}{1004}\right)(6k)^2\right]^2} = \frac{m_b}{5 \cdot \left(\frac{23}{25}\right)^{\frac{1}{2}} \cdot (k)^2} = \frac{m_\tau}{1 \cdot \left[\left(\frac{2}{1450}\right)(5k)^2\right]^2} = \frac{m_s}{\left(\frac{23}{25}\right) \cdot (5k)} = \frac{m_c}{10 \cdot (6k)} = \frac{m_\mu}{5k} = \frac{m_d}{10} = \frac{m_u}{5} = m_e$$

Is this true?

$\frac{m_s}{m_u}\left(\frac{25}{23}\right)$	$= \frac{97.205699127171364309497389896187}{2.5549946390235103880721950029485}\left(\frac{25}{23}\right)$	$= 41.353655699595529713433202094743$
$\frac{1}{6}\frac{m_c}{m_d}$	$= \frac{1}{6}\frac{1267.9004233978873605586616073416}{5.1099892780470207761443900058 97}$	$= 41.353655699595529713433202094744$
$\frac{1}{5}\frac{m_\mu}{m_e}$	$= \frac{1}{5}\frac{105.65836861649061337988846727846}{0.51099892780470207761443900005897}$	$= 41.353655699595529713433202094742$
$\sqrt{\frac{m_b}{m_u}\sqrt{\frac{25}{23}}}$	$= \sqrt{\frac{4190.9426907545271186849743851983}{2.5549946390235103880721950029485}\sqrt{\frac{25}{23}}}$	$= 41.353655699595529713433202094743$
$\frac{1}{6}\sqrt{\frac{1004}{3}\sqrt{\frac{m_t}{m_d}}}$	$= \frac{1}{6}\sqrt{\frac{1004}{3}\sqrt{\frac{172924.17191486611744398343538627}{5.1099892780470207761443900005897}}}$	$= 41.353655699595529713433202094743$
$\frac{1}{5}\sqrt{\frac{1450}{2}\sqrt{\frac{m_\tau}{m_e}}}$	$= \frac{1}{5}\sqrt{\frac{1450}{2}\sqrt{\frac{1776.9680674108457768918379570944}{0.51099892780470207761443900005897}}}$	$= 41.353655699595529713433202094743$

So, is it true, or not?
(use less significant figures if you wish, but the results are still: $k \approx 41.353$)

$$k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\sum_{k=0}^{\infty}\left(-\frac{1}{20}\right)^k = 4\pi^2 + \frac{15}{8} + \frac{1}{4000}\left(\frac{20}{21}\right)$$
$$= 41.353655699595529713433202094743$$

So, clearly k a fundamental dimensionless physical constant

$\Rightarrow$

$m_e = m_e$	$m_\mu = 5km_e$	$m_\tau = \left(\frac{1}{9}\right)^2 k^4 m_e$
$m_u = 5m_e$	$m_c = 6km_d$	$m_t = \left(\frac{27}{251}\right)^2 k^4 m_d$
$m_d = 10m_e$	$m_s = \frac{23}{25}km_u$	$m_b = \sqrt{\frac{23}{25}}k^2 m_u$

$\Updownarrow$

$\Rightarrow$

$m_e = m_e$	$m_\mu = 5km_e$	$m_\tau = \left(\frac{1}{9}\right)^2 k^4 m_e$
$m_u = 5m_e$	$m_c = 12km_u$	$m_t = 2\left(\frac{27}{251}\right)^2 k^4 m_u$
$m_d = 10m_e$	$m_s = \frac{1}{2}\frac{23}{25}km_d$	$m_b = \frac{1}{2}\sqrt{\frac{23}{25}}k^2 m_d$

$\Leftrightarrow$

$m_e = m_e$	$m_u = 5m_e$	$m_d = 10m_e$
$m_\mu = 5km_e$	$m_c = 12km_u$	$m_s = \frac{1}{2}\frac{23}{25}km_d$
$m_\tau = \left(\frac{1}{9}\right)^2 k^4 m_e$	$m_t = 2\left(\frac{27}{251}\right)^2 k^4 m_u$	$m_b = \frac{1}{2}\sqrt{\frac{23}{25}}k^2 m_d$

Now, the Weak Force/Interaction has been theorized as a half-generalized electroweak phenomenology in conjunction with the Higgs Lagrangian to predict boson particles of certain masses.
As the Higgs is unnecessary for establishing the fermion architecture, nor their charges, color, nor masses (via the fundamental dimensionless physical constant k , as above) ; neither the electroweak nor the Higgs theory are necessary for the weak interaction:

$$q + \overline{q} \leftrightarrow Z^0 \quad , \quad q \in \{d,s,t,u,c,b\}$$
$$q\left[\left(\pm\frac{1}{3},\pm\frac{5}{6},\pm\frac{7}{3}\right)\lambda_1\right] \leftrightarrow W^\pm + q\left[\left(\mp\frac{1}{3},\mp\frac{5}{6},\mp\frac{7}{3}\right)\lambda_2\right]$$

(where allowed by energy conservation)

where:

$$|m_W| = 4^3 m_c = 4^3 \times 12km_u = 4^3 \times 60km_e = 81,145.62709746479107575434286986$$
$$|m_Z| = \frac{1}{2}\left(\frac{3}{2}\right)^2 4^3 m_c = \frac{1}{2}\left(\frac{3}{2}\right)^2 4^3 \times 12km_u = 4^2 \times 54km_u = \frac{1}{2} \times (12)^2 \times 60km_e$$
$$= \frac{5}{2}(12)^3 km_e = 91,288.830484647889960223635728593$$

So:

$$\frac{m_W}{m_u}\left(\frac{1}{4^3 \times 12}\right) = \frac{81130}{2.5549946390235103880721950029485}\left(\frac{1}{4^3 \times 12}\right) = 41.3457$$
$$\frac{m_Z}{m_u}\left(\frac{1}{4^2 \times 54}\right) = \frac{91890}{2.5549946390235103880721950029485}\left(\frac{1}{4^2 \times 54}\right) = 41.6259$$

(apparently the actual mass of the Z^0 is much closer to the lower measure than the higher)

$$\frac{m_Z}{m_u}\left(\frac{1}{4^2 \times 54}\right) = \frac{91300}{2.5549946390235103880721950029485}\left(\frac{1}{4^2 \times 54}\right) = 41.358715467474582378365794645146$$

Note:

$$m_u = (m_1, m_2, m_3, m_0) = \left(+\frac{2}{3}, +\frac{5}{3}, +\frac{14}{3}, 0\right)\lambda = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, 0\right)2\lambda$$

$$\Rightarrow \sqrt{\left(\frac{1}{3}\right)^2 + \left(\frac{5}{6}\right)^2 + \left(\frac{7}{3}\right)^2} \times 2\lambda = \frac{5}{2}2\lambda = |m_u| = 5|m_e| \Rightarrow \lambda = |m_e| \Rightarrow m_u = \left(+\frac{2}{3}, +\frac{5}{3}, +\frac{14}{3}, 0\right)|m_e|$$

$$m_d = (m_1, m_2, m_3, m_0) = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\lambda = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\lambda$$

$$\Rightarrow \frac{5}{2}\lambda = |m_d| \Rightarrow \lambda = \frac{|m_d|}{\left(\frac{5}{2}\right)} = \frac{2|m_u|}{\left(\frac{5}{2}\right)} = \frac{10|m_e|}{\left(\frac{5}{2}\right)} = \frac{20}{5}|m_e| = 4|m_e| \Rightarrow m_u = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)4|m_e|$$

$$m_{q_u} = (m_1, m_2, m_3, m_0) = \left(+\frac{2}{3}, +\frac{5}{3}, +\frac{14}{3}, 0\right)\lambda = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, 0\right)2\lambda$$

$$\Rightarrow \frac{5}{2}2\lambda = 5\lambda = |m_{q_u}| \Rightarrow \lambda = \frac{1}{5}|m_{q_u}| \Rightarrow m_{q_u} = \left(+\frac{2}{3}, +\frac{5}{3}, +\frac{14}{3}, 0\right)\frac{1}{5}|m_{q_u}| = \left(+\frac{2}{3}, +\frac{5}{3}, +\frac{14}{3}, 0\right)\frac{|m_{q_u}|}{5}$$

$$m_{q_d} = (m_1, m_2, m_3, m_0) = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\lambda = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\lambda$$

$$\Rightarrow \frac{5}{2}\lambda = |m_{q_d}| \Rightarrow \lambda = \frac{2}{5}|m_{q_d}| \Rightarrow m_{q_d} = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\frac{2}{5}|m_{q_d}| = \left(-\frac{1}{3}, -\frac{5}{6}, -\frac{7}{3}, 0\right)\frac{|m_{q_d}|}{\left(\frac{5}{2}\right)}$$

(The fourth element of a pythagorean quadruple is, of course, the square root of the sum of the squares of first three elements. However, the fourth element of a quark primitive is 0 ; this square root of the sum of the squares of first three elements becomes the divisor of the quark mass as the primitive multiplier λ)

And:

$$m_{W^\pm} = (m_1, m_2, m_3, m_0) = \left(\pm\frac{1}{3}, \pm\frac{5}{6}, \pm\frac{7}{3}, 0\right)\lambda + \left(0, 0, 0, \pm\frac{2}{3}\right)\lambda$$

$$\Rightarrow |m_{W^\pm}| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda\right)^2 + \left(\pm\frac{5}{6}\lambda\right)^2 + \left(\pm\frac{7}{3}\lambda\right)^2\right] + \left(\pm\frac{2}{3}\lambda\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda^2 + \left(\frac{2}{3}\lambda\right)^2} = \sqrt{\left(\frac{5}{2}\lambda\right)^2 + \left(\frac{2}{3}\lambda\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (2 \cdot 2)^2}{6^2}}\lambda = \sqrt{\frac{15^2 + 4^2}{6^2}}\lambda = \frac{\sqrt{225 + 16}}{6}\lambda = \frac{\sqrt{241}}{6}\lambda$$

$$= |m_{W^\pm}| = 4^3 \times 60km_e \Rightarrow \lambda = 4^3 \times \frac{6 \times 60}{\sqrt{241}}km_e$$

$$\Rightarrow m_{W^\pm} = (m_1, m_2, m_3, m_0) = \left(\pm\frac{1}{3}, \pm\frac{5}{6}, \pm\frac{7}{3}, \pm\frac{2}{3}\right)(4^3)\left(\frac{360}{\sqrt{241}}\right)km_e$$

$$m_Z = (m_1, m_2, m_3, m_0) = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, 0\right)\lambda + \left(0, 0, 0, -\frac{1}{3}\right)\lambda$$

$$\Rightarrow |m_Z| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda\right)^2 + \left(\pm\frac{5}{6}\lambda\right)^2 + \left(\pm\frac{7}{3}\lambda\right)^2\right] + \left(\pm\frac{1}{3}\lambda\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda^2 + \left(\frac{1}{3}\lambda\right)^2} = \sqrt{\left(\frac{5}{2}\lambda\right)^2 + \left(\frac{1}{3}\lambda\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (1 \cdot 2)^2}{6}}\lambda = \frac{\sqrt{229}}{6}\lambda$$

$$= |m_Z| = \frac{5}{2}(12)^3 km_e \Rightarrow \lambda = \frac{30}{\sqrt{229}}(12)^3 km_e$$

$$\Rightarrow m_Z = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, -\frac{1}{3}\right)\left(\frac{30}{\sqrt{229}}\right)(12)^3 km_e$$

Note:

Because neutrinos have and interact color-wise and have zero charge, they have architecture like Z^0 particles:

$$\Rightarrow m_\nu = (m_1, m_2, m_3, m_0) = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, 0\right)\lambda_\nu + \left(0, 0, 0, -\frac{1}{3}\right)\lambda_\nu$$

$$\Rightarrow |m_\nu| = \sqrt{\left[\left(\pm\frac{1}{3}\lambda_\nu\right)^2 + \left(\pm\frac{5}{6}\lambda_\nu\right)^2 + \left(\pm\frac{7}{3}\lambda_\nu\right)^2\right] + \left(\pm\frac{1}{3}\lambda_\nu\right)^2}$$

$$= \sqrt{\left(\frac{4}{6^2} + \frac{25}{6^2} + \frac{196}{6^2}\right)\lambda_\nu^2 + \left(\frac{1}{3}\lambda_\nu\right)^2} = \sqrt{\left(\frac{5}{2}\lambda_\nu\right)^2 + \left(\frac{1}{3}\lambda_\nu\right)^2}$$

$$= \sqrt{\frac{(5 \cdot 3)^2 + (1 \cdot 2)^2}{6}}\lambda_\nu = \frac{\sqrt{229}}{6}\lambda_\nu$$

$$\Rightarrow \lambda_\nu = \frac{\sqrt{229}}{6}\lambda_\nu = |m_\nu| \Rightarrow \lambda_\nu = \frac{6}{\sqrt{229}}|m_\nu|$$

$$\Rightarrow m_\nu = \left(+\frac{1}{3}, +\frac{5}{6}, +\frac{7}{3}, -\frac{1}{3}\right)\left(\frac{6}{\sqrt{229}}\right)|m_\nu|$$

For heavy leptons (including electrons): $m_1 = m_2 = m_3 = 0$:

$$\Rightarrow \begin{cases} \begin{pmatrix} \mathbf{I}_2(\partial_0 f^1 - \partial_1 f^0) - \boldsymbol{\sigma}^1(-\partial_3 f^2 + \partial_2 f^3) + \boldsymbol{\sigma}^3 m_0 f^1 \\ \mathbf{I}_2(\partial_0 f^2 - \partial_2 f^0) - \boldsymbol{\sigma}^1(\partial_3 f^1 - \partial_1 f^3) + \boldsymbol{\sigma}^3 m_0 f^2 \\ \mathbf{I}_2(\partial_0 f^3 - \partial_3 f^0) - \boldsymbol{\sigma}^1(-\partial_2 f^1 + \partial_1 f^2) + \boldsymbol{\sigma}^3 m_0 f^3 \\ -\mathbf{I}_2(\partial_1 f^1 + \partial_2 f^2 + \partial_3 f^3 - \partial_0 f^0) - \boldsymbol{\sigma}^3 m_0 f^0 \end{pmatrix} = \mathbf{0} \\ \begin{pmatrix} \mathbf{I}_2\left(\partial_0 f^1_\Updownarrow - \partial_1 f^0_\Updownarrow\right) + \boldsymbol{\sigma}^1\left(-\partial_3 f^2_\Updownarrow + \partial_2 f^3_\Updownarrow\right) - \boldsymbol{\sigma}^3 m_0 f^1_\Updownarrow \\ \mathbf{I}_2\left(\partial_0 f^2_\Updownarrow - \partial_2 f^0_\Updownarrow\right) + \boldsymbol{\sigma}^1\left(\partial_3 f^1_\Updownarrow - \partial_1 f^3_\Updownarrow\right) - \boldsymbol{\sigma}^3 m_0 f^2_\Updownarrow \\ \mathbf{I}_2\left(\partial_0 f^3_\Updownarrow - \partial_3 f^0_\Updownarrow\right) + \boldsymbol{\sigma}^1\left(-\partial_2 f^1_\Updownarrow + \partial_1 f^2_\Updownarrow\right) - \boldsymbol{\sigma}^3 m_0 f^3_\Updownarrow \\ -\mathbf{I}_2\left(\partial_1 f^1_\Updownarrow + \partial_2 f^2_\Updownarrow + \partial_3 f^3_\Updownarrow + \partial_0 f^0_\Updownarrow\right) + \boldsymbol{\sigma}^3 m_0 f^0_\Updownarrow \end{pmatrix} = \mathbf{0} \end{cases} \tag{3}$$

$$
\Rightarrow \begin{cases} \begin{pmatrix} \mathbf{I}_2(\partial_0 f^1 - \partial_1 f^0) - \boldsymbol{\sigma}^1(-\partial_3 f^2 + \partial_2 f^3) + \boldsymbol{\sigma}^3 m_0 f^1 \\ \mathbf{I}_2(\partial_0 f^2 - \partial_2 f^0) - \boldsymbol{\sigma}^1(\partial_3 f^1 - \partial_1 f^3) + \boldsymbol{\sigma}^3 m_0 f^2 \\ \mathbf{I}_2(\partial_0 f^3 - \partial_3 f^0) - \boldsymbol{\sigma}^1(-\partial_2 f^1 + \partial_1 f^2) + \boldsymbol{\sigma}^3 m_0 f^3 \\ -\mathbf{I}_2(\partial_1 f^1 + \partial_2 f^2 + \partial_3 f^3 - \partial_0 f^0) - \boldsymbol{\sigma}^3 m_0 f^0 \end{pmatrix} = \mathbf{0} \\ \begin{pmatrix} \mathbf{I}_2\left(\partial_0 f^1_{\Updownarrow} - \partial_1 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(-\partial_3 f^2_{\Updownarrow} + \partial_2 f^3_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^1_{\Updownarrow} \\ \mathbf{I}_2\left(\partial_0 f^2_{\Updownarrow} - \partial_2 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(\partial_3 f^1_{\Updownarrow} - \partial_1 f^3_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^2_{\Updownarrow} \\ \mathbf{I}_2\left(\partial_0 f^3_{\Updownarrow} - \partial_3 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(-\partial_2 f^1_{\Updownarrow} + \partial_1 f^2_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^3_{\Updownarrow} \\ -\mathbf{I}_2\left(\partial_1 f^1_{\Updownarrow} + \partial_2 f^2_{\Updownarrow} + \partial_3 f^3_{\Updownarrow} + \partial_0 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^3 m_0 f^0_{\Updownarrow} \end{pmatrix} = \mathbf{0} \end{cases}
$$

(Note: $m_0 \leftrightarrow -m_0 \Leftrightarrow F^j \leftrightarrow f^j_{\Updownarrow}$ & $\boldsymbol{\sigma}^1 \leftrightarrow -\boldsymbol{\sigma}^1$)

(in other words, the equation sets are matter/anti-matter images)

Thus, this set of equations may be reduced to a single equation:

$$
\mathbf{I}_2\left(\partial_0 f^j_{s_{m_0}} - \partial_j f^0_{s_{m_0}}\right) - (s_{m_0})\boldsymbol{\sigma}^1\left(\nabla \times f_{s_{m_0}}\right)_j + \boldsymbol{\sigma}^3[(s_{m_0})|m_0|]f^j_{s_{m_0}} = \mathbf{0}
$$

where: $s_{m_0} = \begin{cases} +, m_0 \geq 0 \\ -, m_0 < 0 \end{cases}$ & $f^j_{s_{m_0}} = \begin{cases} f^j, m_0 \geq 0 \\ f^j_{\Updownarrow}, m_0 < 0 \end{cases}$

and: $\begin{cases} (A \times B)_j = \begin{cases} -A^3B^2 + A^2B^3, \ j = 1 \\ -A^3B^1 + A^1B^3, \ j = 2 \\ -A^2B^1 + A^1B^2, \ j = 3 \end{cases} \\ (\nabla \times V)_j = \begin{cases} -\partial_3 V^2 + \partial_2 V^3 = 1 \\ \partial_3 V^1 - \partial_1 V^3 = 2 \\ -\partial_2 V^1 + \partial_1 V^2 = 3 \end{cases} \end{cases}$, (or equivalent notation)

$$
\Rightarrow \begin{cases} \begin{pmatrix} \mathbf{I}_2(\partial_0 f^1 - \partial_1 f^0) - \boldsymbol{\sigma}^1(-\partial_3 f^2 + \partial_2 f^3) + \boldsymbol{\sigma}^3 m_0 f^1 \\ \mathbf{I}_2(\partial_0 f^2 - \partial_2 f^0) - \boldsymbol{\sigma}^1(\partial_3 f^1 - \partial_1 f^3) + \boldsymbol{\sigma}^3 m_0 f^2 \\ \mathbf{I}_2(\partial_0 f^3 - \partial_3 f^0) - \boldsymbol{\sigma}^1(-\partial_2 f^1 + \partial_1 f^2) + \boldsymbol{\sigma}^3 m_0 f^3 \\ -\mathbf{I}_2(\partial_1 f^1 + \partial_2 f^2 + \partial_3 f^3 - \partial_0 f^0) - \boldsymbol{\sigma}^3 m_0 f^0 \end{pmatrix} = \mathbf{0} \\ \begin{pmatrix} \mathbf{I}_2\left(\partial_0 f^1_{\Updownarrow} - \partial_1 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(-\partial_3 f^2_{\Updownarrow} + \partial_2 f^3_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^1_{\Updownarrow} \\ \mathbf{I}_2\left(\partial_0 f^2_{\Updownarrow} - \partial_2 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(\partial_3 f^1_{\Updownarrow} - \partial_1 f^3_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^2_{\Updownarrow} \\ \mathbf{I}_2\left(\partial_0 f^3_{\Updownarrow} - \partial_3 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^1\left(-\partial_2 f^1_{\Updownarrow} + \partial_1 f^2_{\Updownarrow}\right) - \boldsymbol{\sigma}^3 m_0 f^3_{\Updownarrow} \\ -\mathbf{I}_2\left(\partial_1 f^1_{\Updownarrow} + \partial_2 f^2_{\Updownarrow} + \partial_3 f^3_{\Updownarrow} + \partial_0 f^0_{\Updownarrow}\right) + \boldsymbol{\sigma}^3 m_0 f^0_{\Updownarrow} \end{pmatrix} = \mathbf{0} \end{cases}
$$

(Note: $m_0 \leftrightarrow -m_0 \Leftrightarrow f^j \leftrightarrow f^j_{\Updownarrow}$ & $\boldsymbol{\sigma}^1 \leftrightarrow -\boldsymbol{\sigma}^1$)

(in other words, the equation sets are matter/anti-matter images)

Thus, this set of equations may be reduced to a single equation:

$$
\mathbf{I}_2\left(\partial_0 f^j_{s_{m_0}} - \partial_j f^0_{s_{m_0}}\right) - (s_{m_0})\boldsymbol{\sigma}^1\left(\nabla \times f_{s_{m_0}}\right)_j + \boldsymbol{\sigma}^3[(s_{m_0})|m_0|]f^j_{s_{m_0}} = \mathbf{0}
$$

where: $s_{m_0} = \begin{cases} +, m_0 \geq 0 \\ -, m_0 < 0 \end{cases}$ & $f^j_{s_{m_0}} = \begin{cases} f^j, m_0 \geq 0 \\ f^j_{\Updownarrow}, m_0 < 0 \end{cases}$

and: $(\nabla \times V)_j = \begin{cases} -\partial_3 V^2 + \partial_2 V^3 = 1 \\ \partial_3 V^1 - \partial_1 V^3 = 2 \\ -\partial_2 V^1 + \partial_1^2 V^2 = 3 \end{cases}$, (or equivalent notation)

Also:

$$
\begin{cases}
D_B\mathbf{F} = \begin{pmatrix}
\begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}F^1 + \begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}F^2 - \begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}F^3 - \begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix}F^0 \\
-\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}F^1 + \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}F^2 + \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}F^3 - \begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix}F^0 \\
\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}F^1 - \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}F^2 + \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}F^3 - \begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix}F^0 \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix}F^1 - \begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix}F^2 - \begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix}F^3 + \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}F^0
\end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} = \begin{pmatrix}
\begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}f^1 - \begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}f^2 + \begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}f^3 - \begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix}f^0 \\
\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}f^1 + \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}f^2 - \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}f^3 - \begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix}f^0 \\
-\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}f^1 + \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}f^2 + \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}f^3 - \begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix}f^0 \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix}f^1 - \begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix}f^2 - \begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix}f^3 + \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}f^0
\end{pmatrix} = \mathbf{F}
\end{cases}
$$

$$
\begin{cases}
D_B\mathbf{F} = \begin{pmatrix}
\begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}\begin{pmatrix} F_+^1 \\ F_-^1 \end{pmatrix} & \begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^2 \\ F_-^2 \end{pmatrix} & -\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^3 \\ F_-^3 \end{pmatrix} & -\begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix}\begin{pmatrix} F_+^0 \\ F_-^0 \end{pmatrix} \\
-\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^1 \\ F_-^1 \end{pmatrix} & \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}\begin{pmatrix} F_+^2 \\ F_-^2 \end{pmatrix} & \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^3 \\ F_-^3 \end{pmatrix} & -\begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix}\begin{pmatrix} F_+^0 \\ F_-^0 \end{pmatrix} \\
\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^1 \\ F_-^1 \end{pmatrix} & -\begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}\begin{pmatrix} F_+^2 \\ F_-^2 \end{pmatrix} & \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}\begin{pmatrix} F_+^3 \\ F_-^3 \end{pmatrix} & -\begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix}\begin{pmatrix} F_+^0 \\ F_-^0 \end{pmatrix} \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix}\begin{pmatrix} F_+^1 \\ F_-^1 \end{pmatrix} & -\begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix}\begin{pmatrix} F_+^2 \\ F_-^2 \end{pmatrix} & -\begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix}\begin{pmatrix} F_+^3 \\ F_-^3 \end{pmatrix} & \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}\begin{pmatrix} F_+^0 \\ F_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} = \begin{pmatrix}
\begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} - \begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} + \begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} - \begin{pmatrix} D_1^+ & 0 \\ 0 & D_1^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \\
\begin{pmatrix} 0 & D_3^- \\ D_3^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} + \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} - \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} - \begin{pmatrix} D_2^+ & 0 \\ 0 & D_2^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \\
-\begin{pmatrix} 0 & D_2^- \\ D_2^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} + \begin{pmatrix} 0 & D_1^- \\ D_1^+ & 0 \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} + \begin{pmatrix} D_0^- & 0 \\ 0 & D_0^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} - \begin{pmatrix} D_3^+ & 0 \\ 0 & D_3^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix} \\
-\begin{pmatrix} D_1^- & 0 \\ 0 & D_1^+ \end{pmatrix}\begin{pmatrix} f_+^1 \\ f_-^1 \end{pmatrix} - \begin{pmatrix} D_2^- & 0 \\ 0 & D_2^+ \end{pmatrix}\begin{pmatrix} f_+^2 \\ f_-^2 \end{pmatrix} - \begin{pmatrix} D_3^- & 0 \\ 0 & D_3^+ \end{pmatrix}\begin{pmatrix} f_+^3 \\ f_-^3 \end{pmatrix} + \begin{pmatrix} D_0^+ & 0 \\ 0 & D_0^- \end{pmatrix}\begin{pmatrix} f_+^0 \\ f_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{F}
\end{cases}
$$

$$
\left\{
\begin{aligned}
D_B\mathbf{F} &= \begin{pmatrix}
\begin{pmatrix} D_0^+F_+^1 + D_3^-F_-^2 - D_2^-F_-^3 - D_1^+F_+^0 \\ D_0^-F_-^1 + D_3^+F_+^2 - D_2^+F_+^3 - D_1^-F_-^0 \end{pmatrix} \\
-\begin{pmatrix} -D_3^-F_-^1 + D_0^+F_+^2 + D_1^-F_-^3 - D_2^+F_+^0 \\ -D_3^+F_+^1 + D_0^-F_-^2 + D_1^+F_+^3 - D_2^-F_-^0 \end{pmatrix} \\
\begin{pmatrix} D_2^-F_-^1 - D_1^-F_-^2 + D_0^+F_+^3 - D_3^+F_+^0 \\ D_2^+F_+^1 - D_1^+F_+^2 + D_0^-F_-^3 - D_3^-F_-^0 \end{pmatrix} \\
-\begin{pmatrix} -D_1^-F_+^1 - D_2^-F_+^2 - D_3^-F_+^3 + D_0^-F_+^0 \\ -D_1^+F_-^1 - D_2^+F_-^2 - D_3^+F_-^3 + D_0^+F_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} &= \begin{pmatrix}
\begin{pmatrix} D_0^-f_+^1 - D_3^-f_-^2 + D_2^-f_-^3 - D_1^+f_+^0 \\ D_0^+f_-^1 - D_3^+f_+^2 + D_2^+f_+^3 - D_1^-f_-^0 \end{pmatrix} \\
\begin{pmatrix} D_3^-f_-^1 + D_0^-f_+^2 - D_1^-f_-^3 - D_2^+f_+^0 \\ D_3^+f_+^1 + D_0^+f_-^2 - D_1^+f_+^3 - D_2^-f_-^0 \end{pmatrix} \\
-\begin{pmatrix} -D_2^-f_-^1 + D_1^-f_-^2 + D_0^-f_+^3 - D_3^+f_+^0 \\ -D_2^+f_+^1 + D_1^+f_+^2 + D_0^+f_-^3 - D_3^-f_-^0 \end{pmatrix} \\
\begin{pmatrix} -D_1^-f_+^1 - D_2^-f_+^2 - D_3^-f_+^3 + D_0^+f_+^0 \\ -D_1^+f_-^1 - D_2^+f_-^2 - D_3^+f_-^3 + D_0^-f_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{F}
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
D_B\mathbf{F} &= \begin{pmatrix}
\begin{pmatrix} (\partial_0 + m_0)F_+^1 + (\partial_3 - m_3)F_-^2 - (\partial_2 - m_2)F_-^3 - (\partial_1 + m_1)F_+^0 \\ (\partial_0 - m_0)F_-^1 + (\partial_3 + m_3)F_+^2 - (\partial_2 + m_2)F_+^3 - (\partial_1 - m_1)F_-^0 \end{pmatrix} \\
\begin{pmatrix} -(\partial_3 - m_3)F_-^1 + (\partial_0 + m_0)F_+^2 + (\partial_1 - m_1)F_-^3 - (\partial_2 + m_2)F_+^0 \\ -(\partial_3 + m_3)F_+^1 + (\partial_0 - m_0)F_-^2 + (\partial_1 + m_1)F_+^3 - (\partial_2 - m_2)F_-^0 \end{pmatrix} \\
\begin{pmatrix} (\partial_2 - m_2)F_-^1 - (\partial_1 - m_1)F_-^2 + (\partial_0 + m_0)F_+^3 - (\partial_3 + m_3)F_+^0 \\ (\partial_2 + m_2)F_+^1 - (\partial_1 + m_1)F_+^2 + (\partial_0 - m_0)F_-^3 - (\partial_3 - m_3)F_-^0 \end{pmatrix} \\
\begin{pmatrix} -(\partial_1 - m_1)F_+^1 - (\partial_2 - m_2)F_+^2 - (\partial_3 - m_3)F_+^3 + (\partial_0 - m_0)F_+^0 \\ -(\partial_1 + m_1)F_-^1 - (\partial_2 + m_2)F_-^2 - (\partial_3 + m_3)F_-^3 + (\partial_0 + m_0)F_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{0} \\
D_A\mathbf{f} &= \begin{pmatrix}
\begin{pmatrix} (\partial_0 - m_0)f_+^1 - (\partial_3 - m_3)f_-^2 + (\partial_2 - m_2)f_-^3 - (\partial_1 + m_1)f_+^0 \\ (\partial_0 + m_0)f_-^1 - (\partial_3 + m_3)f_+^2 + (\partial_2 + m_2)f_+^3 - (\partial_1 - m_1)f_-^0 \end{pmatrix} \\
\begin{pmatrix} (\partial_3 - m_3)f_-^1 + (\partial_0 - m_0)f_+^2 - (\partial_1 - m_1)f_-^3 - (\partial_2 + m_2)f_+^0 \\ (\partial_3 + m_3)f_+^1 + (\partial_0 + m_0)f_-^2 - (\partial_1 + m_1)f_+^3 - (\partial_2 - m_2)f_-^0 \end{pmatrix} \\
-\begin{pmatrix} -(\partial_2 - m_2)f_-^1 + (\partial_1 - m_1)f_-^2 + (\partial_0 - m_0)f_+^3 - (\partial_3 + m_3)f_+^0 \\ -(\partial_2 + m_2)f_+^1 + (\partial_1 + m_1)f_+^2 + (\partial_0 + m_0)f_-^3 - (\partial_3 - m_3)f_-^0 \end{pmatrix} \\
\begin{pmatrix} -(\partial_1 - m_1)f_+^1 - (\partial_2 - m_2)f_+^2 - (\partial_3 - m_3)f_+^3 + (\partial_0 + m_0)f_+^0 \\ -(\partial_1 + m_1)f_-^1 - (\partial_2 + m_2)f_-^2 - (\partial_3 + m_3)f_-^3 + (\partial_0 - m_0)f_-^0 \end{pmatrix}
\end{pmatrix} = \mathbf{F}
\end{aligned}
\right.
$$

$$\left\{ \begin{array}{l} D_B\mathbf{F} = \begin{pmatrix} \begin{pmatrix} (\partial_0 F_+^1 - \partial_1 F_+^0) + (\partial_3 F_-^2 - \partial_2 F_-^3) + (m_0 F_+^1 - m_1 F_+^0) - (m_3 F_-^2 - m_2 F_-^3) \\ (\partial_0 F_-^1 - \partial_1 F_-^0) + (\partial_3 F_+^2 - \partial_2 F_+^3) - (m_0 F_-^1 - m_1 F_-^0) + (m_3 F_+^2 - m_2 F_+^3) \end{pmatrix} \\ \begin{pmatrix} (\partial_0 F_+^2 - \partial_2 F_+^0) - (\partial_3 F_-^1 - \partial_1 F_-^3) + (m_0 F_+^2 - m_2 F_+^0) + (m_3 F_-^1 - m_1 F_-^3) \\ (\partial_0 F_-^2 - \partial_2 F_-^0) - (\partial_3 F_+^1 - \partial_1 F_+^3) - (m_0 F_-^2 - m_2 F_-^0) - (m_3 F_+^1 - m_1 F_+^3) \end{pmatrix} \\ \begin{pmatrix} (\partial_0 F_+^3 - \partial_3 F_+^0) - (\partial_1 F_-^2 - \partial_2 F_-^1) + (m_0 F_+^3 - m_3 F_+^0) + (m_1 F_-^2 - m_2 F_-^1) \\ (\partial_0 F_-^3 - \partial_3 F_-^0) - (\partial_1 F_+^2 - \partial_2 F_+^1) - (m_0 F_-^3 + m_3 F_-^0) - (m_1 F_+^2 - m_2 F_+^1) \end{pmatrix} \\ \begin{pmatrix} -(\partial_1 F_+^1 + \partial_2 F_+^2 + \partial_3 F_+^3 - \partial_0 F_+^0) + (m_1 F_+^1 + m_2 F_+^2 + m_3 F_+^3 - m_0 F_+^0) \\ -(\partial_1 F_-^1 + \partial_2 F_-^2 + \partial_3 F_-^3 - \partial_0 F_-^0) - (m_1 F_-^1 + m_2 F_-^2 + m_3 F_-^3 - m_0 F_-^0) \end{pmatrix} \end{pmatrix} = \mathbf{0} \\ D_A\mathbf{f} = \begin{pmatrix} \begin{pmatrix} (\partial_0 f_+^1 - \partial_1 f_+^0) - (\partial_3 f_-^2 - \partial_2 f_-^3) - (m_0 f_+^1 + m_1 f_+^0) + (m_3 f_-^2 - m_2 f_-^3) \\ (\partial_0 f_-^1 - \partial_1 f_-^0) - (\partial_3 f_+^2 - \partial_2 f_+^3) + (m_0 f_-^1 + m_1 f_-^0) - (m_3 f_+^2 - m_2 f_+^3) \end{pmatrix} \\ \begin{pmatrix} (\partial_0 f_+^2 - \partial_2 f_+^0) + (\partial_3 f_-^1 - \partial_1 f_-^3) - (m_0 f_+^2 + m_2 f_+^0) - (m_3 f_-^1 - m_1 f_-^3) \\ (\partial_0 f_-^2 - \partial_2 f_-^0) + (\partial_3 f_+^1 - \partial_1 f_+^3) + (m_0 f_-^2 + m_2 f_-^0) + (m_3 f_+^1 - m_1 f_+^3) \end{pmatrix} \\ -\begin{pmatrix} (\partial_0 f_+^3 - \partial_3 f_+^0) + (\partial_1 f_-^2 - \partial_2 f_-^1) - (m_0 f_+^3 + m_3 f_+^0) + (m_2 f_-^1 - m_1 f_-^2) \\ -(\partial_2 + m_2) f_+^1 + (\partial_1 + m_1) f_+^2 + (\partial_0 + m_0) f_-^3 - (\partial_3 - m_3) f_-^0 \end{pmatrix} \\ \begin{pmatrix} -(\partial_1 - m_1) f_+^1 - (\partial_2 - m_2) f_+^2 - (\partial_3 - m_3) f_+^3 + (\partial_0 + m_0) f_+^0 \\ -(\partial_1 + m_1) f_-^1 - (\partial_2 + m_2) f_-^2 - (\partial_3 + m_3) f_-^3 + (\partial_0 - m_0) f_-^0 \end{pmatrix} \end{pmatrix} = \mathbf{F} \end{array} \right.$$

The clear and obvious differences between the Dirac factoring and Helmholtzian factorization are:

1) The Dirac factoring may contain complex numbers in the terms, while the Helmholtzian factorization terms are real numbers (although Pauli matrices contain imaginaries if used (as above))
2) The Helmholtzian refers directly to the $\begin{pmatrix} f^1 \\ f^2 \\ f^3 \\ f^0 \end{pmatrix}$ space 4-vector components
3) The Helmholtzian refers to the $\begin{pmatrix} f_+^j \\ f_-^j \end{pmatrix}$ matter/anti-matter nature directly
4) The Helmholtzian refers to matter/anti-matter simply by change of sign of m_0
5) In the Helmholtzian, matter/anti-matter magnetic 3-space-components are changed left-handed/right-handed (ie.: magnetic moment reversal) (as the use of the Pauli matrices representation points out)
6) The Helmholtzian includes an equation of continuity.

Thus, as expressed above, through the breakthroughs; especially the Helmholtzian factorization, pythagorean quadruple primitives and the fundamental dimensionless physical constant k ; a concrete design is established through which practical engineering constructions may be produced.

Through mathematics we are able to see deeper than measurements alone can, due to Heisenberg uncertainty principle constraints.

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